AUTOMOBILE AND AIRCRAFT ENGINES

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PREFACE TO FOURTH EDITION

THE present volume is based upon the section of the author's book, *Automobile and Aircraft Engines*, entitled "Mechanics of The High Speed Internal Combustion Engines." The original combined volume, published over thirty years ago and subsequently appearing in successive revised editions, had become rather unwieldy in size, so the decision was taken to issue it in two separate volumes, of which this is the first. In this edition the original matter has been revised and extended considerably, much of the earlier material being either deleted or rewritten.

Additional sections included relate to the subjects of engine vibrations, engine mountings, torsional vibrations, balancing of rotating members, modern balancing machines and valve cams and followers.

It is hoped that the information and data given will prove useful to internal combustion engine students, designers, engineers and others concerned with the mechanical design aspects of petrol and Diesel engines, whilst to those desiring to study the subjects dealt with more fully, the various footnote and other references given throughout the book will be found helpful.

The author takes this opportunity of acknowledging the help and advice offered by Mr. R. G. Manley in the preparation of the present edition and for his contribution of the chapter entitled "Torsional Oscillations in Engines." Acknowledgment is also gratefully accorded to certain firms in connexion with the provision of information and illustrations of their products; in particular to Messrs. Edward G. Herbert, Ltd., W. and T. Avery, Ltd., Metalastik, Ltd., André Rubber Co., Ltd., Benrath Machine Tools, Ltd., Westinghouse Electric International Company (U.S.A.), and Tinius Olsen Testing Machine Company (U.S.A.). In concluding these prefatory remarks, the author would mention that any suggestions for the improvement or extension of the material of this book, in its next edition, will be welcomed.

A. W. JUDGE.

FARNHAM, SURREY.
1946.

CONTENTS

AUTOMOBILE AND AIRCRAFT ENGINES

CHAPTER I

PISTON DISPLACEMENT, VELOCITY, AND ACCELERATION

General Considerations

It is proposed in this opening chapter to consider the conversion into useful mechanical work of the heat energy of a mixture of air and fuel, burnt in the combustion chamber of a petrol or high-speed Diesel engine.

In the past there have been many suggestions for mechanical devices to obtain continuous rotary motion from the combustion and expansion of the explosive charge, but the only methods that have survived are the reciprocating engine, swash-plate and cam engines, and the gas turbine. It is with the former type of engine, with its trunk piston, connecting rod, and crank method of converting reciprocating into circular motion, that this book is concerned, since by far the greatest proportion of all internal combustion engines operate on this principle.

It is, however, realized that the reciprocating engine, with its numerous sliding and rubbing surfaces, its difficulties of engine-balance and vibration problems, is by no means ideal and may eventually be displaced to a greater or less extent by the efficient gas turbine where continuous rotary motion, uniform torque, and accurate balance are readily attainable.

Although convenient and comparatively simple in its conception, the application of the piston-connecting-rod-crank mechanism to high-speed engines involves a certain amount of advanced mathematics concerning the position, velocity and acceleration of the piston when a uniform crankshaft speed is the object. If it were possible to use very long connecting rods for relatively short cranks the problems would be simplified to those associated with a piston having a simple harmonic motion, the formulae for which are comparatively easy to apply. With the enforced use of relatively short connecting rods, or small connecting rod-to-crank ratios, the motion of the piston is more complex, and it is necessary to analyse this motion in order to obtain formulae that can be employed in the actual design of the piston, connecting rod and crankshaft; and also in connexion with connecting rod and main crankshaft bearings design.

The method adopted in the present chapter is first to obtain expressions for the piston displacement, from which the corresponding velocity can be derived by mathematics or graphical methods. The piston's acceleration can then be deduced and the results applied to the determination of the inertia of the reciprocating parts and the resultant forces due to inertia and cylinder gas pressures. From these resultant forces the value of the torque on the crankshaft at any position can be determined. The results can then be applied to the design of the moving parts and the engine bearings.

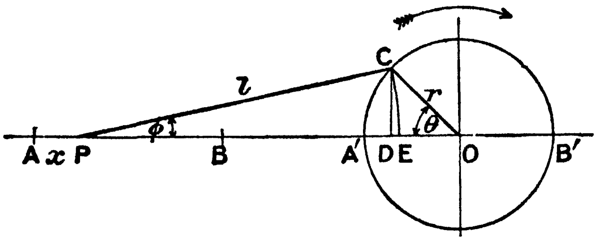


FIG. 1

Piston Position and Crank Angle

It is of importance to the designer to know the exact piston position corresponding to any given crank angle.

This can be obtained either mathematically or graphically.

Dealing with the former method first, let P represent the position of the piston (which replaces the crosshead and piston rod of steam-engine practice) along its stroke AB ($= 2r$), Fig. 1.

Let r = radius of crank pin circle

l = connecting rod length

θ = angle of crank with line of stroke

and x = displacement AP of piston.

We then have $$x = \text{AP} = \text{A}'\text{E}$$

where E is the point in which an arc of radius PC $= l$, from P as centre, cuts the line of stroke.

And
$$\begin{aligned} \text{A}'\text{E} &= \text{A}'\text{D} + \text{DE} \\ &= (\text{A}'\text{O} - \text{DO}) + \text{DE} \\ &= (r - r\cos\theta) + l - \sqrt{(l^2 - r^2\sin^2\theta)} \\ &= r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}\right\} \end{aligned}$$

If $n =$ the ratio $\dfrac{\text{connecting rod}}{\text{crank}} = \dfrac{l}{r}$

then $x = r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \dfrac{\sin^2\theta}{n^2}}\right\}$

This may be written—

$$x = r(1 - \cos\theta) + l\left\{1 - \left(1 - \tfrac{1}{2}\frac{\sin^2\theta}{n^2}\right)\right\} \text{ approximately}$$

$$= r(1 - \cos\theta) + \frac{r}{4n}(1 - \cos 2\theta)$$

The above expression is very nearly true, since the other terms of the series on the right-hand side become very small.

If the motion of the piston were a simple harmonic one (S.H.M.), that is, if the connecting rod were of infinite length, then the piston displacement would be given by

$$x = r(1 - \cos\theta)$$

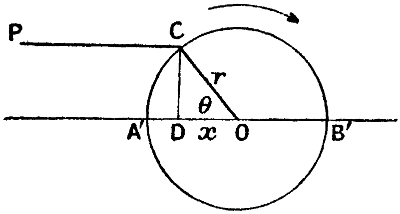


FIG. 2. SIMPLE HARMONIC MOTION

so that the effect of a finite length of connecting rod is to introduce the second term, and the resulting piston displacement may be regarded as due to an initial S.H.M. due to the revolution of a point of radius r and a superimposed S.H.M. due to the revolution of a point in a circle of radius $\dfrac{r}{4n}$, and with an angular velocity double that of the original one.

Further, the "error" introduced in the piston's position, due to the obliquity of the connecting rod, is represented by the term

$$y = \frac{r}{4n}(1 - \cos 2\theta)$$

This is a maximum when $\dfrac{dy}{d\theta} = O$, that is, when $\theta = 90°$ or $270°$, and its value then becomes $\dfrac{2r}{4n}$, that is $\dfrac{\text{the piston stroke}}{4n}$

It is a minimum when $\theta = 0°$ or $180°$, and is therefore zero at the two ends of the piston stroke.

When $\theta = 45°$, the value of y is $\dfrac{r}{4n}$.

More Accurate Expression for Piston Position

The expression for the piston displacement x previously given, namely,

$$x = r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{\sin^2\theta}{n^2}}\right\}$$

can be written as

$$x = l + r - r\cos\theta - l\left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}$$

Now the right-hand expression in the brackets can be expanded by the aid of the binomial theorem, thus—

$$\left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}} = 1 - \frac{\sin^2\theta}{2n^2} - \frac{\sin^4\theta}{8n^4} - \frac{\sin^6\theta}{16n^6} - \text{(and so on)}$$

Thus the more accurate relation for the piston displacement may be written as follows—

$$x = r(1 - \cos\theta) + \frac{l\sin^2\theta}{2n^2} + \frac{l\sin^4\theta}{8n^4} + \frac{l\sin^6\theta}{16n^6} + \text{(and so on)}$$

The error due to the approximation $x = r(1 - \cos\theta) + \frac{l\sin^2\theta}{2n^2}$, which is the same as $x = r(1 - \cos\theta) + \frac{r}{4n}(1 - \cos 2\theta)$, is therefore represented by the terms

$$\frac{l\sin^4\theta}{8n^4} + \frac{l\sin^6\theta}{16n^6} + \dots$$

In actual applications these terms are extremely small. Thus, if $n = 5$ the value of $\frac{l}{8n^4}$ is $\frac{l}{5000}$, and for $\frac{l}{16n^6}$ it is equal to $\frac{l}{250,000}$, so that for all practical purposes these higher order terms may be neglected.

Piston Displacement for Offset Cylinder

It is important in dealing with questions of balance, etc., to be able to determine the piston position at any crank angle. In several examples of petrol engines the line of stroke of the piston is displaced at right angles to its normal position by a small amount, for certain reasons which will be dealt with later.

In the diagram, Fig. 3, let b = the amount of offset and, for convenience, the distance PA $= x$ the piston position, and the distance OP $= a$.

With the notation given in the diagram (Fig. 3), and considering the triangle POC, we have—

$$l^2 = a^2 + r^2 - 2ar\cos\theta \quad . \quad (1)$$

Also $$\theta = \beta - \alpha \quad . \quad . \quad (2)$$

and $$\cos\alpha = \frac{x}{a}, \text{ and } \sin\alpha = \frac{b}{a} \quad . \quad (3)$$

$$x^2 + b^2 = a^2 \quad . \quad . \quad (4)$$

Further from (2)—

$$\cos\theta = \cos(\beta - \alpha) = \cos\beta \,.\, \cos\alpha + \sin\beta \,.\, \sin\alpha \quad . \quad . \quad (5)$$

Substituting for θ and a in Equation (1), the values given in (3), (4), and (5), it becomes—

$$x^2 - (2r\cos\beta)\,.\,x + (b^2 + r^2 - 2br\sin\beta - l^2) = 0$$

which is a quadratic equation in x.

The solution of this equation by the usual algebraic rule is—

$$x = r\cos\beta \pm \sqrt{r^2\cos^2\beta + l^2 - b^2 - r^2 + 2br\sin\beta}$$

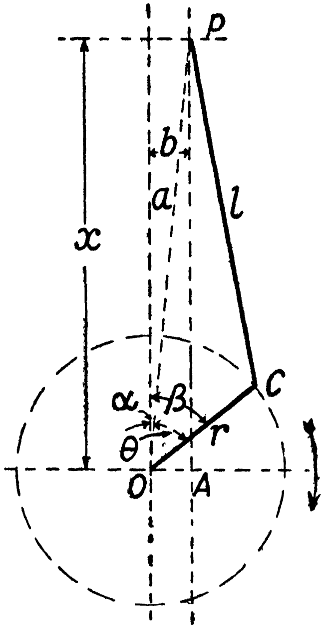


FIG. 3

It will be evident from the figure that the negative root is inadmissible, and so, in terms of the cosine only, we get, since $\sin^2\beta = 1 - \cos^2\beta$,

$$x = r\cos\beta + \sqrt{l^2 - b^2 - r^2 + r^2\cos^2\beta + 2br(\sqrt{1 - \cos^2\beta})}$$

The second expression under the root sign can be expressed as a series for

$$(1 - \cos^2\beta)^{\frac{1}{2}} = 1 - \frac{\cos^2\beta}{2} - \frac{\cos^4\beta}{8} - \frac{\cos^6\beta}{16} - \text{etc.}$$

and we have

$$x = r\cos\beta + \sqrt{(l^2 - b^2 + 2br - r^2) + (r^2 - br)\cos^2\beta - \frac{br}{4}\cos^4\beta + \text{etc.}}$$

This expression can be further simplified by neglecting higher powers of $\cos\beta$, and expanding out as a series the expression under the surd sign, and we have, after substituting $\mathrm{A} = l^2 - b^2 + 2br - r^2$ and $\mathrm{B} = r(b - r)$,

$$x = r\cos\beta + \mathrm{A}^{\frac{1}{2}} + \frac{\mathrm{A}^{\frac{5}{2}}}{2\mathrm{B}^2}\cos^2\beta + \frac{\mathrm{A}^{\frac{9}{2}}}{2\mathrm{B}^4}\cos^4\beta + \text{etc.}$$

This expression represents the position of the piston for any given crank angle β very approximately, and can be employed for differentiation purposes in connexion with the velocity and acceleration.

Stroke of Offset Piston

The principal object of offsetting is to diminish the obliquity of the connecting rod during the firing stroke and hence reduce the mean piston thrust on the cylinder walls. The motion of the piston is also slower at the commencement of the firing stroke, which is advantageous from the combustion point of view. In practice, the amount of offset varies from $\frac{1}{5}$ to $\frac{1}{6}$ of the cylinder bore.

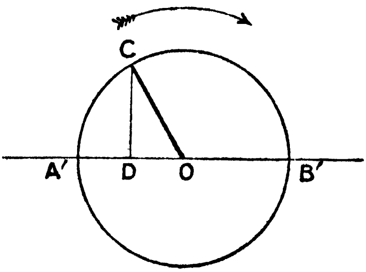


FIG. 4

The effect of offsetting the cylinder axis by a small amount can be readily understood if two diagrams of the normal and offset types be drawn, and the piston position found, in the usual manner, for different crank angles.

It will be found that for the same crank angle position, the offset piston is behind the other one during the downstrokes, that is, the firing and suction strokes; whilst during the upstrokes, namely, the compression and exhaust ones, the offset piston is ahead of the normal one.

Further, the stroke of the offset piston is rather greater than that of the normal type, although the total distance between the piston at the top of its stroke and the crank centre is actually a little less, so that the offset cylinder can be made slightly shorter.

Fig. 218 on page 284 illustrates the corresponding piston positions for the normal engine (as shown by the dotted lines) and the offset engine (as shown by the full line); the above-mentioned points will be readily followed from this diagram.

It can be shown, in a simple manner, that the length of stroke of the offset piston l' is given by the following expression—

$$l' = 2r\left\{1 + \frac{b^2}{2(l^2 - r^2)}\right\} \text{ very nearly}$$

where l = connecting-rod length, r = crank radius, and b = the amount of offset.

For a connecting-rod crank ratio of 4, the ordinary offset piston stroke is about 0·15 per cent greater.

Graphical Method for Piston Displacement

If the connecting rod be regarded as of infinite length, it is a simple matter to obtain the position of the piston corresponding to any crank angle.

Thus if the crank be in the position OC (see Fig. 4), and a perpendicular be dropped from C to the line of stroke at D, then A′D represents the distance of piston from the outer end of its stroke.

For $$A'D = r(1 - \cos\theta) \text{ where } \theta = \text{Angle } A'OC$$

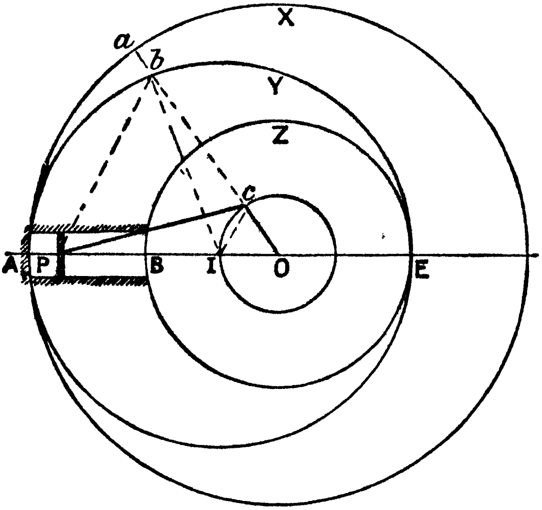


FIG. 5. MULLER'S CIRCLES FOR PISTON DISPLACEMENT

From this it is evident that the piston displacement plotted against crank angle will give a sine curve.

Next, consider the connecting rod of finite length PC (Fig. 1).

The position P of the piston for any crank angle θ is obtained by sweeping an arc of radius CP $= l$ from C as centre, the corresponding piston position P being where this arc cuts the line of stroke.

If it is required to find the piston position corresponding with each position of the crank, this operation is repeated for each crank position.

Muller's Circles

A somewhat more convenient method of obtaining, in an easy manner, the piston displacement for any crank position, due to Müller, is as follows—

Let ABE be the line of stroke (Fig. 5) and Oc the crank radius, cP being the connecting rod.

With centre O and radius $OI(=r)$ *describe the circle* IcI

" " O " $OA(=l+r)$ " " X

" " O " $OB(=l-r)$ " " Z

Now with radius l and centre I describe the circle Y.

The piston displacement for any crank angle IOc is given by the intercept ab on the crank arm, between the circles X and Y.

In order to prove this, join bI, bP and Ic respectively. Then in the triangles Icb and IcP we have the sides bI and cP both equal to l, and cI is common to both, and

$$\angle bcI = \angle PIc$$

So that the triangles are equal in all respects

and $\quad cb = PI$

Also $\quad ca = IA = l$

Hence $\quad ab = AP$ the piston displacement.

Curves of Piston Displacement

It is often convenient, in the case of any given engine, to be able to find quickly the position of the piston for a given crank angle, and vice versa.

The polar curve is very suitable for this purpose.

The principle of the use of this curve is that the intercept of the crank arm on the curve represents the distance of the piston from its mid-position.

The curves are constructed graphically by first finding the point E, corresponding to the piston's position, and then intercepting the distance OE upon the crank arm, thus obtaining a point R.

The locus of R for all crank angles gives the polar curve.

An example of a polar displacement curve is shown in Fig. 6, and it will be evident that the intercept of the crank OR represents the corresponding piston position from the mid-point of its stroke, or CR its distance from the extreme point of its stroke.

For an infinite connecting rod this polar curve becomes a pair of circles of diameter equal to the crank radius, and with their centres on the line of stroke, and the greater the obliquity of the connecting rod, the more distorted do these initially circular curves become.

Another method of representing conveniently the piston displacement for different crank positions is to plot upon a crank angle base the distances of the piston from the centre of its stroke as ordinates.

Angular Velocity Relationships

Referring to Fig. 4, if the crank OC rotates around the centre O and if the angle A′OC = θ

Then angular velocity of OC $= \dfrac{d\theta}{dt}$

If w = angular velocity of OC in radians per second, r = radius in

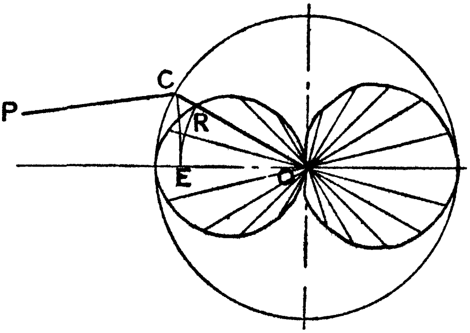


FIG. 6. POLAR CURVES OF PISTON DISPLACEMENT

feet, N = revolutions per minute of OC and v = linear velocity of C in feet per second.

Then $$w = \frac{2\pi N}{60} = 0{\cdot}10472N \text{ radians per second}$$

$$v = wr = 0{\cdot}10472N \text{ feet per second}$$

Also $$N = \frac{60w}{2\pi} = 9{\cdot}5493w$$

Piston Velocity

It is required to obtain an expression for the velocity of the piston at any moment in terms of the crank angle, stroke, and connecting-rod length.

Dealing with the case where the connecting rod is very long, so that the motion of the piston follows a simple harmonic law, we have, with the notation in the diagram, Fig. 2—

The piston displacement from the centre $x = r \cos \theta$.

The velocity $$V_p = \frac{dx}{dt} = -r \sin \theta \frac{d\theta^*}{dt}$$

* As the direction of rotation is of no significance, the negative sign can be ignored.

But $\frac{d\theta}{dt}$ is the rate of change of the angle θ, that is, its angular velocity, and the linear velocity of the crank pin C $= r\frac{d\theta}{dt} = V_c$.

Hence $$V_p = V_c \sin\theta$$

So that the velocity of the point P varies according to the law of sines, and is a maximum when $\theta = 90°$, it is then equal to the velocity of the crank pin, and the velocity is zero at the two extreme ends of its path of travel.

Effect of Connecting-rod Obliquity

Consider next, the case for a finite length of connecting rod l.

Referring again to Fig. 1, we have already obtained an expression for the position of the piston in terms of the crank angle, and in order to obtain the velocity it is necessary to differentiate this expression in regard to the time.

Thus we have given $x = r(1 - \cos\theta) + l\left\{1 - \frac{1}{l}\sqrt{l^2 - r^2\sin^2\theta}\right\}$

Then $V_p = \frac{dx}{dt} = r\sin\theta\frac{d\theta}{dt} + \frac{r^2 \quad 2\sin\theta \quad \cos\theta}{2\sqrt{l^2 - r^2\sin^2\theta}} \cdot \frac{d\theta}{dt}$

Writing $V_c = r\frac{d\theta}{dt}$ and simplifying, we have for the velocity of the piston—

$$V_p = V_c\sin\theta\left\{1 + \frac{r\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right\}$$

It will be seen that the effect of the obliquity of the connecting rod is to introduce the second term

$$\frac{r}{2}V_c\frac{\sin 2\theta}{\sqrt{l^2 - r^2\sin^2\theta}}$$

The velocity is a maximum at a position which can be determined from the above expression by substituting actual numerical values, or by differentiating the expression.

Thus by differentiating the expression for V_p we obtain

$$\frac{dV_p}{dt} = \frac{V^2_c}{r}\left\{\cos\theta + \frac{rl^2\cos 2\theta + r^3\sin^4\theta}{(l^2 - r^2\sin^2\theta)^{\frac{3}{2}}}\right\}$$

and by equating to zero, and simplifying, the expression for the values of the crank angle at which the velocity is a maximum is obtained, and we then have

$$\sin^6\theta - \frac{l^2}{r^2}\sin^4\theta - \frac{l^4}{r^4}\sin^2\theta + \frac{l^4}{r^4} = 0$$

The solution of this cubic equation gives the values of the crank angle for maximum velocity.

As an example of the application of this expression to practical purposes, the following values for connecting rod to crank ratios occurring in practice have been calculated.

TABLE I

Ratio $\frac{l}{r}$	Crank Angle (in degrees) from top dead centre at which the velocity of the piston is a maximum
3	73·18
4	76·71
5	79·11
6	80·60

From these results it will be evident that the greater the obliquity of the connecting rod, the earlier in the stroke is the velocity of the piston a maximum; and the longer the connecting rod in relation to the crank, the nearer to its mid-position does the maximum velocity of the piston occur—that is, the motion of the piston approaches a simple harmonic one.

In a similar manner the velocity at any point of the stroke of the piston having an offset or "desaxé" cylinder can be obtained.

It is only necessary to differentiate the expression for the piston displacement in order to arrive at an expression for the piston's velocity at any crank angle.

Approximate Method for Piston Velocity

It has been shown that the piston displacement may be expressed approximately by the following relation—

$$x = r(1 - \cos\theta) + \frac{r}{4n}(1 - \cos 2\theta)$$

Since θ is the angle of rotation of the crank in time t from its outer dead-centre position, then $\theta = wt$ where $w =$ angular velocity of the crankshaft.

Thus, $x = r(1 - \cos wt) + \dfrac{r}{4n}(1 - \cos 2wt)$.

The velocity of the piston is given by

$$V_p = \frac{dx}{dt} = wr \sin wt + \frac{wr}{2n}\sin 2wt$$

$$= wr \sin\theta + \frac{wr}{2n}\sin 2\theta$$

The velocity is a maximum when $\frac{dV_p}{dt} = 0$.

Thus, max. V_p is given by the angle θ in the following equation, which is obtained by differentiating the velocity expression and equating to zero—

$$n \cos \theta + \cos 2\theta = 0$$

Substituting $\cos 2\theta = 2 \cos^2 \theta - 1$, we get

$$2 \cos^2 \theta + n \cos \theta - 1 = 0$$

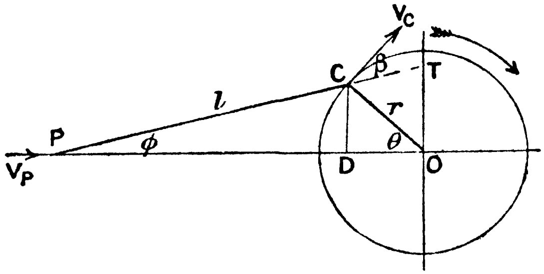


FIG. 7

From which the crank angle θ for maximum piston velocity for any ratio of connecting rod to crank n can be computed.

Graphical Method for Piston Velocity

With the notation employed in Fig. 7, let V_p represent the piston's velocity at any point P, and V_c the uniform crank pin linear velocity.

It will be evident that at the crank pin C its direction of motion will be tangential to the crank pin circle.

Further, the resolute of the piston's velocity along the connecting rod must be equal to the resolute of the crank pin's velocity along this rod, since it is a rigid mass.

We thus have $V_p \cos \phi = V_c \cos \beta$

or
$$\frac{V_p}{V_c} = \frac{\cos \beta}{\cos \phi} = \frac{\sin OCT}{\sin OTC} = \frac{OT}{OC} = \frac{\sin (\phi + \theta)}{\cos \phi}$$

Hence the intercept OT of the connecting rod upon the vertical diameter of the crank pin circle at once gives a measure of the piston velocity for this position of the crank (the radius of the crank pin circle being proportional to the uniform crank pin velocity).

It will be evident from this that the piston velocity is a maximum when the angle OCT is 90° on either side of the line of stroke, and is equal in value to $\frac{V_c}{\sin OTC}$ or $\frac{V_c}{\cos \phi}$.

This will be seen to occur before the crank is perpendicular to the line of stroke, as was shown previously mathematically.

Velocity Curves

If the method of intercepts be employed to obtain the piston velocity, relative to the crank pin, and if, further, the intercept OT be marked off upon the crank arm, for all positions of the crank, then the locus

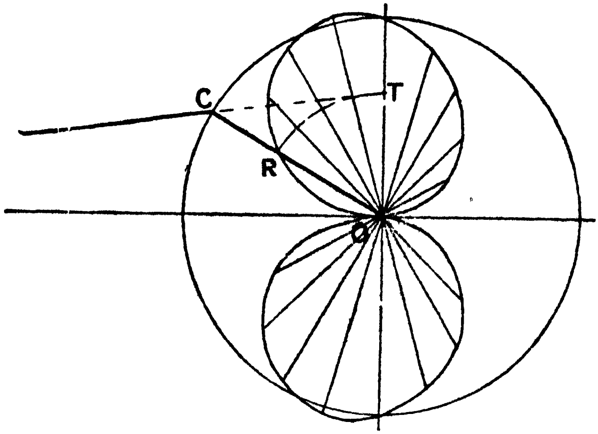


FIG. 8. POLAR CURVES OF PISTON VELOCITY

of all such points on the crank arm will give a polar curve of piston velocity.

The method of construction is indicated and an example of the curves for a connecting-rod crank ratio of 4 given in Fig. 8.

To obtain the velocity of the piston for any crank angle from these curves, it is only necessary to measure the length of the crank pin intercept OR, then, if OC represents the crank pin velocity to scale, OR will to a similar scale represent the piston's velocity.

If the connecting rod is very long, the two polar curves will approach true circles having diameters equal to the crank pin radius, and their centres upon a diameter of the crank pin circle perpendicular to the line of stroke.

A general method of construction of the velocity curve upon a piston stroke base, due to Unwin, is given in Fig. 9; the polar velocity curve has been described from this, and is also given in the same figure.

In this figure, Od represents the crank length r, and dc the connecting rod l.

Produce Od to f, making df equal to the crank pin velocity V_c. Draw fg parallel to the connecting rod to cut the perpendicular from c in g.

Then cg represents the piston velocity when the piston is at c.

For all positions of the crank ordinates such as cg can be determined, and a curve of piston velocity upon the piston stroke base obtained.

When the connecting rod is very long, this curve approaches an ellipse.

Angular Velocity of the Connecting Rod

It is required to obtain an expression for the angular motion of the connecting rod about the gudgeon pin, as this at once gives a measure of the rubbing velocity at the bearing surfaces of the latter.

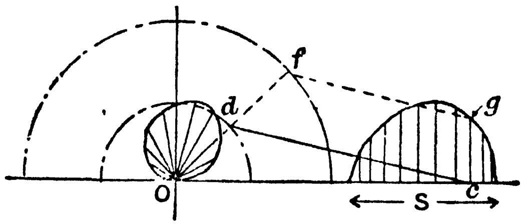


FIG. 9. CURVES OF PISTON VELOCITY

Using the diagram of Fig. 7, we have

$$l \sin \phi = r \sin \theta = \text{CD}$$

or

$$l \cos \phi . \, d\phi = r \cos \theta \, . \, d\theta$$

So that

$$\frac{d\phi}{d\theta} = \frac{r \cos \theta}{l \cos \phi} = \frac{\text{DO}}{\text{DP}} = \frac{\text{CT}}{\text{CP}}$$

Hence the ratio of the connecting rod to crank pin angular velocities is given by the segments into which the crank pin C divides the line PT.

It follows from this that the angular velocity of the connecting rod will be zero when CT $= 0$, that is, when the crank is perpendicular to the line of centres.

The angular motion of the rod will be a maximum when CT $= r$, that is, when the crank is on the dead centres, and its value will then be

$$W_R = W_C \cdot \frac{r}{l} = \frac{W_C}{n}$$

where W_R and W_C represent the angular velocities of the connecting rod and crank respectively.

Rubbing Velocity of Gudgeon and Crank Pins

The maximum rubbing velocity of the small end upon the gudgeon pin is then given by

$$V_G = W_R \, . \, a = a \, . \, \frac{W_C}{n}$$

where V_G is the rubbing velocity, a the radius of the gudgeon pin, and W_C the angular velocity of the crank.

Expressed in a more convenient form, in terms of the revolutions per minute N of the crankpin, we have $V_G = \frac{a}{n} . 2\pi N$.

Similarly, the crank pin rubbing velocity at any moment is V_N = angular velocity of OCP $\times b$, where b = radius of crank pin.

We thus have: Angular velocity of OC relative to PC given by

$$\frac{d(\phi + \theta)}{d\theta} = \frac{PC + CT}{CT} = \frac{PT}{CT} \quad \text{(Fig. 7)}$$

Then rubbing velocity of crank pin $= b . \frac{PT}{CT} W_C$.

This is a maximum when $CT = r$, that is, when the crank is in the dead centre position.

Expressed in terms of the revolutions per minute, the maximum rubbing velocity of crank pin $= b . \frac{l + r}{r} . 2\pi N$.

If the lengths be expressed in feet, the rubbing velocity will be in feet per minute.

Mean Piston Velocity

If the crank pin velocity be V_C and the crank pin radius r, then the crank pin moves through a distance $2\pi r$, whilst the piston moves through $4r$ (twice the stroke).

Hence the mean piston velocity $V_m = \frac{4r}{2\pi r} . V_C = \frac{2}{\pi} V_C$.

On the polar curves this mean velocity can be represented by a circle of radius $\frac{2}{\pi} V_C$ drawn with O as centre, and in the case of linear curves by a straight line of height $\frac{2}{\pi} V_C$.

A more convenient expression for the mean piston velocity, readily deduced from the above relation, is as follows—

$$\text{Mean piston velocity} = 4rN \text{ ft. per min.}$$

where N = r.p.m. of crankshaft and r is in feet. This may be written as

$$\text{Mean piston velocity} = 2 \text{ (stroke) } N$$

It follows from this that for *a limiting rubbing speed* of the piston *the r.p.m. of any engine will vary inversely as the piston stroke,* so that engines with shorter strokes will operate at higher r.p.m. than those with longer strokes.

The *present average piston speeds* at r.p.m. corresponding to full

output vary from 2000 to 2800 ft. per min., so that if an average value of 2400 ft. per min. is taken, it follows that an engine of 6 in. stroke will run at a maximum of 2400 r.p.m. and one of 3 in. stroke at 4800 r.p.m.

Piston Acceleration

For *approximate purposes* the piston acceleration can be obtained from the piston displacement relation, previously given,* namely

$$x = r(1 - \cos wt) + \frac{r}{4n}(1 - \cos 2wt)$$

By differentiating twice we obtain

$$\frac{d^2x}{dt^2} = \frac{w^2r}{n}\cos 2wt + w^2r \cos wt$$

or, since $wt = \theta$, this can be written as follows—

$$\text{Piston acceleration } \frac{d^2x}{dt^2} = \frac{w^2r}{n}\cos 2\theta + w^2r\cos\theta$$

The maximum value of the acceleration occurs when $\theta = 0°$, and is given by

$$\text{Maximum acceleration} = \frac{w^2r}{n} + w^2r \text{ or } w^2r\left(1 + \frac{1}{n}\right)$$

When $\theta = 180°$, the value of the acceleration at the other, or inner, end of the piston stroke is as follows

$$\text{Acceleration} = -w^2r\left(1 - \frac{1}{n}\right)$$

The negative value here denotes a deceleration or retardation.

The zero value of the acceleration, corresponding to the maximum piston velocity, occurs at an angle θ, obtained from the expression†

$$2\cos^2\theta + n\cos\theta - 1 = 0$$

For example, in the case of an engine of 4 in. stroke with connecting rod 8 in. long and speed of 3000 r.p.m., $n = 4$ and maximum acceleration is given by

$$\frac{w^2 . 2}{12}(1 + \tfrac{1}{4}) = \frac{5}{24}w^2$$

$$\text{The angular velocity } w = \frac{3000 \times 2\pi}{60} = 314{\cdot}16 \text{ radians per sec.}$$

Thence, max. acceleration $= \frac{5}{24} \times (314{\cdot}16)^2 = 20{,}550$ ft. per sec. per sec.

* *Vide* page 3. † For solution of this, see page 18.

For *more exact purposes* the complete expression for the piston displacement must be employed. Thus—

$$x = r\,(1 - \cos\theta) + l\left\{1 - \frac{\sqrt{l^2 - r^2\sin^2\theta}}{l^2}\right\}$$

Then piston velocity

$$\frac{dx}{dt} = \left\{r\sin\theta + \frac{r^2}{2}\cdot\frac{2\sin\theta\,.\,\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right\}\frac{d\theta}{dt}$$

Since $\frac{d\theta}{dt} = w$, the angular velocity, this may be written as

$$\frac{dx}{dt} = wr\sin\theta\left(1 + \frac{r\cos\theta}{\sqrt{l^2 - r^2\sin^2\theta}}\right)$$

It may be noted that wr is the linear velocity of the crank pin, namely, the term V_c previously employed in this chapter.

The acceleration is

$$\frac{d^2x}{dt^2} = w^2r\left\{\cos\theta + \frac{rl^2\cos 2\theta + r^3\sin^4\theta}{(l^2 - r^2\sin^2\theta)^{\frac{3}{2}}}\right\}$$

If this expression be compared with the approximation given on page 16, it will be observed that the harmonic term $\frac{w^2r}{n}\cos 2\theta$ is replaced by the right-hand expression in the brackets of the above formula, but multiplied by n.

The effect of using the approximate formula* is to *cause a small error* which increases rapidly as the ratio of connecting rod to crank diminishes. This error is zero for crank angles θ of 0°, 180°, 360°, etc., and is also zero at angles of about 60°, 120°, 240°, and 300°.

The greatest error occurs when $\theta = 90°$ and 270°. The usual magnitude of the greatest error, for values of $n = 3{\cdot}5$ to $4{\cdot}5$, is of the order of less than 1 per cent, namely, 0·7 to 0·3 per cent.

Accelerations at Ends of Stroke

When the crank angle $\theta = 0°$, the value of the expression given for the acceleration of the piston, obtained by substituting $\cos 0° = 1$ and $\sin 0° = 0$, is as follows—

$$\text{Max. acceleration (top dead centre)} = w^2r\left(1 + \frac{r}{l}\right)$$

$$= \left(\frac{2\pi N}{60}\right)^2 . r . \left(1 + \frac{r}{l}\right) f.s.s.$$

where N = r.p.m. and r and l are in feet.

* "Piston Acceleration. Graphic Representation of the Error Involved in the Usual Formulae." A. W. Newman, *Automobile Engineer*, January, 1944.

Similarly, the acceleration at bottom dead centre is given by

$$\text{Max. acceleration (bottom dead centre)} = -\left(\frac{2\pi N}{60}\right)^2 . r . \left(1 - \frac{r}{l}\right) f.s.s.$$

The effect of connecting rod obliquity is to increase the acceleration at the top dead centre over that of a simple harmonic motion by the fraction $\frac{r}{l}$ or $\frac{1}{n}$, and to reduce it at the bottom dead centre by a like amount. Further,* the smaller the value of $\frac{1}{n}$, i.e. the longer the connecting rod for a given crank radius, the more nearly does the piston's motion approximate to a simple harmonic one.

Acceleration when Crank is at 90°

The value of the acceleration for the piston position corresponding to the crank being at right angles to the cylinder axis produced is obtained by substituting $\theta = 90°$ in the general expression for the piston acceleration. Thus, substituting $\sin 90° = 1$ and $\cos 90° = 0$, we get

$$\text{Acceleration} = w^2 r \left\{0 + \frac{0 + r^3}{(l^2 - r^2)^{\frac{3}{2}}}\right\}$$

$$= \frac{w^2 r^4}{(l^2 - r^2)^{\frac{3}{2}}}$$

This expression can be reduced to a function of $\frac{l}{r}$, thus showing that the acceleration at $\theta = 90°$ depends only upon the angular speed and ratio of connecting rod to crank.

Position of Crank for Zero Acceleration

When the velocity is a maximum, the acceleration is zero, and, as shown previously, this occurs when the values of θ are given by the following relation—

$$\sin^6 \theta - \frac{l^2}{r^2} \sin^4 \theta - \frac{l^4}{r^4} \sin^2 \theta + \frac{l^4}{r^4} = 0$$

This expression is obtained by equating that for the piston acceleration to zero. If this is done for various connecting-rod-to-crank ratios, n, from 3 : 1 to 5 : 1 the results given in Fig. 10* will be obtained. As would be expected, the larger the value of n the nearer to a right angle does θ become; at n = infinity, corresponding to a simple harmonic motion, $\theta = 90°$.

* *Vide* footnote on page 17.

Graphical Method for Piston Acceleration

If the velocity of the piston be known at different positions of the stroke, the corresponding values for the acceleration can be deduced, since acceleration is the time rate of change of velocity.

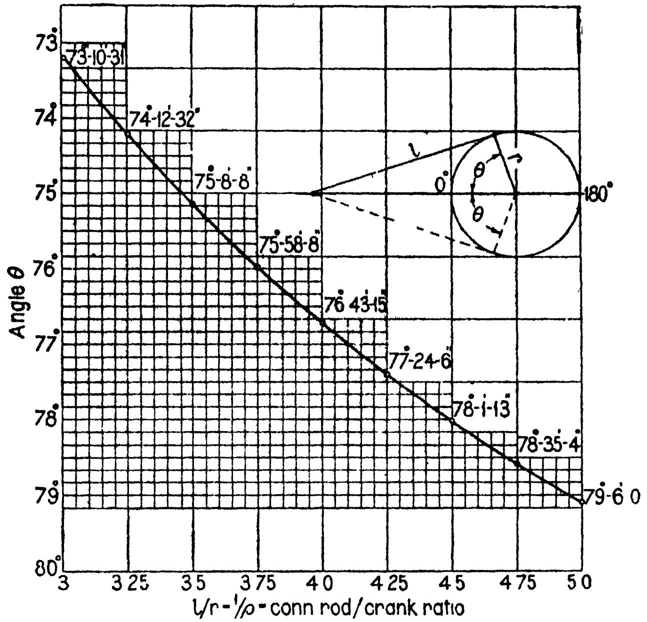


FIG. 10. CRANK POSITIONS FOR ZERO ACCELERATION

In the case of the polar curve of piston velocity, consider a number of radial vectors drawn at equal time intervals, that is, imagine the crank pin circle to be divided up into a number of equal parts (since the crank pin velocity is uniform).

Then consider three consecutive radius vectors drawn in this manner. Let OA, OB, and OC, Fig. 11, represent the velocities for these positions of the crank (that is the intercepts upon the polar velocity curve).

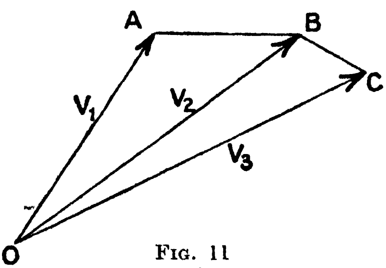


FIG. 11

If now V_1, V_2, and V_3 represent the magnitudes of these respective velocities, we have—

$$\left.\begin{array}{l}\text{Average acceleration between}\\ \text{crank positions OA and OB}\end{array}\right\} = \frac{V_2 - V_1}{\delta t} \qquad \delta t = \text{the time interval}$$

$$\text{and between OB and OC} = \frac{V_3 - V_2}{\delta t}$$

Now in the limit when the time interval is exceedingly small the points A, B, and C will be very close together. The acceleration at the point B will be represented by the slope of the tangent to the curve at that point.

Similarly, in the case of the linear velocity curve upon a crank angle base, the ordinates at equal intervals will represent equal time intervals, and the difference in height of consecutive ordinates will represent the average acceleration over the intermediate time interval.

Thus, referring to Fig. 12, if the time intervals be δt, then the average acceleration between A and A′ will be represented by $\left(\frac{y_1 - y}{\delta t}\right)$, and by erecting an ordinate midway between A and A_1, proportional to $y_1 - y$, a curve of acceleration upon a crank angle base is obtained. Alternatively, if a series of tangents are drawn to the velocity curve at the points B, B′, B″, etc., the values of the tangents of the angles made with the base line AA″ will be proportional to the accelerations at these points.

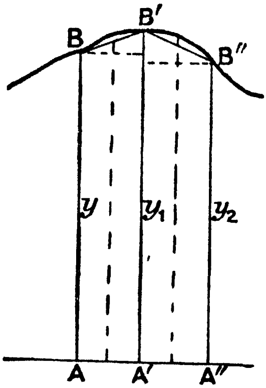


FIG. 12

If the base line represents piston displacement, it should be divided into equal time interval positions, the divisions, of course, being unequal in this case.

The above general method for obtaining the acceleration graphically can be applied to the velocity curves of any mechanism, but it is only approximate in practice, since it is difficult to draw a tangent at a point accurately, and the time intervals, necessarily, have to be finite.

The method usually adopted, in the case of the engine mechanism, is to plot the acceleration of the piston upon a piston-stroke base.

The values of the acceleration can be calculated at the two extreme ends of the stroke, and the position of no acceleration obtained by methods already considered.

By calculating values of the acceleration from the general expression for one or two other intermediate positions, sufficient points are obtained to enable the curve of acceleration to be drawn with fair accuracy.

An example of a piston acceleration curve is given in Fig. 13, and it will be observed that the acceleration is a maximum at the beginning of the downward stroke of the piston (that is, towards the crankshaft), and gradually diminishes to zero before half of the stroke is accomplished, after which a negative acceleration (or retardation) of increasing magnitude occurs, finally attaining a maximum value at the inner end of the stroke.

The dotted line shows the corresponding acceleration curve for an infinitely long connecting rod, i.e. simple harmonic motion.

The greater the obliquity of the connecting rod, the greater will

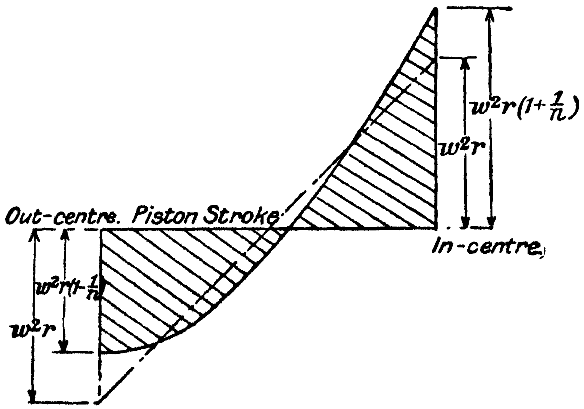


FIG. 13. CURVE OF PISTON ACCELERATION ON STROKE BASE

be the value of the initial maximum acceleration, and the difference between the maximum positive and negative accelerations.

It should be mentioned that it is the initial value of the piston

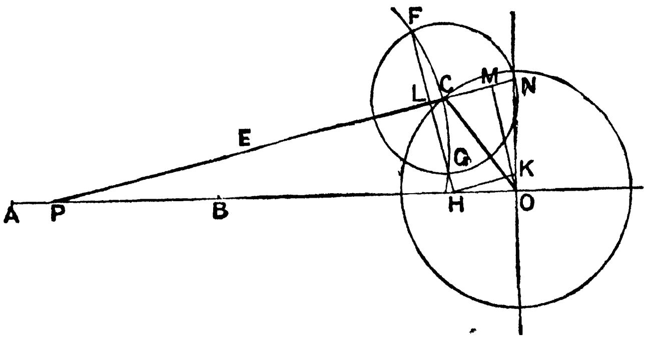


FIG. 14. KLEIN'S CONSTRUCTION FOR PISTON ACCELERATION

acceleration which determines the maximum magnitude of the inertia stresses due to the piston's weight, etc., to which the moving parts are subjected.

Klein's Piston Acceleration Method

A graphic method for finding the acceleration of the piston for any position is shown in Fig. 14.

The connecting rod PC produced cuts the vertical diameter in N. With C as centre and CN as radius, a circle, is described.

Bisect the connecting rod in E and sweep out the arc FG to cut the smaller circle at these points, the radius of the arc being EC.

Join FG, and, if necessary, produce it to cut the line of stroke in H.

$$\left.\begin{array}{c}\text{Then the acceleration of the}\\ \text{piston at P}\end{array}\right\} = \text{HO} \times \left\{\begin{array}{c}\text{angular velocity}\\ \text{of crank}\end{array}\right\}^2$$

The proof of this method is arrived at by dropping a perpendicular OM on CN and drawing HK parallel to LM.

It has previously been shown that if V_c be the crank pin velocity, and W_R and W_C the angular velocities of the connecting rod and crank, respectively,

$$\text{Then } W_R = W_C \cdot \frac{\text{CN}}{\text{PC}}$$

The acceleration of C along CO is $W_C{}^2$CO, and the resolute of this acceleration along the connecting rod is $W_C{}^2$CM.

The acceleration of C along the connecting rod, due to the component of the connecting rod's angular acceleration, is $W_R{}^2$CP.

$$\text{Now } W_R{}^2\text{CP} = \frac{W_C{}^2 \,.\, \text{CN}^2}{\text{PC}} = \frac{W_C{}^2\text{CF}^2}{\text{PC}} = W_C{}^2\text{LC}$$

The total acceleration of the piston along the connecting rod is made up of the resolute of the crank's acceleration in its direction, and of its own acceleration in the direction of its length.

$$\begin{aligned}\text{Hence acceleration of piston along PC} &= W_C{}^2(\text{LC} + \text{CM})\\ &= W_C{}^2 \,.\, \text{LM} = W_C{}^2\text{HK}\end{aligned}$$

Resolving this acceleration along the line of stroke PO we finally obtain the acceleration of the piston in its direction of motion as $W_C{}^2\text{HK} \,.\, \dfrac{\text{HO}}{\text{HK}} = W_C{}^2\text{HO}$.

Inertia of Reciprocating Parts

It has already been shown that the piston experiences an acceleration during the earlier period of its inward movement, and a retardation towards the latter end of the stroke.

In order to accelerate the mass of the piston, a certain force is required.

Thus, if M be the mass of the piston in pounds and f the acceleration in feet per second per second, then the force F required to accelerate it is given by

$$\text{F} = \frac{\text{M}f}{g}$$, where g is the acceleration due to gravity (= 32·19 feet per sec.^{2})

Now it is possible to calculate f for any position of the piston, and therefore the corresponding value of F can be ascertained.

Evidently, since M and g are constant, F will vary directly as f, and the diagram of piston acceleration is also, to a suitable scale, a diagram of piston accelerating force F.

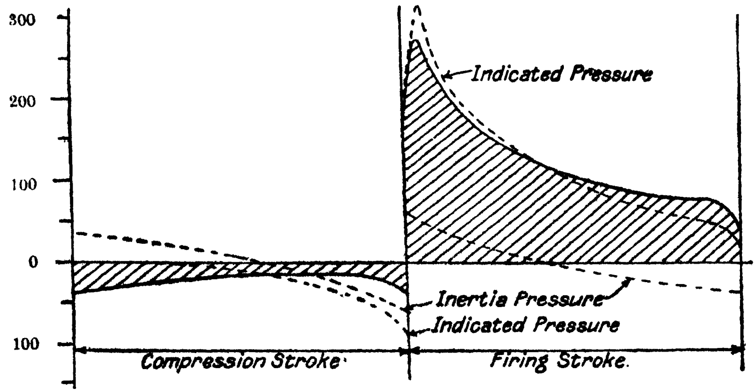


FIG. 15. PISTON PRESSURES CORRECTED FOR INERTIA [1000 REVS. PER MIN.]

Hence the inertia force at the beginning of the down-stroke (that is, for the in-centre crank position) is given by

$$F_i = \frac{M\omega^2 r}{g}\left(1 + \frac{1}{n}\right), \text{ where } n = \frac{l}{r}$$

and for the out-centre—

$$F_o = \frac{M\omega^2 r}{g}\left(1 - \frac{1}{n}\right)$$

This may be expressed in terms of the engine revolutions per minute by writing—

$$_iF_o = \frac{M}{g} \cdot \frac{4\pi^2 N^2}{3600} \cdot r\left(1 \pm \frac{1}{n}\right)$$

By reference to Fig. 13, which may now be regarded as a diagram of accelerating force, it will be seen that whereas the initial acceleration is equivalent to a reduction in the effective pressure upon the piston during the earlier part of its stroke, yet during the latter portion this pressure is enhanced by the retardation occurring.

This effect during the working (or firing) stroke of the piston can be more clearly understood by reference to the diagram of piston pressures upon a piston stroke base, as plotted from an indicator diagram, and shown in Fig. 15.

The shaded curve shows the resultant effective pressure per sq. in. upon the piston when corrected for the piston inertia effect.

Inertia Pressures

In order to obtain the real correction for the inertia effect, the accelerating force F at any point must be divided by the area of the piston.

Using the previous notation, if d = diameter of cylinder (ins.).

$$\text{Then piston area } A = \frac{\pi d^2}{4}$$

Denoting the piston stroke by $s = 2r$, then—

$$\text{Inertia force at top of stroke } F_1 = \frac{M}{g} \cdot \frac{4\pi^2 N^2}{3600} \cdot \frac{s}{2}\left(1 + \frac{1}{n}\right)$$

$$\text{Inertia pressure at top of stroke} = \frac{F_1}{A} = \frac{M}{g} \cdot \frac{\pi N^2}{450} \cdot \frac{s}{d^2}\left(1 + \frac{1}{n}\right)$$

$$= 0{\cdot}0002168 \frac{MN^2 s}{d^2}\left(1 + \frac{1}{n}\right)$$

$$\text{Inertia pressure at bottom of stroke} = 0{\cdot}0002168 \frac{MN^2 s}{d^2}\left(1 - \frac{1}{n}\right)$$

Note. Acceleration due to gravity $g = 32{\cdot}19$ ft. per sec.^{2}

Further, in the case of vertical engines the weight, W, of the piston itself has to be added to the force causing acceleration, during the downward motion, and subtracted during the upward motion.

This is equivalent to adding or subtracting a small constant pressure $\frac{4W}{\pi d^2}$ to the effective explosion pressure. Since, however, in most car and aircraft engines the dead weight of the piston is only a matter of a pound or two, and the area of the piston several square inches, the dead weight equivalent pressure will only be a fraction of a pound per square inch as compared with explosion and inertia presşures (at high speeds) of one or two hundred pounds per square inch.

It will be observed from Fig. 15, which illustrates an actual example, to scale, that one effect of piston inertia is to tend to render the effective explosion pressure more uniform during the firing stroke; also during the compression stroke the effect of the retardation of the piston towards the end of the stroke is to diminish the actual effort due to the energy stored in the flywheel upon the compressed gas.

This retardation force, called into play, actually diminishes the pressure between the small end of the connecting rod and the gudgeon pin.

In order to reduce these inertia pressures, for a given maximum speed, *the reciprocating parts* have, necessarily, to *be made as light as possible,* since at any given speed the inertia force due to the acceleration or retardation of the piston varies as the mass of the accelerated or retarded parts.

It must also be remembered that whatever energy is absorbed or used up in accelerating the reciprocating parts is not lost, since an equal amount of energy is given back during the ensuing retardation

This will be evident from the curve of piston inertia given, for the areas for the acceleration and retardation periods are equal.

Lightness of reciprocating parts is an invaluable aid in reducing the bearing pressures between the working surfaces, for with heavy pistons and rods these pressures may become excessive, and since friction is an irreversible process, the loss of power is continuous for both acceleration and retardation. At high speeds, with heavy parts, the lubrication tends to become ineffective, since the lubricant becomes squeezed out of the bearings.

Further, as will be shown later, that the degree of vibration due to unbalanced parts varies directly as the mass of the reciprocating parts, and is a source of inefficiency, wear and tear, and discomfort.

In high-speed engines the inertia pressures often become so great that they sometimes exceed the explosion pressures and cause a marked change in the distribution of resultant pressure and torque.

At speeds of 2500 r.p.m. and over in the case of car and motor-cycle engines, the inertia pressures may begin to equal or exceed the explosion pressures, and the actual wearing effect of the explosion pressures becomes insignificant in comparison with inertia wearing effects; in fact, practical experience in the case of high-speed engines shows that the side of the crank pin remote from the piston during the firing stroke (i.e. nearest to the crankshaft centre) wears the most.

Fig. 16 represents to scale the piston pressure corrected for inertia effects for a speed of 2500 r.p.m., the reciprocating mass being 3 lb. per cylinder as before.

It will be observed that the inertia pressures are very high, and that they completely alter the character and shape of indicator diagram (shown on a stroke base).

In the diagram, areas above the base line represent work done by the piston, whilst areas below this line represent work done upon the piston either to accelerate the reciprocating mass or to compress the gas.

It will be noticed that a considerable amount of inertia energy is given back towards the end of the compression stroke, greatly in excess of that needed for compression purposes.

This diagram may also be taken to represent the force on the gudgeon pin (to a suitable scale, that is, by multiplying by the piston area).

The ordinates of the shaded areas above the zero pressure line represent positive forces towards the crankshaft, whilst those below the line correspond to forces away from the crankshaft, along the cylinder axis in both instances.

Sudden changes from downward to upward pressures on the piston at the ends of the compression strokes indicate a reversal of load on

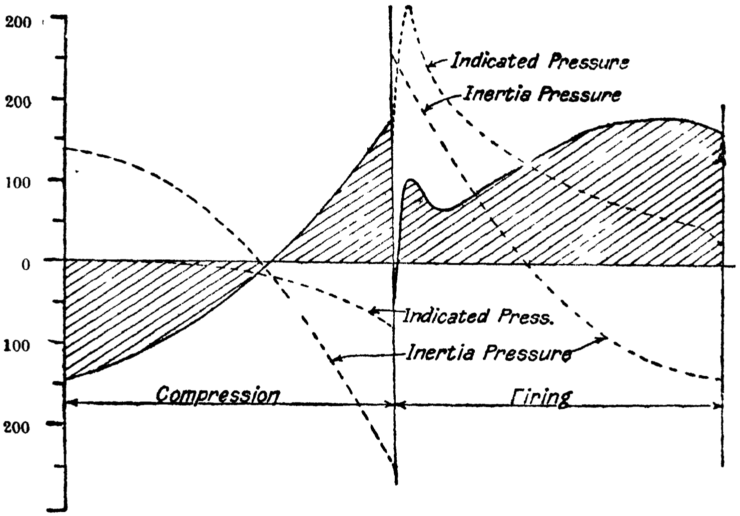


FIG. 16. PISTON PRESSURES CORRECTED FOR INERTIA [2500 REVS. PER MIN.]

the gudgeon pin bearing. If there is any appreciable clearance between the gudgeon pin and small end bearing this will give rise to a knock.

Inertia Pressure Data

The inertia pressures, expressed in lb. per sq. in. of piston area, it has been shown, will depend upon (*a*) the weight of the reciprocating parts; and (*b*) the acceleration and retardation forces which, in turn, depend upon the connecting-rod-to-crank ratio and the maximum crankshaft speed.

In the case of a six-cylinder 260 h.p. aircraft engine, the weight of the complete piston per sq. in. of piston area was 0·344 lb.; whilst the total reciprocating weight per cylinder was 0·424 lb.

At a speed of 1400 r.p.m., the following were the inertia pressure values in terms of piston area—

Top centre	106·70 lb. sq. in.
Bottom centre	60·50 ,,
Mean	83·60 ,,

In this case the inertia pressure, at its maximum value, was equal to 0·9 of the brake M.E.P.

By employing short length aluminium pistons, as in aircraft practice, it is possible to reduce the total reciprocating mass weight to about 0·18 to 0·22 lb. per sq. in. of piston area.

When aluminium pistons of the slipper type are employed, this value is reduced to about 0·17 to 0·18 lb. per sq. in. of piston area.

In the case of a high-speed petrol engine of 4-in. bore by 6 in. stroke, the compression ratio was 5, and the indicated M.E.P. 132 lb. per sq. in.

The maximum explosion pressure was about 420 lb. sq. in.

The weight of the reciprocating masses was 2·5 lb , that is, 0·20 lb. per sq. in. of piston area.

The average gas pressure over a complete four-stroke cycle (720°) at 3000 r.p.m. is approximately given by—

$$\text{Aver. press.} = \tfrac{1}{4}\{\text{Useful mean pressure} + \text{twice mean compression pressure} + \text{the fluid pumping pressure}\}$$

$$= \frac{132 + 52 + 6}{4} = 47\cdot 5 \text{ lb. sq. in.}$$

The average inertia pressure is given by—

$$F = \frac{0\cdot 00017 \times 0\cdot 2 \times (3000)^2 \times 0\cdot 5}{2}$$

$$= 76\cdot 5 \text{ lb. sq. in.}$$

Of this inertia pressure, about one-third is assumed to be balanced by fluid pressure, as viewed from the standpoint of piston-friction, so that the net effective inertia pressure is given by—

$$F_1 = \tfrac{2}{3} \times 76\cdot 5 = 51 \text{ lb. sq. in.}$$

It will be seen that the average values of the inertia and gas pressures are approximately equal, during parts of the cycle they act together, whilst at other intervals they almost cancel out.

Inertia Forces in Aircraft Engine

In order to illustrate the method of working out inertia forces, in practice, it is proposed to consider in some detail the case of the inertia forces and bearing pressures in the case of a twelve-cylinder Vee-engine of 400 h.p., with an included angle of 45°.

The following are the chief particulars of the engine which concern the present considerations—

Bore = 5 in.
Stroke = 7 in.
Stroke-bore ratio = 1·4 in.
Total piston-swept volume per cylinder = 137·445 cub. in.

Compression ratios: (*a*) Low $= 5{\cdot}0$; (*b*) High $= 5{\cdot}4$	
Normal b.h.p.: 400 at 1750 r.p.m.	
Approximate weight	= 806 lb.
Weight of aluminium piston and rings	= 3·8 lb.
,, upper end of connecting rod	= 1·225 lb.
,, gudgeon pin and retainers	= 1·038 lb.
,, reciprocating parts	= 6·063 lb.
Length of connecting rod	= 12 in.
Crankshaft diameter at main bearings	= 2·625 in.
,, ,, crank pin	= 2·375 in.
,, bearing, intermediate, length	= 1·75 in.
,, ,, front length	= 4·375 in.
,, ,, rear length	= 1·875 in.
Number of bearings	= 7

1. Indicator Diagram. The cylinder pressure diagram is given in Fig. 17; this diagram of pressures will be used in the following considerations.

2. Inertia Forces. The inertia forces seriously modify the forces due to the cylinder pressures, and therefore also modify the torque diagram.

In order to consider the resultant effect due to the pressure and inertia forces, the best method probably is to construct separate diagrams of each force on a piston-stroke, or crank-angle base, and then to combine, algebraically, the two sets of forces.

The inertia forces may be fairly accurately estimated from a formula which takes the fundamental and the first harmonic forces only into account, namely—

$$F = 0{\cdot}00034\ W\ .\ N^2 r\ .\ (\cos\theta + \frac{r}{l}\cos 2\theta)\ \text{lb.}$$

where W = weight of reciprocating parts (= 6·063 lb. in this case)

and N = 1700 r.p.m. for the normal speed at which the rated output is given.

The other notation is the same as that employed before.

It is now a simple matter to work out corresponding values of F for different crank angles θ, and to plot values of F on a crank-angle base for a total period of 720°.

The modified formula, obtained by substituting values for W, N, r, and l becomes—

$$F = 1738\,(\cos\theta + 0{\cdot}2917\cos 2\theta)\ \text{lb.}$$

Values of F have been calculated from this formula, and are shown plotted on a crank-angle base in Fig. 18, together with the pressure diagram obtained from the indicator diagram. The signs of the forces

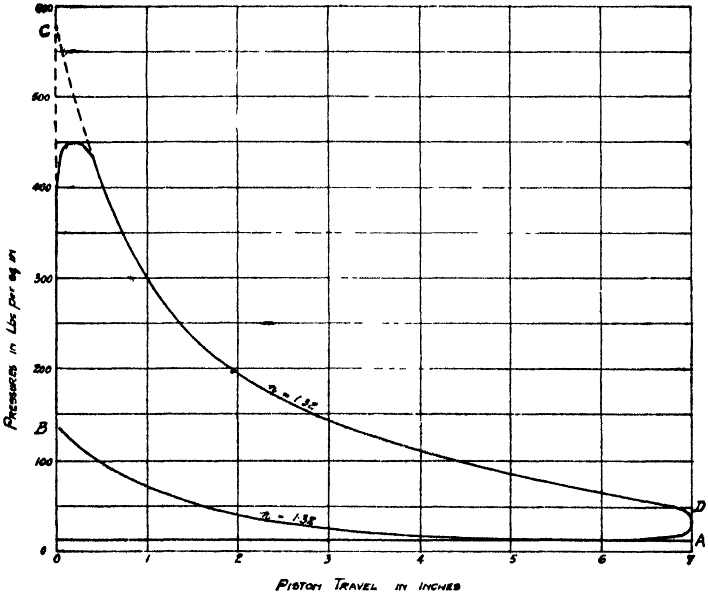


FIG. 17. ASSUMPTION INDICATOR DIAGRAM

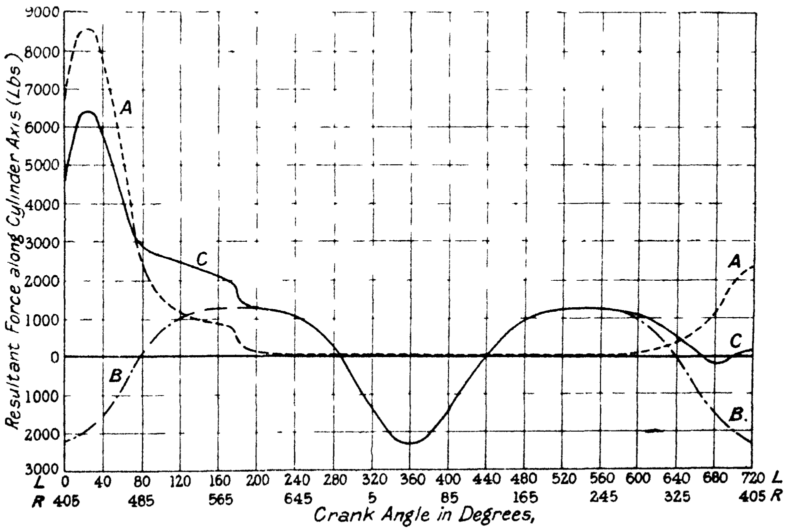


FIG. 18. RESULTANT FORCES ALONG CYLINDER AXIS OF 12-CYLINDER VEE-ENGINE

are so arranged here that downward-acting forces lie below the zero line, whilst upward-acting forces are above the zero line: it should be remembered also that these forces act in a direction along the cylinder axis in all cases, and refer to one cylinder only.

An examination of the diagrams in Fig. 18 will show that the result of algebraically adding the inertia forces of curve B to the pressure forces of curve A, gives the resultant axial force diagram of curve C.

During the first part of the piston stroke up to the point where the crank and connecting rod are at right angles, the piston-pressure forces are modified by the inertia forces; from this position to bottom dead centre the absorbed inertia effect is restored, less a certain amount lost in the frictional losses. After the crank passes the dead centre position, the inertia forces are considerably greater than those due to the pressure, and act against the turning direction of the crank until the crank and connecting rod are at right angles on the upward exhaust stroke; from this point to the top-dead centre, the inertia forces (or absorbed crank-turning moment) are given back as it were, and act in the direction of rotation as useful crank effort.

During the idle suction stroke (i.e. from 360° to 540°) the inertia forces entirely predominate, and in the following compression stroke they predominate as a resisting torque during the first period of 540° to about 630°, and thereafter the compression pressure forces are practically balanced by them.

The maximum value of the inertia forces during the initial acceleration periods is about equal to the mean pressure force on the piston, namely, 2400 lb. per sq. in., and is about one-half of the maximum resultant pressure value.

3. Estimation of Bearing Pressures. In order to estimate the bearing pressures on the crank pin, it is necessary to know the values at any angle of—

(*a*) The gas pressure forces along the cylinder axis.

(*b*) The inertia forces along the cylinder axis.

(*c*) The centrifugal forces along the crank radius.

The combined effect of (*a*) and (*b*), as given by the ordinates of curve C (Fig. 18), may be more conveniently employed for this purpose.

The centrifugal forces are due to the rotating part of the connecting-rod big end, are constant, and always act radially along the crank.

The centrifugal force is given by—

$$F_C = 0{\cdot}00034\ WN^2r$$

where W = weight of lower end of connecting rod (that is, 6·8 lb. for the engine rods of the present engine),

N = r.p.m. (1700 in the present example),

r = crank radius in feet.

In order to find the resultant force due to the combined influence of (*a*), (*b*), and (*c*) above, and to both of the inclined cylinders, since their connecting rods act on a common crank, it is necessary to combine each of these forces as shown in Fig. 19.

In this diagram, the centre lines of the pair of cylinders are marked L and R respectively, and are at 45° to each other. The main crank is shown at 60° past the top dead centre of the L.H. cylinder, that is, 15° past that of the R.H. cylinder; in this position the turning-moment for the former is nearly a maximum.

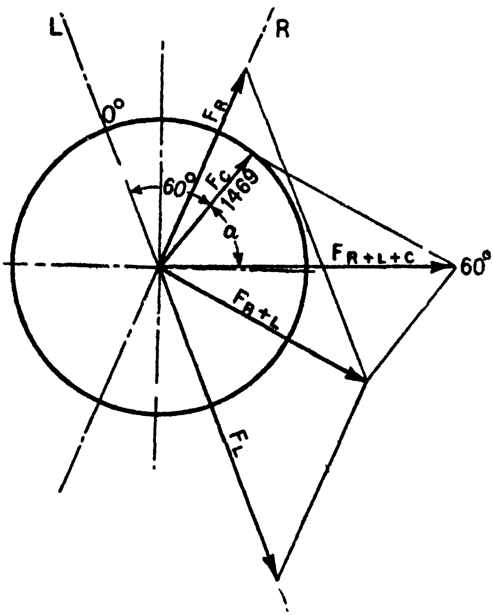


FIG. 19. RESULTANT FORCE ON CRANK

It will be seen that there are three forces acting, namely—

(1) The resultant of the gas and inertia forces for the L.H. cylinder, which is indicated by F_L;

(2) The resultant of the gas and inertia forces for the R.H. cylinder, as shown by F_R; and

(3) The centrifugal force F_C acting along the crank.

It is now a straightforward matter to find (by the parallelogram of forces) the resultant F_{R+L} of F_R and F_L, and then to find in a similar manner the resultant of this resultant F_{R+L} and F_C; the total resultant is shown by F_{R+L+C} and, in the position of the main crank chosen, acts at an angle α ahead of the main crank. The value of this angle α varies continuously during the complete cycle of operations of 720° of engine crank rotation.

If, now, the resultant F_{R+L+C} be computed for all positions of the main crank, and vectors be drawn in the direction of the resultant and proportional, in length, to the value of F_{R+L+C}, then a polar diagram of resultant forces will be obtained similar to that shown in Fig. 20.

The direction and magnitude of the resultant force on the crank pin bearing for any crank angle is given by the direction and length of a line joining the crank-angular position marked on the polar curve to the origin or crank pin centre.

An examination of the polar diagram shows that maximum bearing pressures occur at crank angles of 15° to 30°, 150° to 180°, 420° to 450°, and 540° to 585°.

The minimum values occur at 0°, 300°, and 405°.

The actual maximum bearing force is 4980 lb., and occurs at 435°.

The projected area of the crank pin is given by*—

$$A = 2{\cdot}375 \times 2{\cdot}25 = 5{\cdot}34 \text{ sq. in.}$$

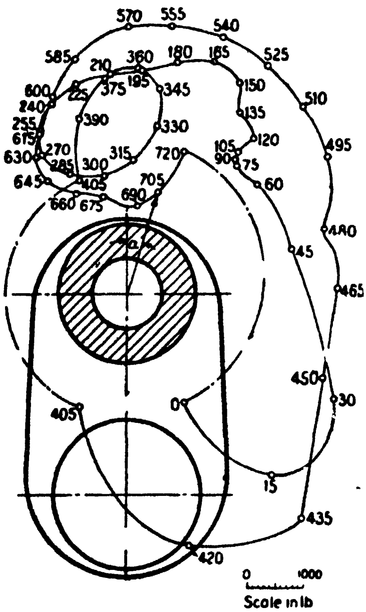


FIG. 20. DIAGRAM OF RESULTANT PRESSURES ON CRANK PIN

The value of the maximum bearing pressure on the crank pin is then given by—

$$P = \frac{4980}{5{\cdot}34} = 932 \text{ lb. sq. in.}$$

The mean resultant force on the crank pin is 3426 lb., and the mean crank pin pressure,

$$P_m = \frac{3426}{5{\cdot}34} = 642 \text{ lb. sq. in.}$$

The mean rubbing velocity on the crank pin is 17·7 ft. per sec., so that the mean rubbing factor is $17{\cdot}7 \times 642 = 13{,}500$ lb. per sq in.-ft. per sec.

It is this latter value that determines the behaviour of the bearing, and which is limited by the coefficient of friction and the viscosity of the oil.

Position of Bearing-cap Split. It may be of interest to mention that the results given in Fig. 20 may be employed to indicate the line along which the bearing-cap split should occur. This should obviously be the line corresponding to the least pressure, and in the present example would be at 45° to the length of the connecting rod; actually, for manufacturing reasons, it is made at 90°.

It should be noted that the resultant forces on the connecting rod are obtained, for each crank-angle position, by resolving the resultant crank pin force along and at right angles to the rod, and by plotting on a polar diagram the former forces.

4. FORCES ON MAIN BEARINGS. The total load on any of the main bearings is made up of the following forces, namely—

(1) The centrifugal force due to two webs (one on each side).

* See p. 28 for dimensions of bearings.

(2) The centrifugal force due to one-half of each crank pin.

(3) One-half of the total load on each crank pin.

In the case of the centre main bearing of the engine, the webs on each side of the bearing are in line. The resultant pressures on the crank pins are 360° apart as regards the four-cycle phases, but are in phase with each other as regards the dynamic stresses, and the polar diagram (Fig 21) is completed by 360° of crankshaft rotation.

In the consecutive plottings of any feature of the polar diagram given in Fig. 21, the members on one crank pin create a vector during the first 360°, whilst the same members create the vectors tracing during the second 360°.

The diagram itself will serve to indicate the method of obtaining the resultant vector.

It will be noticed that there is a discontinuity between the points 360° and 0°, and at 45° and 45°, on the curve, indicating that there is a loss of bearing pressure at the centre main bearing when the crank moves past the top dead centre line of either line of cylinders; this is due to the inertia pressure of two sets of reciprocating parts coming on the bearing as a reversed, or negative, loading. The evenness of the polar diagram indicates a fairly uniform bearing pressure throughout each cycle.

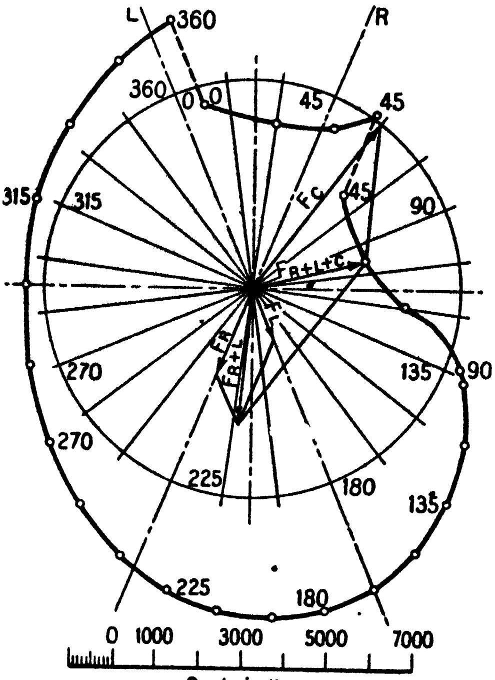


FIG. 21. POLAR DIAGRAM OF FORCES ON CRANKSHAFT MAIN BEARING

The maximum load on the centre main bearing is 7700 lb.

The projected area = 2·625 × 1·75 in. = 4·59 sq. in., so that the maximum bearing pressure is given by—

$$P = \frac{7700}{4 \cdot 59} = 1675 \text{ lb. sq. in.}$$

The average load on the bearing as given by the mean radius vector

of the polar curve in Fig. 21 is 5810 lb., so that the mean bearing pressure is given by

$$P_m = \frac{5810}{4 \cdot 59} = 1265 \text{ lb. sq. in.}$$

The rubbing velocity at this bearing is 19·5 ft. per sec., and therefore the mean load factor is—

$$f = 1265 \times 19 \cdot 5 = 24{,}670 \text{ lb. per sq. in., ft. per sec.}$$

5. Side Thrust of Piston. It is a fairly simple matter to estimate the piston side thrust forces for all crank angles, from a knowledge of the resultant of the cylinder pressure and inertia forces, and from the crank to connecting rod ratio.

The former factor is given in curve C (Fig. 18), whilst the piston thrust is given by the relation—

$$\mathrm{R} = \mathrm{Q} \sin \phi$$

$$= \mathrm{Q} \cdot \frac{r}{l} \cdot \sin \theta, \text{ since } \frac{r}{\sin \phi} = \frac{l}{\sin \theta}$$

Also

$$\mathrm{Q} = \frac{\mathrm{P}}{\cos \phi} = \frac{\mathrm{P}}{\sqrt{1 - \sin^2 \phi}} = \frac{\mathrm{P}}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}}$$

so that

$$\mathrm{R} = \frac{\mathrm{P}r \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

The piston thrust R is then given in terms of the resultant pressure P and the crank angle θ.

In the case under consideration, the maximum value of the piston thrust is given when the crank arm and connecting rod are at right angles; in this position the value of ϕ is given by—

$$\tan \phi = \frac{r}{l} = \frac{3 \cdot 5}{12} = 0 \cdot 29166$$

so that

$$\phi = 16° \ 16'$$

The corresponding crank angle θ is given by*—

$$\sin \theta = \frac{r}{l} \sin \phi = \frac{12}{3 \cdot 5} \sin 16° \ 16'$$

whence

$$\theta = 73° \ 44'$$

Values of R, the piston thrust, have been estimated for various crank angles θ, using values of P given by the ordinates of curve A (Fig. 21).

* θ is the complement of ϕ.

These values are shown plotted on a crank angle base in Fig. 22 for the 720° complete cycle period.

It will be seen that the side thrust is a maximum at about 35°, and during the firing stroke it is always in the same direction.

During the idle exhaust and suction strokes, owing principally to inertia effects, the piston thrust changes from side to side, being first on the side opposite to the initial thrust, then on the same side for the interval 282° to 360°, then opposite for 360° to 437°, and so on.

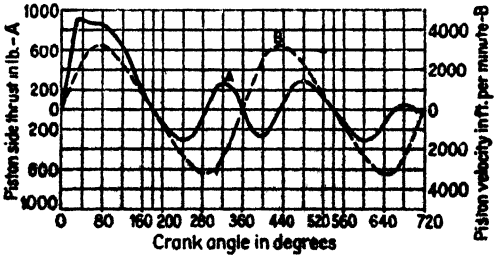


FIG. 22. PISTON-THRUST DIAGRAM

It is noticeable that the compression stroke hardly gives any side thrust as shown by the low ordinates of the curve for the period 540° to 720°, and is almost zero towards the end of the stroke.

On the same diagram (Fig. 22), a dotted curve of piston velocity is also shown, so that the load-velocity product can be at once obtained for any crank position.

The curve shows that the maximum piston thrust is about 930 lb., this pressure may be considered as being carried by the portion of the piston below the rings.

The projected area of this portion is given by—

$$A = 3{\cdot}625 \times 5{\cdot}0 \text{ in.} = 18{\cdot}125 \text{ sq. in.}$$

Hence the maximum pressure between the piston and cylinder walls is—

$$P = \frac{930}{18{\cdot}125} = 51{\cdot}2 \text{ lb. sq. in.}$$

The mean piston velocity is given by—

$$V_P = \frac{2}{\pi} . V_C \text{ where } V_C = \text{crank pin velocity}$$

$$= \frac{4N . r}{60l} \text{ where } N = \text{r.p.m.}$$

$$= \frac{4 \times 1700 \times 3{\cdot}5}{60 \times 12} = 33{\cdot}1 \text{ f.s.}$$

Hence the load factor on the side walls is given by

$$L = 51{\cdot}2 \times 33{\cdot}1$$
$$= 1690 \text{ lb. sq. in. ft. per sec.}$$

The relative movement in this case is one of sliding.

Knowing the forces in the various members, it is possible to obtain the stresses in these parts.

CHAPTER II

ENGINE TORQUE AND TORQUE DIAGRAMS

Crank Effort or Torque

IT is necessary for the purposes of crankshaft design and in connexion with considerations of torsional vibrations in engine-driven systems to know the value of the engine torque for any given crank angle.

Referring to Fig. 23, P denotes the total pressure or load on the piston, Q the resultant connecting rod thrust, and R the normal thrust

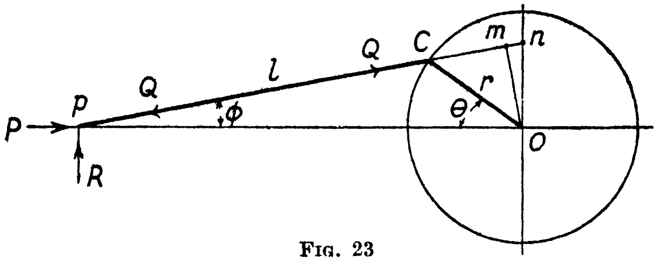


FIG. 23

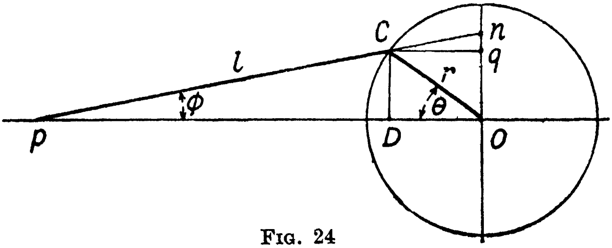


FIG. 24

between the piston and cylinder wall. From the fact that the triangles pnO and Onm are similar, it follows that

$$\mathrm{P} = \mathrm{Q} \cos \phi = \mathrm{Q}\frac{\mathrm{O}p}{pn} = \mathrm{Q}\frac{\mathrm{O}m}{\mathrm{O}n}$$

The crank effort is P . On, which has been shown to be equal to Q . Om.

Referring to Fig. 24, the value of On, in terms of l, r, and θ, is obtained as follows—

$$\mathrm{O}n = \mathrm{O}q + qn = \mathrm{CD} + qn = r \sin \theta + qn$$

Also $$\frac{qn}{\mathrm{C}q} = \frac{\mathrm{CD}}{p\mathrm{D}}, \text{ whence } qn = \frac{\mathrm{C}q \,.\, \mathrm{CD}}{p\mathrm{D}}$$

Substituting in this latter expression $Cq = r \cos \theta$, $CD = r \sin \theta$ and $pD = \sqrt{l^2 - r^2 \sin^2 \theta}$, we get

$$qn = \frac{r^2 \cos \theta \, . \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}$$

Hence the crank effort $P \, . \, On = P(Oq + qn)$

$$= Pr \sin \theta \left(1 + \frac{r \cos \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}\right)$$

It will be observed that this expression is similar to that for the velocity of the piston, given on page 17, for in each case the multiplying factor is a term proportional to the intercept of the connecting rod on the vertical diameter of the crank pin circle.

The total pressure P, which is the resultant of the gas and inertia pressure loads, varies throughout the cycle of operations. Its value can be obtained by the method given in the previous chapter. A diagram of crank effort or torque can then be drawn by multiplying corresponding values of P and the intercept On (Fig. 23) together and plotting the result on a piston or crank angle base.

The area of this diagram to a suitable scale of units will represent the work done per cycle, since the area of the torque curve on a crank angle base (or polar diagram) is equal to $\int_0^{360} T \, . \, d\theta$ where θ is the crank angle. Another expression for the torque, which can readily be obtained from Fig. 23, is as follows—

$$\text{Torque} = \frac{Pr \sin (\theta + \phi)}{\cos \phi}$$

It should be noted that in the preceding calculations it is the theoretical torque on the crankshaft that has been considered. The effective or actual torque will be less than this owing to engine frictional losses between the cylinder and crankshaft, which have the effect of reducing the value of the cylinder gas pressure as given by the indicator diagram.

Mean Torque

The *mean torque at the crankshaft* can be obtained from the brake horse power and corresponding engine speed as follows—

Torque $\times$ angle of rotation (radians per min.) $=$ b.h.p. $\times$ 33,000

where the torque is in lb.-ft.

If $N =$ r.p.m. of engine, angle of rotation per min. $= 2\pi N$.

Thus $$\text{Torque} = \frac{33{,}000}{2\pi} \, . \, \frac{\text{b.h.p.}}{N} = 5252 \, . \, \frac{\text{b.h.p}}{N}$$

Torque and M.E.P. Relationship

It is useful to note that since the b.h.p. of an engine is proportional to the product of the brake mean effective pressure (b.m.e.p.) and the engine speed N, it follows that the engine torque will vary as the b.m.e.p., so that a curve representing b.m.e.p. values on a speed base will also represent engine mean torque values, but to a different scale.

Brake Horse Power Calculation

Although it is usual to measure the brake horse power of a petrol or Diesel engine with some type of dynamometer it is useful to be able to calculate the probable value of the b.h.p. from the engine dimensions and other factors. If N = r.p.m., d = diameter of cylinder in inches, l = length of stroke in inches, n = number of cylinders, and p = b.m.e.p. at the speed N,

$$\text{Then} \quad \text{b.h.p.} = \frac{p \times \frac{l}{12} \times \frac{\pi d^2}{4} \times \frac{N}{2} \times n}{33{,}000}$$

$$= 9{\cdot}91664 \times 10^{-7}\, pld^2Nn$$

The reason for using $\frac{N}{2}$ in this formula is because there is one firing stroke per two engine revolutions for a four-cycle engine. In the case of a two-cycle engine the value N would be used so that the constant of the preceding formula must then be doubled.

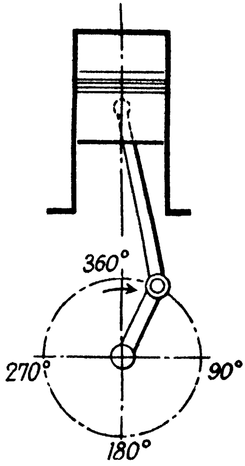


FIG 25. SINGLE CYLINDER ENGINE

The value of p, the b.m.e.p., is usually deduced from the indicated m.e.p. by multiplying the latter by the mechanical efficiency. For modern high-speed internal combustion engines the value of the mechanical efficiency varies from 80 to 90 per cent.

The value of the indicated m.e.p. can also be worked out from the theoretical indicator diagram, arrived at by thermodynamical considerations of the particular conditions, such as compression ratio, cooling losses, mixture ratio, etc.

Torque Diagrams for Different Types of Engine

It is now proposed to consider a few of the more important cases of the application of crank effort or torque curves to examples occurring in practice.

For the purposes of comparison, it will be more convenient to

assume that all the types of engine have the same size of cylinder, the engine speed being constant in each case.

In the examples considered, the bore and stroke have been taken as 3 and 4 inches respectively, and the engine as running at 1000 r.p.m.

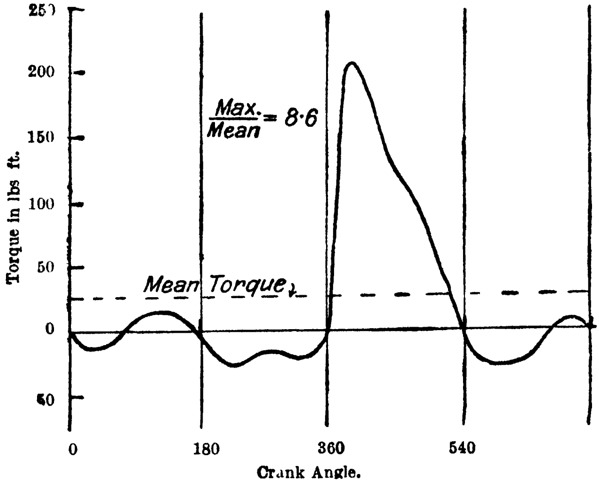


FIG. 26 SINGLE CYLINDER ENGINE TORQUE CURVE

Further, a connecting-rod-crank ratio of 4 ($= n$) has been assumed, and the weight of the reciprocating parts taken to be 4 lb.

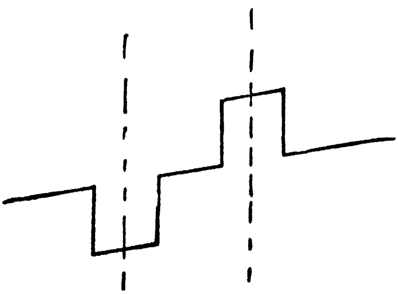


FIG. 27. TWO-CYLINDER CRANKSHAFT

A standard example of indicator diagram has been taken, plotted out on a crank angle base, and corrected for inertia effects in the manner previously explained.

The equivalent inertia pressures, in lb. per sq. in., have been estimated to be 59 and 36 for the in- and out-centres respectively.

In each case the mean value of the torque for one complete cycle

of operations of a single cylinder or two crank revolutions has been obtained from the mean height of the torque diagram, and, further, the ratio of the maximum to the mean torque is given in each case.

Case I. The single-cylinder type, as illustrated diagrammatically in Fig. 25, gives a torque diagram as illustrated in Fig. 26.

It will be noticed that the variation in the crank effort during the firing stroke is very marked in this case, the ratio of maximum to mean torque being 8·6.

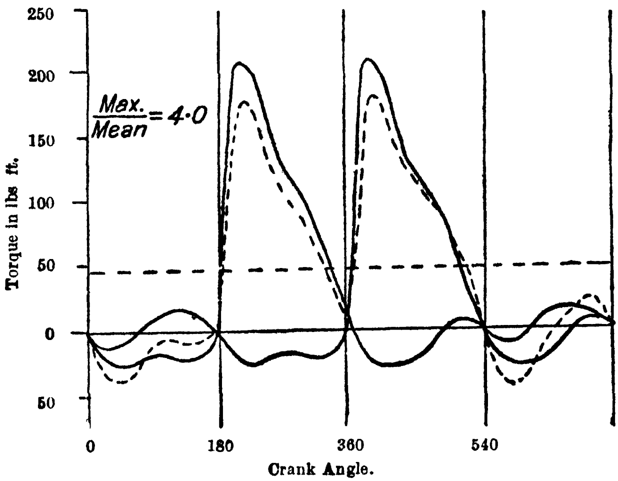


FIG 28 TWO CYLINDER VERTICAL ENGINE TORQUE DIAGRAM

It will be seen later that such a high value of this ratio necessitates relatively larger sizes of the torque transmission parts and a heavier flywheel than in the succeeding cases.

Case II The diagrams given in Fig. 28 illustrate the torque curves for a twin-cylinder side-by-side type of engine with cranks at 180°, and firing alternately, shown in Fig. 27.

The dotted curve in the diagram represents the resultant torque due to the combination of the two separate torques of the respective cylinders

In this case the maximum torque is just four times the mean, which is represented by the dotted horizontal line.

Further, it will be seen that two explosions follow each other consecutively, and then there is an idle period of one revolution, and two more firing strokes during the next revolution, and so on.

Case III. Two opposed cylinders, with cranks at 180°. The crank effort curves for this type of engine, which is illustrated in Figs. 198 and 199, are shown in Fig. 29.

It is noticeable that the frequency of occurrence of the firing strokes is quite regular, so that the maximum torque periods will also be regular.

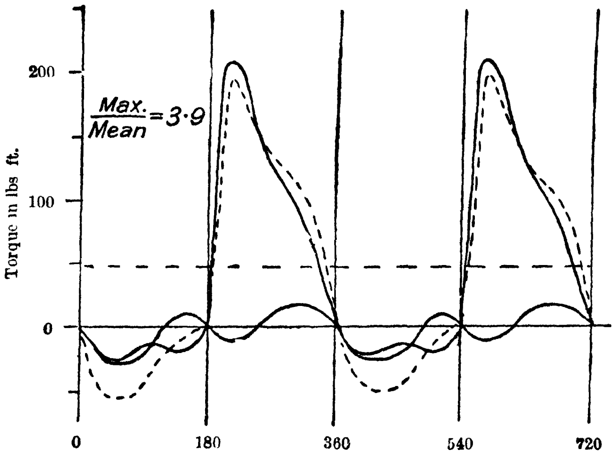


FIG 29. TWO-CYLINDER OPPOSED ENGINE TORQUE DIAGRAM

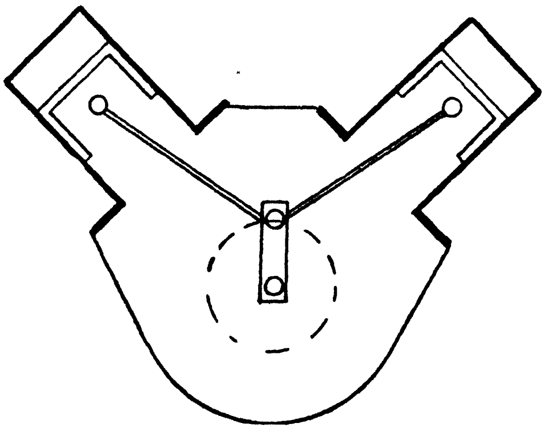


FIG. 30. 90° TWIN ENGINE

The ratio of maximum to mean torque during two revolutions in this case is 3·9.

Case IV. Next in consideration is the *Vee* type of engine, commonly employed in motor-cycle practice.

The diagrams in Figs. 30 and 31 relate to cylinders set at 90° and firing alternately.

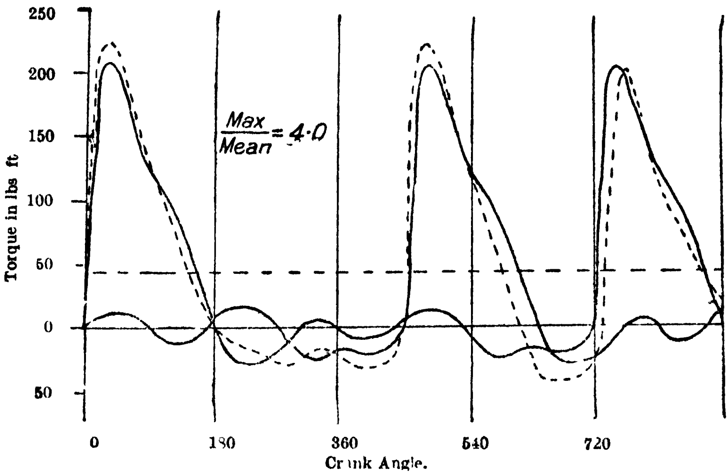
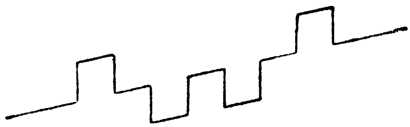


FIG 31 90° TWIN ENGINE TORQUE DIAGRAM



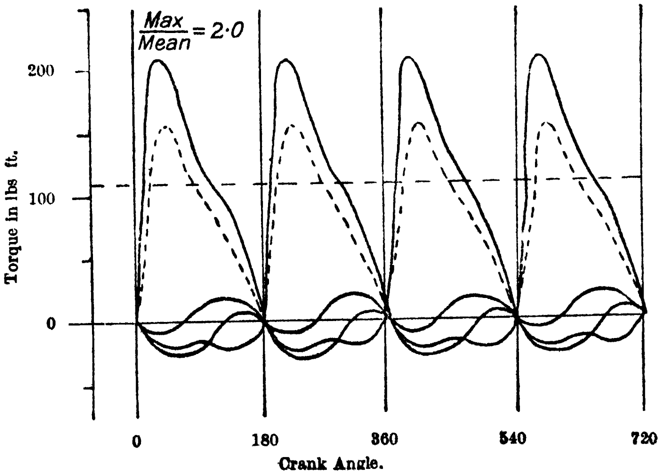


FIG. 32. FOUR-CYLINDER ENGINE TORQUE DIAGRAM

The interval between consecutive explosions is not quite regular, the intervals being alternately $\frac{3}{4}$ and $\frac{1}{4}$ of a revolution.

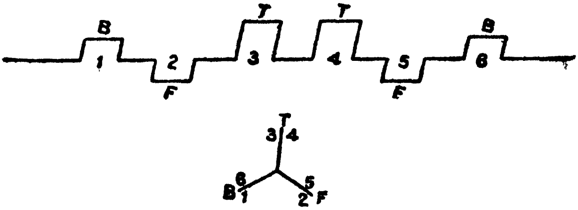


FIG. 33 SIX-CYLINDER VERTICAL ENGINE CRANK ARRANGEMENT

The ratio of maximum to mean torque is 4·0.

It will be apparent that the smaller the angle between the cylinders, the nearer does the total or combined torque curve approach that

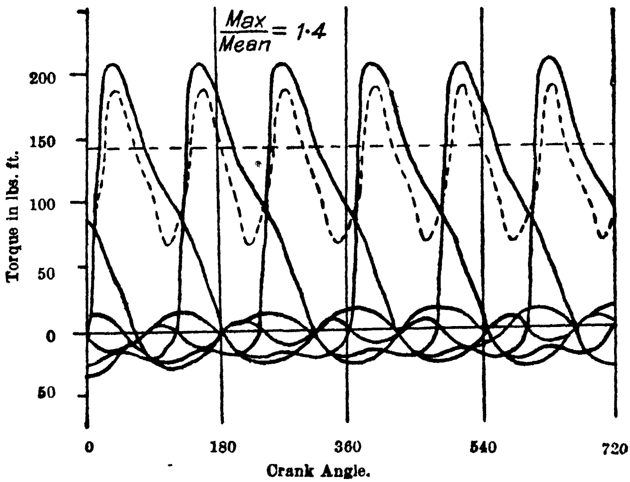


FIG. 34. SIX-CYLINDER TORQUE DIAGRAM

of a two-cylinder engine with one crank only, and the more irregular do the firing periods become, that is, the less uniform the periods of maximum crank effort.

Case V. Four cylinders with cranks in pairs at 180°, as shown in Fig. 32, the order of firing being 1, 2, 4, 3—this being an ordinary motor-car engine arrangement.

The crank effort diagram indicates that the firing periods are quite regular and follow on consecutively with no irregular intervals.

There are four periods of firing or useful crank effort during every two revolutions, and consequently four peaks to the combined torque curve.

The ratio of the maximum to the mean torque in this case is 2·0 approximately.

Case VI. The six-cylinder type with cranks at 120°, the firing order being 1, 4, 2, 6, 3, 5, and representing an ordinary type of six-cylinder car engine.

There are now six firing periods during every two revolutions, and consequently six maximum torque peaks, as shown in Figs. 33 and 34.

The ratio of maximum to the mean torque in this case is 1·4, so that one effect of cylinder duplication is evidently to raise the height of the mean torque line, and to cause it to approach more nearly the maximum torque height.

From the examples considered, it should be a simple matter to determine the resultant crank effort curve for any other arrangement of cylinders.

Polar Torque Diagrams

An alternative method of plotting torque diagrams is the polar one, in which the instantaneous torque values are plotted as radii vectors at angles to the zero radius line, equal to the crank angles.

A typical polar torque curve* is shown in Fig. 35.

This curve refers to an engine of the four-cylinder vertical type, with the following characteristics—

Bore = 5·046-in. Stroke = 6 in.
Connecting rod length = 12 in.
Indicated m.e.p. = 100 lb. sq. in.
Engine speed = 1909·9 r.p.m., or 200 radians per sec.
Reciprocating weight = 3·22 lb. per cylr. = 0·162 lb. per sq. in. of piston area.

The maximum explosion pressure was 330 lb. sq. in., and the compression ratio about 5 : 1. The expansion and compression curves were of the form $PV^{1.33}$ = const.

The torque values given include both pressure and inertia effects.

The torque curve for the single cylinder must be drawn first, and, as in the preceding cases, the torque curve for any other number of cylinders, or cylinder arrangements, can then be obtained by

* See "Unbalanced Forces in Reciprocating Engines," by J. L. Napier: *The Autom. Engineer* (Feb. and Mar.), 1920.

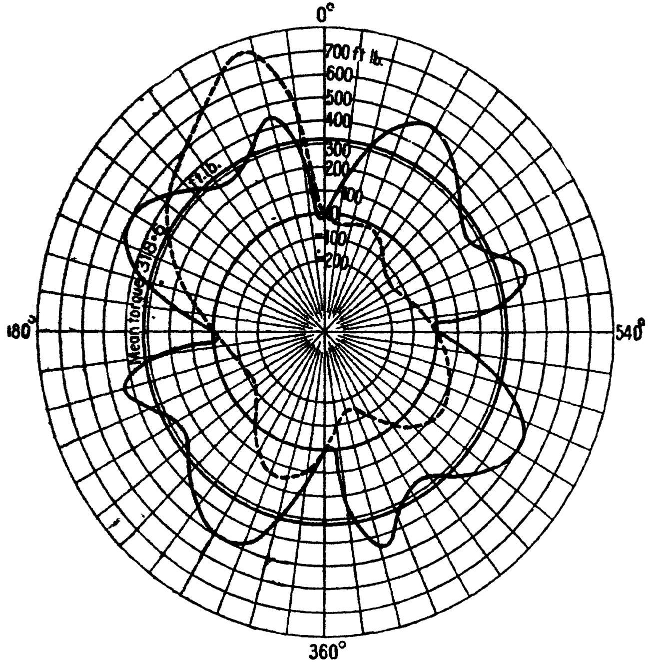


Fig. 35. Polar Torque Curves for Single- and Four-cylinder Engine

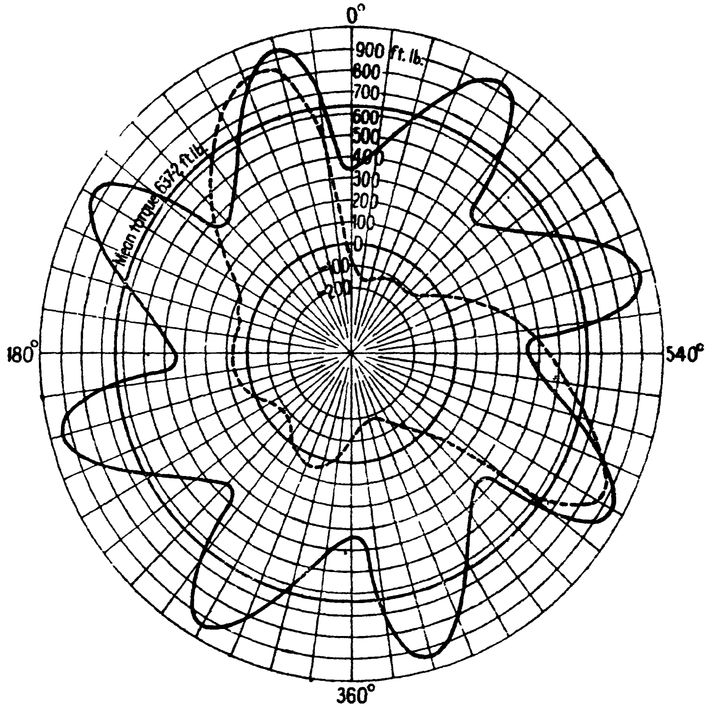


Fig. 36. Polar Torque Curve for Eight-cylinder Vee-type Engine

replotting the torque curves in their proper phase relations, and algebraically summing the ordinates, or radii vectors.

The dotted curve in Fig. 35 gives the torque curve for a single cylinder; the radii vectors are drawn at intervals of 15°.

The maximum torque value due to both gas pressure and inertia

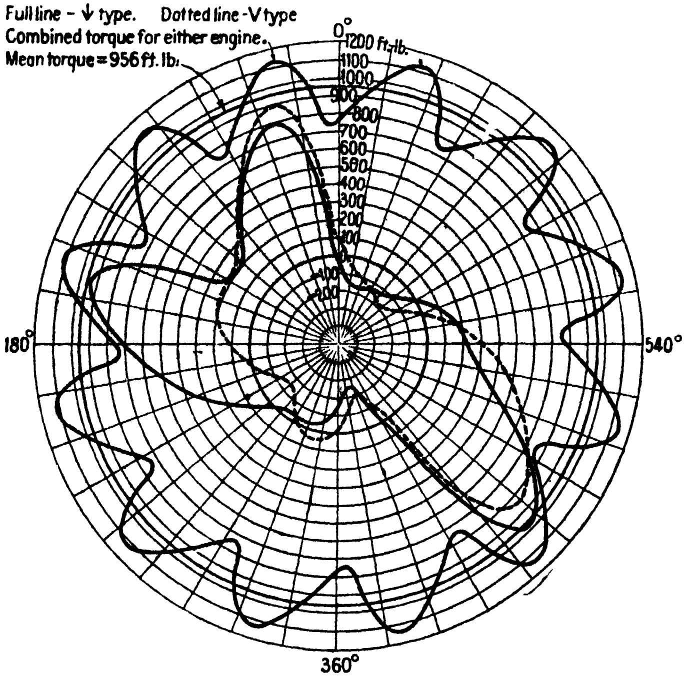


FIG. 37. POLAR TORQUE CURVES FOR TWELVE-CYLINDER VEE AND ARROW TYPES OF ENGINE

forces is about 570 lb.-ft., whilst the mean torque value is about 79·6 lb.-ft.

$$\text{The ratio} = \frac{\text{maximum torque}}{\text{mean torque}} = 7{\cdot}15$$

In the case of the four-cylinder vertical engine with cranks at 180°, as in normal practice, the values of the maximum and mean torques are 526·6 and 318·6 lb.-ft. respectively, giving a maximum to mean ratio of 1·653.

The following table shows the values of the maximum torque variation,* and mean torques for other well-known petrol engine

* The greater fluctuation of torque due to the gas pressure in this case is balanced by the opposite greater inertia fluctuations.

cylinder arrangements, each cylinder having the same characteristics as the one mentioned on page 45; the m.e.p. and engine speed are the same as in the previous case—

TABLE II

TORQUE RELATIONS FOR DIFFERENT ENGINES (*Napier*)

(*a*) Type of Engine	(*b*) Load	(*c*) Connecting Rods Crank Ratio	(*d*) Maximum‡ Torque Variation	(*e*) Mean Torque	(*f*) Ratio $\frac{(d)}{(e)}$
			lb.-ft.	lb.-ft.	
Single cylinder . .	Full	4	750	79·6	9·3
Four-cylinder vertical .	,,	4	526·6	318·6	1·653
Six-cylinder vertical . .	,,	4	475·6	474	0·990
Cranks at 120° standard arrangement . . .	One-half	4	359·9	239	1·502
,, ,, ,,	One-quarter	4	398·1	119·5	3·331
,, ,, ,,	No Load	4	576·24	0	Infinity
Eight-cylinder standard Vee-type engine . .	Full Load	4	583·72	637·2	0·916
,, ,, ,,*	,,	3	584	637	0·916
Twelve-cylinder Broad Arrow engine (four per line)	,,	4	398	956	0·407
Twelve-cylinder Vee-type (six per line)† . .	Full Load	4	398	956	0·407

* The greater fluctuation of torque due to the gas pressure in this case is balanced by the opposite greater inertia fluctuations.

† Both types fire at equal intervals of 60° each, and the complete torque diagram is the same for each type.

(Examples of the eight- and twelve-cylinder polar torque diagrams are given in Figs. 36 and 37 from Napier's results.)

‡ Note that the maximum torque variation = max.–min. torque, and is not the same as the max. torque.

Twelve-cylinder Vee-type Engine Torque

In the case of the twelve-cylinder Vee-type engine referred to on page 27, using the same pressure and inertia data, the torque curves for each pair of Vee cylinders may be separately drawn upon a crank-angle base diagram, and the ordinates algebraically summed in the usual manner.

Fig. 38 shows the combined torque diagram for this engine for a 720° period, the dotted lines referring to the separate torque curves for six pairs of cylinders; the full-line diagram gives the resultant curves for the six pairs of cylinders, and the horizontal line cutting the peaks of this diagram shows the mean torque curve.

The maximum torque peak occurs at an angle of 73° 44′ from the top position, and at every 120° thereafter.

The minimum torque peak occurs at 48° 44′ and at every 120° thereafter.

The maximum peak occurs at the crank position at which the crank and connecting rod are at right angles during the explosion stroke of the left-hand cylinder of a pair; a simultaneous explosion also occurs

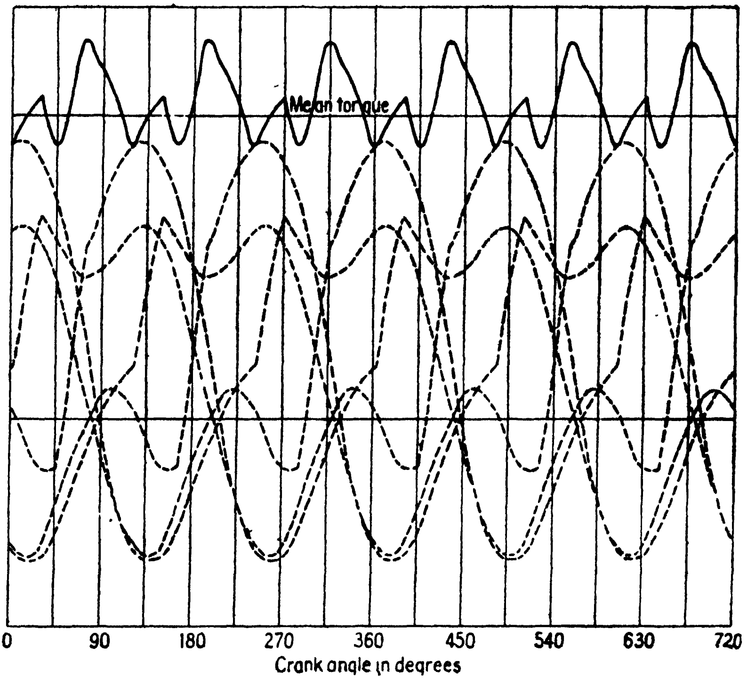


FIG. 38. TORQUE CURVES FOR TWELVE-CYLINDER VEE ENGINE

on a right-hand cylinder elsewhere, but the crank in this case is only 28° 44′ past top dead centre.

The maximum value of the engine torque, corresponding to 123 lb. m.e.p., is given by—

$$T_{max} = \frac{123 \times 19{\cdot}6 \times 7 \times 6}{12 \times 2\pi} = 1345 \text{ lb. ft.}$$

which is equivalent to a force of 5760 lb. acting tangentially at crank pin radius.

The ratio of the maximum to the mean torque value in this case is 1·25.

The ratio of the maximum torque variation to the mean torque is about 0·35.

Bearing Loads of Six- and Twelve-cylinder Engines

An interesting comparison has been made between the bearing loads of a six-cylinder engine and a twelve-cylinder Vee-type one.* The former engine, of the usual vertical type, had a bore of 81·5 mm. and stroke of 114 mm., giving a cylinder capacity of 3568 c.c. and an R.A.C. horse-power rating of 24·7. The compression ratio was 5·3 : 1. The reciprocating weight per cylinder was 2·625 lb. and the rotating weight of the big end 1·735 lb. The big-end bearing area was 2·52 sq. in.

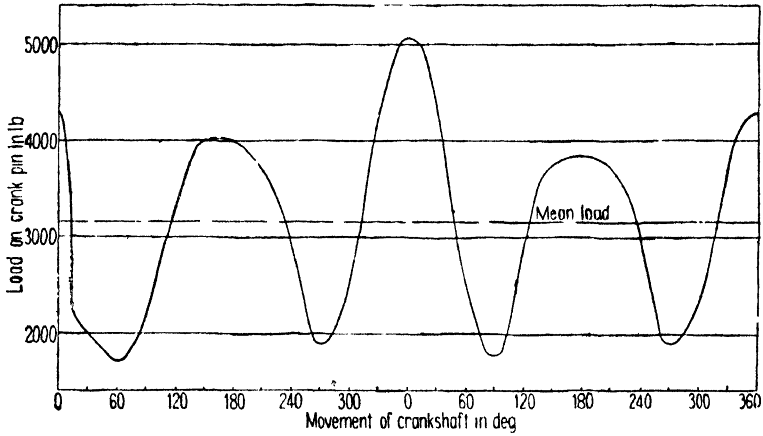


FIG. 39. BIG-END BEARING LOADS FOR SIX-CYLINDER ENGINE

In the case of the twelve-cylinder engine the bore was 65 mm. and stroke 94 mm., giving a cylinder capacity of 3744 c.c. and an R.A.C horse-power rating of 31·4. The compression ratio was 6·2 : 1. The reciprocating weight per cylinder was 1·34 lb., and the rotating weight of the big end was 2·06 lb. on the near side and 0·99 lb. on the off side. It should be mentioned that the connecting rods on opposite pairs of cylinders worked on the same crank pin.

Fig. 39 shows the actual big-end bearing load of the six-cylinder engine at 4000 r.p.m. under full load conditions for a complete cycle of two crankshaft revolutions. It will be observed that the maximum load acting on the big-end bearing occurs at the top of the exhaust stroke and is about 5000 lb. The average load over the complete cycle is 3140 lb. The maximum bearing pressure is equal to the maximum load divided by the big-end bearing area, and works out at 2000 lb. per sq. in. The mean bearing pressure, P, is 1245 lb. per sq. in. The diameter of the crankshaft is 1·89 in., so that the rubbing

* "The Double-six Engine," L. H. Pomeroy, *Proc. Inst. Autom. Engrs.*, 1930.

velocity V is 33 ft. per sec. at 4000 r.p.m. The product of bearing pressure and rubbing velocity, known as the PV factor, is 41,000 lb. per sq. in., ft. sec. at 4000 r.p.m.

Fig. 40 (upper diagram) shows how the big-end bearing load of the twelve-cylinder engine varies during a complete cycle under the same running conditions. This graph represents the combined loads of the two connecting rods and pistons on the main big-end bearing. It commences at the point where the crank for the near-side piston is 60°

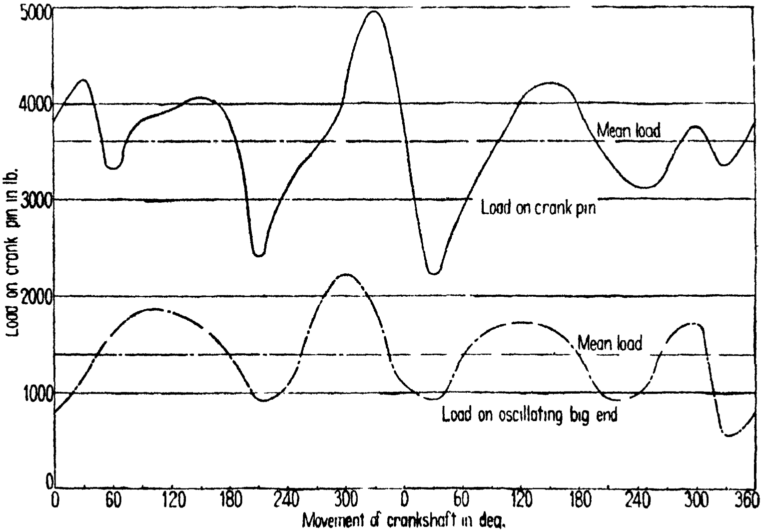


FIG. 40. BIG-END BEARING LOADS FOR TWELVE-CYLINDER VEE TYPE ENGINE

after the top of the combustion stroke. The maximum load on the big-end bearing is 4950 lb., and the average load is 3600 lb. The maximum and mean bearing pressures corresponding to these loads work out at 2000 and 1465 lb. per sq. in., respectively. The crank-pin diameter is 1·73 in., giving a rubbing velocity of 30 ft. per sec.

The average PV factor for the big-end bearing is therefore 44,000 lb. per sq. in., ft. sec.

The lower curve in Fig. 40 represents the load on the big-end of the oscillating rod. This rod is located in the off-side cylinder bank, and takes its bearing on the outside of the main-bearing shell in the centre of the forked rod, to which the shell is fixed. The maximum load on this bearing is 2200 lb., and the mean load 1400 lb. The effective projected area of the bearing is 1·64 sq. in., so that the maximum bearing pressure is 1340 lb. per sq. in. and the mean pressure 850 lb. per sq. in. The relative movement between this rod and the forked

rod is 24°. The diameter of the bearing is 2·00 in. The mean PV factor for this bearing is 4370 lb. per sq. in., ft. sec.

Although the PV factors for the two engines are very similar at 4000 r.p.m., tests upon these engines revealed the fact that the six-cylinder engine would not operate for any appreciable period at this speed owing to bearing trouble, whereas the twelve-cylinder engine gave no trouble. The explanation appears to be that the polar diagram of bearing loading for the latter engine approximated to that of a circle, whereas that of the six-cylinder engine was distinctly elliptical, the major axis being about 2·7 times the mean axis.

Torque Variation of Similar Capacity Engines

The previous considerations have been confined to the comparisons of engines having a different number of cylinders, each of the latter being of *the same size.*

An important case occasionally occurring in practice, when the advisability of changing from one design to another is being considered, is that of engines having *the same total cylinder capacity* but a different number of cylinders; each of the latter will, therefore, be smaller as the number of cylinders is increased.

An interesting comparison that has been made in the case of 4, 6, 8, and 12 cylinder automobile four-stroke engines is given in the table below. The torque values refer to engines of equal total cylinder capacity.

One is thus able to compare the merits of engines in the same cubical capacity class—

TABLE III

TORQUE VARIATIONS FOR EQUAL CUBIC CAPACITIES

No. of Cylinders	Variation of Torque from Mean Value (per cent)	Relative Variation of Torque
4	125	100
6	45	36
8	20	16
12	10	8

Effect of Speed Variations on Torque Values

From the designer's point of view it is important to be able to examine the mode of variation of the torque values at different engine speeds, and to consider not only the mean, but also the maximum working speeds.

Hitherto the ratio of the maximum to mean torque at some intermediate engine speed only, has been considered, whereas at the highest working speed a much higher ratio occurs.

The following tabulated values refer to a specific example* of a four-cylinder petrol engine of 3-in. bore and 5-in. stroke with reciprocating parts weighing 2 lb. per cylinder—

TABLE IV

EFFECT OF SPEED ON TORQUE

Revs. per minute	2000	2500	3000	3500
Max. torque lb.-ft.	230	355	516	712
Min. torque lb.-ft.	0	− 119	− 273	− 466
Mean torque lb.-ft.	103·7	103·7	103·7	103·7
Ratio $\frac{\text{Max.}}{\text{Mean}}$	2·23	3·44	5·01	6·29

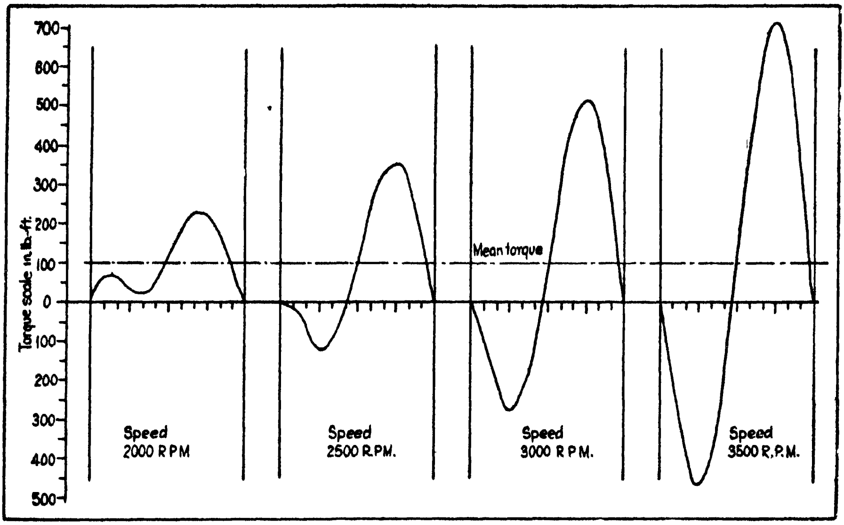


FIG. 41. SHOWING EFFECT OF SPEED VARIATIONS ON TORQUE VALUES

The manner in which the torque varies during the stroke is shown in Fig. 41 for the four cases tabulated above. These diagrams show in a very striking manner the important effect of speed on torque. In another typical case the torque values were worked out for speeds of 1500 r.p.m. and 3000 r.p.m. respectively, in the case of a similar four-cylinder petrol engine with 3 lb. weight for the reciprocating parts per cylinder.

At 3000 r.p.m. the range of torque was 6·75 times as great as at 1500 r.p.m., and the ratio of maximum to mean torque was 4·0 times the value at the lower speed.

* "Torque Calculations," H. A. Golding, *Proc. Inst. Autom. Engrs.* (1925–26).

Another interesting comparison between the torque curves of a certain design of engines of equal cylinder capacity, but with 6, 8, and 12 cylinders, respectively, is shown in Figs. 42 and 43.* In the former case the torque curves have been taken at a crankshaft speed of 200 r.p.m., at which speed the inertia forces are practically negligible, so that the torque variations are due to gas pressure effects. The graphs

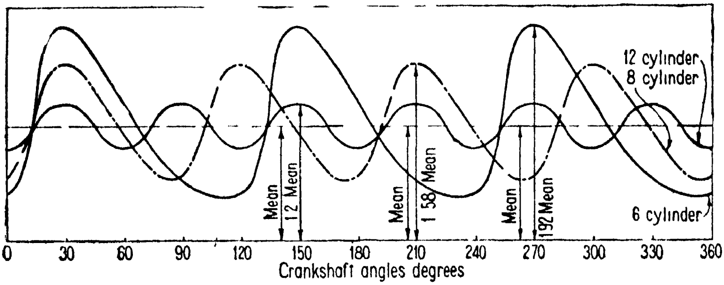


FIG. 42. TORQUE CURVES FOR THREE TYPES OF ENGINES AT 200 R.P.M.

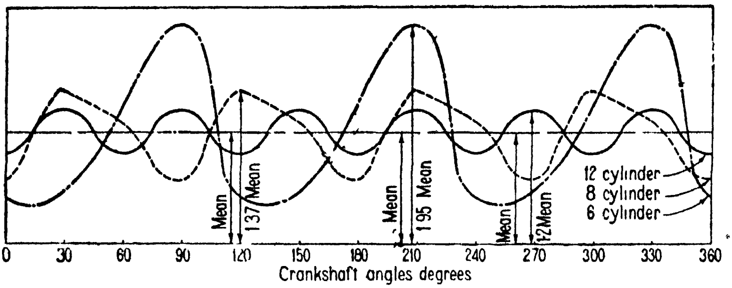


FIG. 43. TORQUE CURVES FOR THREE TYPES OF ENGINES AT 3000 R.P.M.

shown in Fig. 43 are for the same three engines but for a speed of 3000 r.p.m.

The following values indicate the variations of torque in the two cases mentioned.

TABLE V

TORQUE COMPARISONS FOR DIFFERENT ENGINES

Engine Speed, r.p.m.	Ratio of Maximum to Mean Torque		
	Six-cylinder	Eight-cylinder	Twelve-cylinder
200	1·92	1·58	1·20
3000	1·95	1·37	1·20

* "The Double-six Engine," L. H. Pomeroy, *Proc. Inst. Autom. Engrs.*, 1930.

Thrust on Cylinder Walls

Case I. The obliquity of the connecting rod produces a side thrust on the cylinder walls. The value of this thrust increases as $\frac{l}{r} = n$ decreases, assuming a definite value for the motor couple or torque.

Let T be the side thrust,

F the force at the gudgeon pin,

and R the force exerted along the connecting rod.

By taking moments about O, Fig. 44—

$$\begin{aligned} T \,.\, OB &= R \,.\, OC \\ &= \text{Torque of engine} \end{aligned}$$

The investigation may be proceeded with analytically. Considering the equilibrium of the three forces at B

$$\frac{T}{F} = \tan \phi$$

Putting ϕ in terms of l, r, and θ; since $r \sin \theta = l \sin \phi$

$$\tan \phi = \frac{\sin \phi}{\sqrt{1 - \sin^2 \phi}} = \frac{\frac{r}{l} \sin \theta}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}} = \frac{r \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}$$

That is $$\frac{T}{F} = \frac{r \sin \theta}{\sqrt{l^2 - r^2 \sin^2 \theta}}$$

Case II. When the engine is offset by an amount b, Fig. 45 the balance of the moments about O is

$$T \,.\, OB' + F \,.\, b = R \,.\, OC'$$

As before, the alternative consideration of the forces at B gives

$$\frac{T}{F} = \tan \phi^1$$

Determining ϕ^1 in terms of $\theta \,.\, l \,.\, r \,.\, b$ from $l \sin \phi^1 = r \sin \theta - b$, we get

$$\tan \phi^1 = \frac{r \sin \theta - b}{\sqrt{l^2 - (r \sin \theta - b)^2}}$$

Now the numerator is less and the denominator greater in *Case II* as compared with *Case I*, so that the value of $\frac{T}{F}$ is less. Hence for a

given value of F, the thrust T is smaller, so long as sin θ has a positive value, that is, during down strokes. When sin θ is negative—corresponding with up strokes—the value $\frac{T}{F}$ is greater; but as the real value of F for compression or exhaust strokes is small the increase in

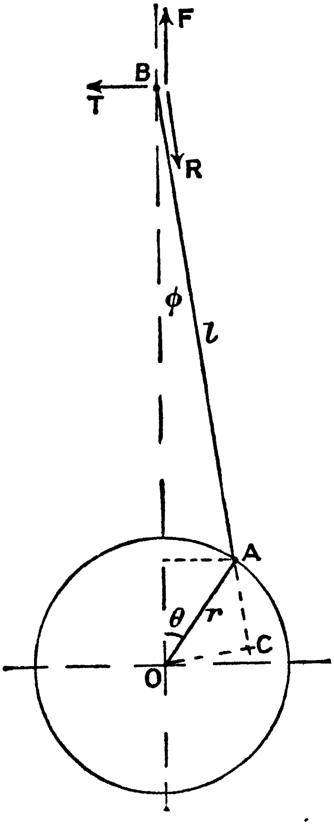


FIG. 44. ORDINARY ENGINE

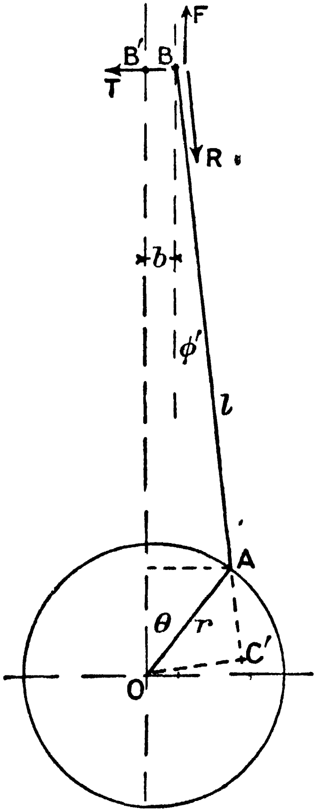


FIG. 45. OFFSET ENGINE

T is not of great moment. The diminution of T on the firing stroke when F is large, is, however, quite appreciable. It is to this factor that the adoption of offsetting may be put down.

The piston-thrust diagram for the offset cylinder may be readily obtained graphically.

The procedure consists in first obtaining graphically a diagram of crank angles and corresponding piston positions, similar to that shown by the full lines in Fig. 46. All that it is necessary to do is to divide the piston stroke into a number of parts and, using these points

as centres and the connecting-rod length as radius, to mark off the corresponding crank positions on the crank circles.

Next, it is necessary to obtain the corresponding connecting-rod angles (made with the cylinder axis) either by the above graphical method or by calculation from the above relation between ϕ^1 and θ.

The piston pressure diagram, corrected for inertia, gives the corresponding values of F, the axial force on the gudgeon pin for different points along the stroke, the corresponding connecting-rod angles to which are now known.

The piston thrust on the cylinder walls is then given by the relation—

$$T = F \tan \phi^1$$

If values of T be plotted upon a piston-stroke base, a diagram (*B*, Fig. 46) will be obtained similar to that shown in Fig. 47, which is drawn for the case of an offset engine, with an offset equal to one-sixth of the cylinder diameter and a connecting rod to crank ratio of 4.

The dotted line *A* shows the corresponding normal cylinder diagram.

The relative values of the two maximum and mean piston thrusts are as follows—

$$\frac{\text{Maximum thrust (normal engine)}}{\text{Maximum thrust (offset engine)}} = 1{\cdot}40$$

$$\frac{\text{Mean thrust (normal engine)}}{\text{Mean thrust (offset engine)}} = 1{\cdot}26$$

The compression-stroke piston-thrust diagram has been drawn upon the same side as the firing stroke diagram, for convenience, although actually it should have been drawn below.

The present considerations are only concerned, however, with quantitative values, so that the curves shown fulfil these requirements. The only difference, then, between the firing and the compression-stroke piston-thrusts is that, in the former case, the explosion pressure causes the thrust against one side of the cylinder wall, whereas, in the latter case, the momentum of the engine causes the thrust whilst compressing the charge, and it occurs upon the opposite side of the cylinder wall.

An examination of Fig. 47 shows that the firing period piston-thrust is reduced by about 35 per cent, due to the smaller connecting-rod obliquity; but during the compression period, when the obliquity is greater than in the normal type, the piston thrust is increased by about 30 to 40 per cent. This increase, as the previous results show, is considerably outweighed by the firing period gain.

It may here be mentioned that in the case of the slipper or short skirt type piston the relative surface areas of the firing and compression

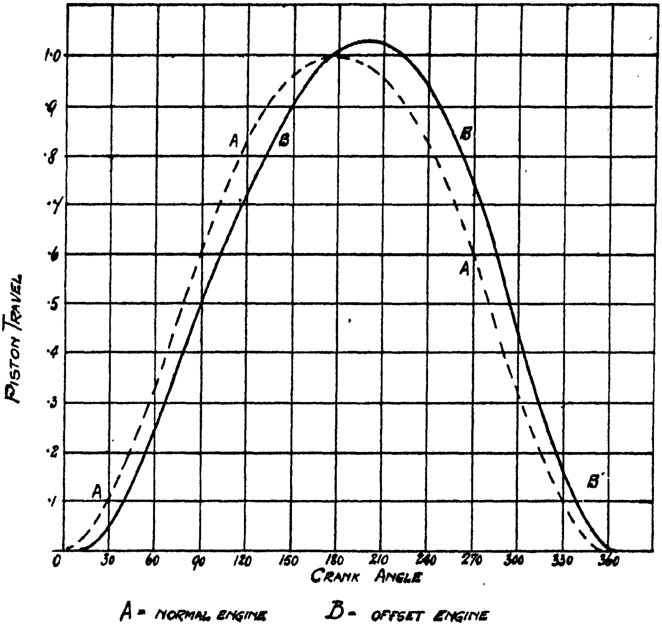


FIG. 46. PISTON POSITIONS FOR NORMAL AND OFFSET ENGINES

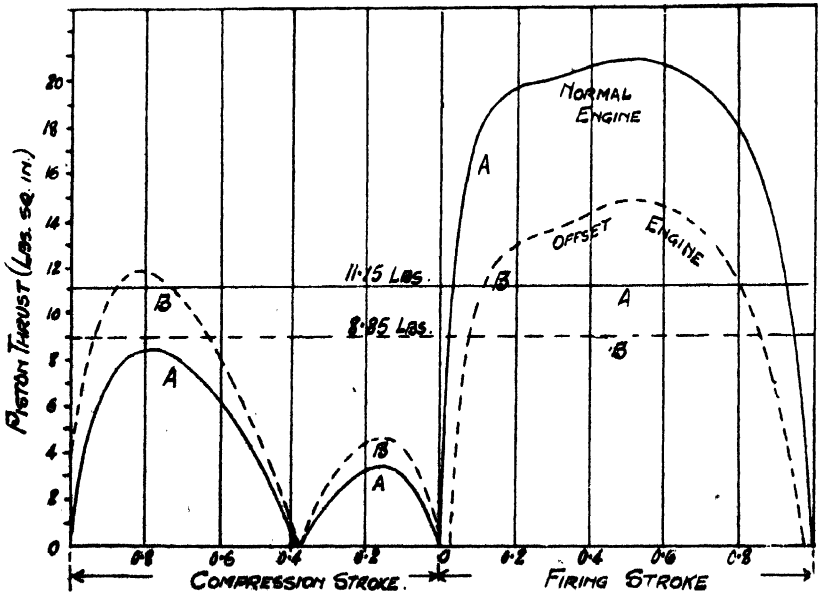


FIG. 47. PISTON THRUST CURVES FOR NORMAL AND OFFSET ENGINES

sides may be estimated from the mean areas of the piston-thrust diagrams for these two periods. The ratio in the present examples is 2·8 : 1.

Radial Engines. Master Connecting Rods

A problem confronting the designer of radial engines is that of finding the turning moment exerted on the master connecting rod by gas pressure on the articulated rods. In this connexion a graphical analysis has been made by Y. Hara and N. Kanzo* on the torques due to gas forces on a seven-cylinder radial engine of 4·63 in. bore and 4·63 in. stroke. The length of the master connecting rod was 8·8 in. and the average radius of the articulated connecting rod hinge pins was 2·2 in. An assumed typical indicator diagram of cylinder pressures was used.

The results of this analysis are shown in Fig. 48, for turning moments on a crank angle base. The master rod cylinder is considered as cylinder No. 1. From these results it is apparent that No. 6 cylinder provides the maximum torque. The resultant torque curve is shown by the full black lines.

An approximate formula, due to R. K. Mueller, for the torque exerted by an articulated connecting rod on a master connecting rod when the articulated rod hinge pin angle is equal to the cylinder angle, as is usually the case with radial engines, is as follows—

$$M = P\left(\frac{rr'}{l - r'}\right) \sin\theta + \sin(\alpha - \theta) \quad . \quad . \quad . \quad (1)$$

where M = torque in lb. in.

P = total gas pressure force on piston in lb.

r = crank radius in inches

r_1 = distance of the articulated connecting rod hinge pin centre from the crank pin centre, in inches

l = length of master connecting rod, in inches

θ = crank angle from master connecting rod top dead centre, in degrees, and

α = angle between master cylinder axis and articulated cylinder, in degrees

The results given by this approximate formula agree very closely with those from the graphical method, illustrated in Fig. 48, and the formula is recommended for torque calculations as being less complicated and less liable to error than the graphical method.

* "Master Connecting Rods," Prof. C. F. Taylor, *Journ. Society Automotive Engineers*, 1933.

An approximation to the maximum torque for any seven-cylinder, or double-row fourteen-cylinder, radial engine with articulated rod hinge pin angles equal to cylinder angles, which is sufficiently accurate for most engineering purposes, is as follows—

$$M_{max} = 1{\cdot}23\, P_{max} = b^2\left(\frac{rr'}{l - 4}\right)$$

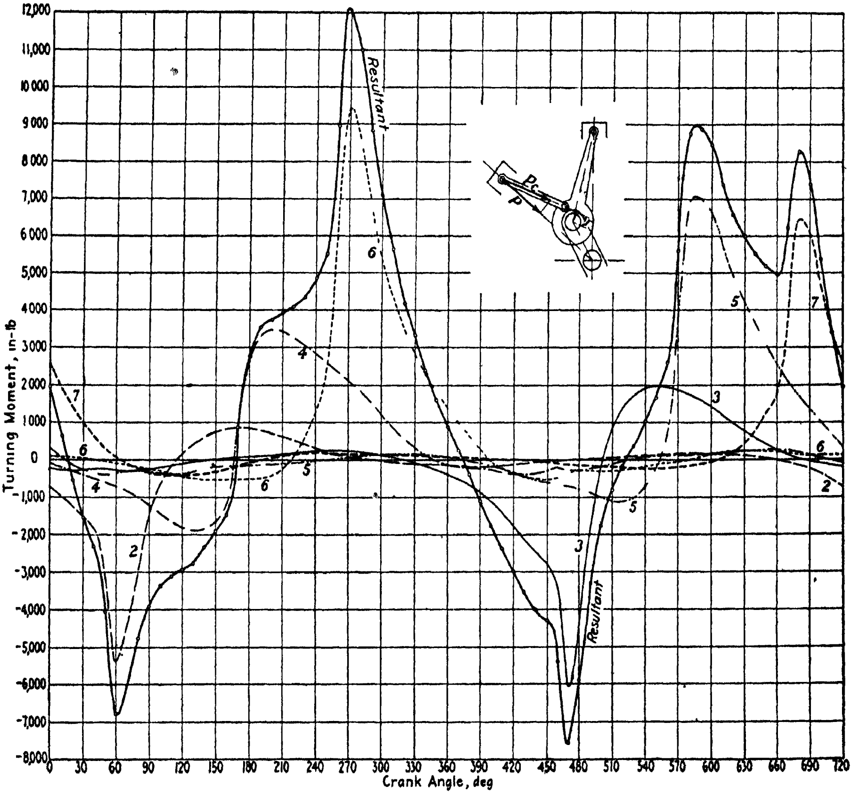


FIG. 48. TORQUE CURVES FOR RADIAL ENGINE (SEVEN-CYLINDER)

where P_{max} = maximum cylinder pressure in lb. per sq. in. and b = cylinder diameter, in inches.

For a *nine-cylinder engine* the corresponding relation is—

$$M_{max} = 1{\cdot}14\, P_{max} = b^2\left(\frac{rr'}{l - r'}\right)$$

It has also been shown that in any radial engine with articulated connecting rod hinge pin angles equal to cylinder angles, and with normal spark timing (maximum pressure at 10 to 15° after top dead centre), the maximum moment due to gas pressure will occur when

the crank is at the maximum pressure position of a cylinder whose axis is near 270° from the master connecting rod cylinder axis. This "critical" crank position for various numbers of engine cylinders is as follows—

TABLE VI

CRITICAL CRANK POSITIONS

Degrees After Top Dead Centre	No. of Cylinders
250 to 255	9
267 to 272	7
298 to 303	5
250 to 255	3

Measurement of Engine Torque

It has been shown how the actual torque at any part of the engine cycle can be estimated from a knowledge of the engine dimensions and the indicator diagram pressures, so that a theoretical diagram of torque values during a cycle can be obtained. Further, a simple formula has been given for calculating the mean engine torque during a cycle.

It is often necessary to be able to measure the engine torque, both in regard to *instantaneous* and also *mean* values. For the former purpose an instrument, known as a *torsiograph*,* is employed to give actual records of torque values during the cycle. The mean torque values can be obtained by three principal methods, namely, as follows.

(1) *From* b.h.p. *Measurements*. If the b.h.p. values are measured at various speeds, the mean torques at these speeds can be computed from the formula given on page 38.

(2) *From Dynamometer Measurements*. Most of the modern engine brakes or dynamometers belong to the torque-reaction type in which the framework of the power-absorbing member is not bolted down to the ground but is mounted in trunnions so that, unless restrained, it can rock or rotate about the axis of the dynamometer. The framework is, however, provided with a lever-arm, upon the end of which is a scale-pan or spring balance. During a b.h.p. test the end of the weighing lever-arm is loaded so that it "floats" between a pair of fixed stops. Under these circumstances the mean torque exerted by the engine can be measured by noting the balancing load on the lever-arm and multiplying this by the effective length of the lever-arm. Thus, it is the mean torque at each speed which is measured in these b.h.p. tests.†

(3) *From Torsiometer Readings*. In this method an instrument,

* *Vide* page 38.

† For a fuller account *vide Testing of High-speed Internal Combustion Engines*, A. W. Judge (Chapman & Hall, Ltd.), Third Edition.

known as a torsiometer or torquemeter, is employed to measure the torque. The principle of most torsiometers is that of measuring the difference between the angles of twist of a short shaft member coupled between the engine and the unit which it has to drive. The twist angle in question measured over a given length of shaft, enables the corresponding mean torque to be calculated from the elastic properties of the shaft, or obtained by applying a torque of known amount and

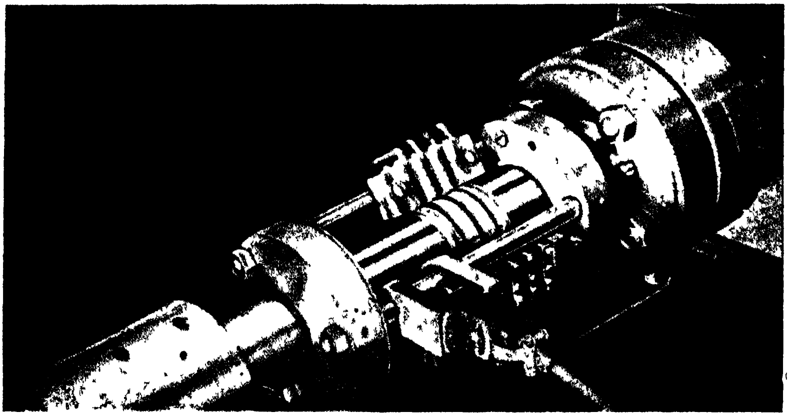


FIG 49. THE BALDWIN SOUTHWARK SR–4 ELECTRIC STRAIN GAUGE TYPE TORSIOMETER

measuring the angle of twist between the two reference planes, at right angles to the axis of shaft.

In one well-known torsiometer mirrors are fixed to the shaft at a given distance apart. These rotate with the shaft, but once every revolution pick up and reflect beams of light on to a semi-transparent scale. Any difference in the angle of twist is at once revealed by the relative displacements of the two light spots on the scale.

It is not possible, for space reasons, to describe these instruments,* but in passing mention may be made of the use of electric strain gauges on the surface of the shaft, to measure the angle of twist. A typical example is that of the SR–4 torsiometer* (Fig. 49), consisting of a short length of shaft built into the power transmission system containing the strain gauges and electrical pick-up cables, etc. From electrical measurements taken during runs the mean torque is readily ascertained. The *instantaneous torque values* can also be recorded on a strip chart pen-and-ink recorder. Readings can be obtained at speeds up to 60 ft. per sec. (surface) and torsional stresses up to 8000 lb. per sq. in.

* Baldwin Southwark Div. Philadelphia, U.S.A.

CHAPTER III

VALVE CAMS AND FOLLOWERS

Valve Acceleration

In connexion with the design of poppet valves for high-speed petrol engines, the question of the inertia forces due to the weight and acceleration (or deceleration) is of much importance in valve-spring and cam design.

Consider, first, the acceleration forces acting on the valve during the opening period. The rate of acceleration, other things being the same, will depend upon the design of the valve operating cam, as is shown later.

A form of cam which is much used is of harmonic or sine curve shape, this gives a fairly uniform acceleration.

If the lift of the valve be denoted by h ft., and the valve period be $\theta°$, then for an engine speed of N r.p.m. the time of valve opening is given by—

$$t = \frac{\theta}{360} \times \frac{60}{\mathrm{N}} = \frac{\theta}{6\mathrm{N}} \text{ sec.} \qquad . \quad . \quad . \quad (a)$$

The time taken for the valve to be lifted through its height h will be one-half of this amount, that is $\frac{\theta}{12\mathrm{N}}$ sec.

For a uniform acceleration through its lift h ft., the acceleration f is given by—

$$h = \tfrac{1}{2}ft^2$$

or

$$f = \frac{2h}{t^2} = 2 \times 12^2 \,.\, \frac{h\mathrm{N}^2}{\theta^2}$$

$$= 288 \,.\, \frac{h\mathrm{N}^2}{\theta^2} \text{ ft. sec.}^2 \qquad . \quad . \quad . \quad . \quad (b)$$

For example, if the valve lift $h = \frac{1}{4}$ in. $= \frac{1}{48}$ ft., N $= 2400$ r.p.m. and $\theta = 180°$.

Then $f = 288 \times \frac{1}{48} \times \left(\frac{2400}{180}\right)^2 = \frac{3200}{3} = 1066{\cdot}6$ ft. sec.^{2}

From this typical example of a high-speed engine valve, it will be seen that the mean acceleration is considerable.

Further, the greater the valve lift, the greater will be the acceleration for a given engine speed, also the longer the opening period, the smaller the acceleration.

Consider next the forces required to accelerate the valve, which must be provided by the cam.

If W = the weight of the valve with its collar, and cotter, including the push rod from the cam, in lb.

w = the weight of the valve spring in lb.

then $W + \frac{w}{3}$ = weight to be accelerated (approximately).

Hence, mean force F required to lift valve $= f \cdot \dfrac{W + \frac{w}{3}}{g}$

$$= \frac{288 \times h \times N^2\left(W + \frac{w}{3}\right)}{32 \cdot 19 \times \theta^2} \text{ lb.} \quad . \quad . \quad . \quad (c)$$

For example, in the case previously considered, the weight $\left(W + \frac{w}{3}\right)$ was 0·8 lb.

Then $$F = \frac{0 \cdot 8}{32 \cdot 19} \times \frac{3200}{3} = 26 \cdot 5 \text{ lb.}$$

The valve spring, which controls this acceleration force by preventing the valve from jumping off the cam during the acceleration period, should have a rather greater strength than the acceleration force, moreover, it should ideally give a rather greater compression strength at each part of the lift than the actual acceleration force at this lift. In other words, the curve of acceleration force-lift should really be the spring compression-lift curve.

During the deceleration period the spring must keep the valve parts in contact with the cam, and for this reason also the spring should be stronger than the acceleration force value.

The rate of acceleration in many cam designs is not uniform, so that the maximum value may be appreciably higher than that given by the constant acceleration method; this is another reason in favour of stronger springs.

From the results given in Equation (c) above, it will be evident that, in order to keep the acceleration and deceleration forces down to a minimum to reduce wear, noise, and power loss, it is necessary to keep the weight of the valves down as much as possible, and to increase the opening period defined by θ.

It is not possible to vary θ very much in practice, owing to considerations of volumetric efficiency and power output, but the weight should be minimized. The lift h can be kept down by utilizing larger diameter valves of small lift.

Some General Considerations

In the preceding example, the average acceleration only has been dealt with. If, however, the motion of a poppet valve be considered in detail, it will be seen that during the lifting period of the valve its velocity is increased from zero to some maximum value, the velocity then diminishing to zero when the valve reaches its fully-opened position, so that the valve is first accelerated to its maximum axial velocity and then decelerated to zero velocity. After passage through the full-lift position the valve is again accelerated and then decelerated to its zero or rest position.

The nature of the upward or lifting accelerations is determined by the shape of the cam and the lifting gear, e.g. whether direct, with a roller-ended tappet, or by means of a flat-faced or mushroom tappet.

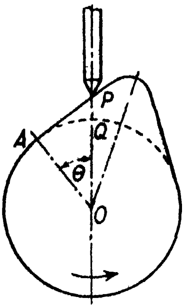


Fig. 50

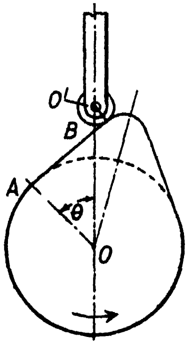


Fig. 51

Direct Lift Tappet. The simplest case is that of a tappet giving point contact, as in Fig 50, in which the cam is shown after having rotated through an angle θ from the cam base circle or no-lift position

If r = radius OA of cam base circle and $x = \text{OP} = \dfrac{r}{\cos\theta}$

$$\text{Then velocity of P} = \frac{dx}{dt} = \frac{r\sin\theta}{\cos^2\theta}\cdot\frac{d\theta}{dt} = w\,.\,r\,.\,\frac{\sin\theta}{\cos^2\theta}$$

$$\text{and acceleration of P} = \frac{d^2x}{dt^2} = w^2r\left(\frac{\cos^3\theta + 2\cos\theta\,.\,\sin^2\theta}{\cos^4\theta}\right)$$

$$= w^2r\left(\frac{2-\cos^2\theta}{\cos^3\theta}\right)$$

$$\text{The actual lift of the valve} = \text{PQ} = r\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

In these considerations it has been assumed that the face and flank of the cam are flat and tangential to the cam base and top or nose circles.

Roller-ended Tappet. In the case of a roller-ended tappet arrangement as shown in Fig. 51, the method of treatment* is as follows—

With the notation shown, let r = radius OA of cam base circle and

* "Experiments on Cams and Poppet Valves," G. E. Scholes, *Proc. Inst. Autom. Engrs.*, Vol. 16, p. 515.

a = radius of roller on tappet. It will be evident that the point of contact of the roller on the tappet and the cam face is not in line with the axis of the tappet and subtends an angle of $(\theta + \beta)$ on the cam shaft axis.

Then distance $x = OO_1 = OB + O_1B$

$$= \frac{r}{\cos\theta} + \frac{a}{\cos\theta} = \frac{r + a}{\cos\theta}$$

The valve lift $= x - (r + a)$

$$= (r + a)\left(\frac{1 - \cos\theta}{\cos\theta}\right)$$

It will be seen on comparison of this result with that of the previous example that the effect of the roller is to give $(r + a)$ instead of r, so that the expressions for the valve velocity and acceleration become—

$$\text{Valve velocity} = w(r + a) \cdot \frac{\sin\theta}{\cos^2\theta}$$

$$= w(r + a)\frac{\sin\theta}{1 - \sin^2\theta}$$

and $$\text{Valve acceleration} = w^2(r + a) \cdot \left(\frac{2 - \cos^2\theta}{\cos^3\theta}\right)$$

Flat-faced or Mushroom Tappet The valve lift, velocity, and acceleration characteristics are quite different if a flat-faced tappet is used instead of a roller one. Referring to Fig. 52, which shows a cam having a face and flank of radius R and a base circle of radius r, if the lift be denoted by x.

Then $$x = DH = OH - OD = EP - OD$$

$$= O_1P - O_1E - OD$$

$$= (R - r)(1 - \cos\theta)$$

$$\text{The valve velocity } \frac{dx}{dt} = w(R - r)\sin\theta$$

and $$\text{acceleration } \frac{d^2x}{dt^2} = w^2(R - r)\cos\theta$$

When the tappet is on the round nose of the cam—

Lift $= (r + l - r_1)\cos\theta_1 - (r - r_1)$

Velocity $= - w(r + l - r_1)\sin\theta_1$

Acceleration $= - w^2(r + l - r_1)\cos\theta_1$

There θ_1 = angular displacement of the tappet axis from the centre line of the cam.

Constant Velocity Cams

It is a comparatively simple matter to ascertain the shape of a cam to give any desired motion to the cam follower or tappet during the valve opening and closing periods.

One common type of cam is that which gives a uniform velocity of lift to the follower. Its shape can be determined graphically, assuming a constant velocity of rotation of the camshaft. Thus, for equal angular intervals of the cam, during the lifting curve the distance

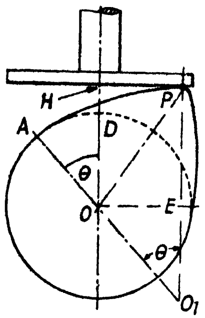


FIG 52 MUSHROOM TAPPET

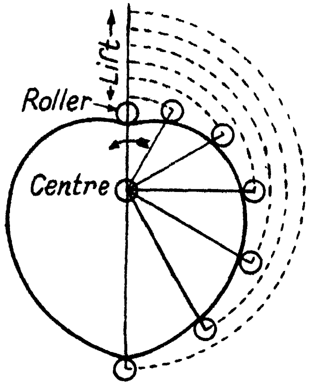


FIG 53. CONSTANT VELOCITY HEART-SHAPED CAM

from the centre of the cam base circle to the point of contact of the follower or roller-ended tappet will increase by equal increments so that the shape of the cam will be that of an *Archimedean spiral*, a polar curve giving equal increments in radii for corresponding equal increments in angle. The general shape of this cam is shown in Fig. 53; this is often referred to as a "heart-shaped" cam, and as illustrated gives a uniform velocity of lift and descent to the tappet; the latter may be either of the direct or roller-ended type.

Constant Acceleration Cams

In this type, the velocity must increase by equal amounts for equal time intervals. Since the cam rotates at a uniform rate, equal time intervals correspond to equal angles of rotation.

The cam radius on the lift portion or face must increase as the velocity and as the time, since distance = velocity × time.

Also, since with constant acceleration the velocity varies as the time, it follows that the lift or cam radius of the face will vary as the square of the time or square of the angle of rotation.

The curves of the cam face must therefore be of *parabolic shape*

for the acceleration and also the deceleration portions of the lifting face of the cam. Thus, if the cam face for the lifting portion be plotted on a linear cam circle base (Fig. 54), it consists of two parabolas AB and BC for the acceleration and deceleration periods, respectively, assuming equal acceleration and deceleration. One parabola is concave upwards, for acceleration, and the other concave downwards, for deceleration.

The constant acceleration cam has *certain specific advantages* for petrol engine valves, more particularly where maximum power outputs at high engine speeds are required, since the valve is lifted to its maximum position as quickly as possible. As compared with the usual flat and round-faced cams it gives a greater lift with the same diameter of base circle and same total angle of engagement; thus, for a given valve lift the camshaft dimensions can be reduced to a minimum.

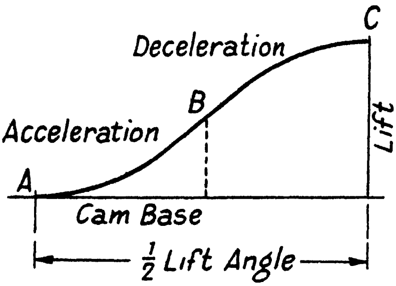


FIG. 54. CONSTANT ACCELERATION CAM SHAPE

The accelerating forces on the tappet are apt to be high with this type of cam, but the deceleration is a function of the valve spring force and the equivalent mass of the valve system

As an instance of the high accelerations experienced,* in one particular example of a valve with a lift of ½ in., the acceleration of the valve and tappet with the camshaft running at 1000 r.p.m. was about 3700 ft.-sec.^{2}, whilst the deceleration was 1600 ft.-sec.^{2}

The equivalent mass of the valve system was 1 lb., and the spring pressure required to maintain contact with the cam worked out at $\frac{3600}{32 \cdot 19} = 112$ lb. It is, however, necessary to arrange for a somewhat higher spring pressure than this in order to take into account valve stem friction in the guide.

The acceleration of the valve during the first portion of its lift is greater than the deceleration during the second portion. If the former value be taken as being K times the latter, and if the total cam operation angle be denoted by 2A, then

$$\text{Angular movement of cam during acceleration} = \frac{A}{K+1}$$

$$\text{and} \quad ,, \quad ,, \quad ,, \quad ,, \quad \text{deceleration} = \frac{KA}{K+1}$$

* *Vide* footnote page 65.

As the mean velocity is the same during acceleration and deceleration, the lift of the valve during these periods will be proportional to the time, i e. to the angle turned through by the camshaft. Thus, for a total valve lift l

$$\text{Lift during acceleration period} = \frac{l}{K+1}$$

$$\text{,, ,, deceleration ,,} = \frac{Kl}{K+1}$$

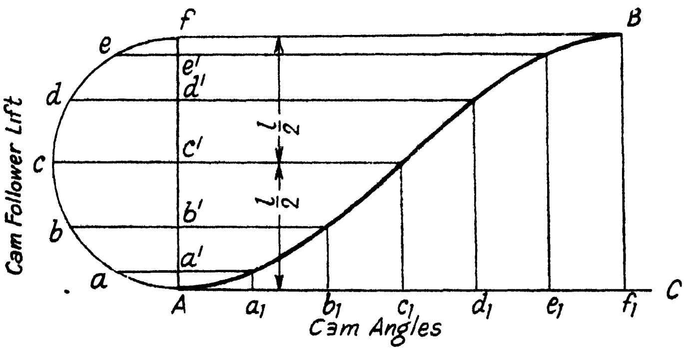


FIG. 55. HARMONIC MOTION CAM

The mean effective lift ratio can be shown to be equal to $\frac{2K+1}{3(K+1)}$. Thus, for equal acceleration and deceleration periods $K = 1$ and the mean effective lift ratio is 0·5 If $K = 2$, 3, 4, or infinity, the corresponding values for this ratio are 0·555, 0·583, 0·600, and 0·666, respectively, a result which indicates the small gain for values of K greater than about 3.

Harmonic Motion Cams

As distinct from the constant velocity and constant acceleration types of cam there is another which gives a somewhat similar motion to that of a petrol engine piston, namely, a maximum acceleration at the beginning and end of the movement. The cam follower starts gradually, the velocity increasing to a maximum about midway in its stroke and thereafter decreasing to zero at the end of the lift, before again accelerating on the downward or return stroke and repeating the first half of the motion but in the reverse direction.

The shape of the harmonic motion cam lift curve is readily found by projection from the constant speed circle Acf (Fig. 55), in which Af represents the lift of the cam follower. The half-circle Acf is divided into an equal number of parts, say, six, at a, b, c, d, and e, and these

are projected on to the diameter Af at a', b', c', d', and e', respectively. Each of the lifts Aa', $a'b'$, $b'c'$, etc., represents the distances moved through by the cam follower in equal intervals. If, therefore, the base line AC be divided into six equal intervals Aa, a_1b_1, b_1c_1, etc., and ordinates be erected on these equal to the cam follower lifts, Aa', Ab', Ac', etc., the shape of the harmonic motion cam curve for the lifting portion is obtained on a linear base. The latter is readily convertible into a cam base-circle base, in order to obtain the true shape of the cam. The curve shown in Fig. 55 is for the lift portion of the cam; that of the return part will be of similar or "looking-glass" image shape.

With this shape of cam the angular movements of the cam during acceleration and deceleration, from A to B, will be equal.

If the cam rotates through an angle α whilst the follower is lifted through its complete rise, i.e. from A to f (Fig. 55), and the constant cam velocity is N r.p.m.

Then, Angular velocity $w = \dfrac{360\pi N}{60\alpha}$ radians per sec.

$$= \frac{6\pi N}{\alpha}$$

The maximum acceleration at A or f is equal to $\dfrac{w^2 l}{2}$, where l is total lift of the follower.

Thus, Maximum acceleration $= \dfrac{(6\pi N)^2 l}{2\alpha^2}$ ft. sec ^{2}

If W is the weight, in pounds, of the valve components which are accelerated, then

$$\text{Maximum force on follower} = \frac{W}{g} \times \text{max. acceleration}$$

$$= \frac{W(6\pi N)^2 \,.\, l}{2g\alpha^2} \text{ lb.}$$

This force, which occurs at the end of the lift, must be less than the sum of the valve component's weight* and the spring forces, or the follower will leave the cam. Actually, the force in question is a retarding one, corresponding to a negative acceleration, or deceleration, and it is the value of the force that must be exerted by the follower, valve weight and the spring pressure to maintain the follower on the cam.

Forces on Cam and Follower

When a cam operates the valve tappet or cam follower, with the aid of a roller, as shown in Fig. 56, the force exerted by the cam is normal

* Assuming the valve axis to be vertical.

to its surface at the point of contact A. If this force is denoted by F, then the vertical component will be F cos ϕ and the horizontal one F sin ϕ. The force F cos ϕ will be equal to the resultant of the vertical forces due to (1) the acceleration of the valve components, (2) the weight of the latter, and (3) the valve spring pressure. This resultant force induces a compressive stress in the cam follower.

The force F sin ϕ tends to cause a bending action on the cam follower and a side thrust on the guide in which the follower reciprocates. In considerations of valve-actuating cams it is usual to refer to the angle ϕ (Fig. 56) as the *pressure angle*; the latter varies in value according to the position of the cam.

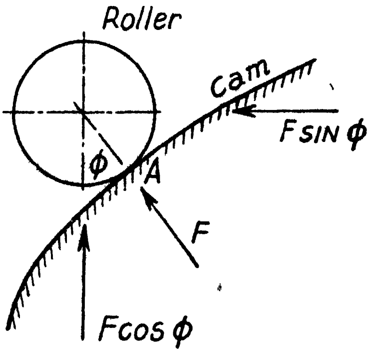


FIG. 56 FORCES ON CAMS AND FOLLOWERS

Notes on Cams and Followers

The pressure angle ϕ (Fig. 56) should not, at its maximum value, exceed about 30°, although in special circumstances 40° is permissible. If these values are exceeded excessive bending action on the cam follower is liable to occur. By making the cam base circle as large as possible, the value of the pressure angle can be reduced to a minimum, but the size of the cam will be increased accordingly.

In regard to the use of rollers on cam followers the smaller the diameter of the roller the closer will the actual cam profile approach to the theoretical one for a direct-lift or pointed end follower.

An important consideration in connexion with the roller is that it must have a smaller radius of curvature than any part of the cam contour. If larger than this, it will not follow the cam and will bridge over the region of smaller radius of curvature. For a similar reason the cam curve should not have any abrupt changes of direction, since it may not be possible for the roller to follow these, i.e. to maintain line contact with the cam all the time. The roller diameter should be kept as small as practical requirements dictate and the base circle as large as expedient in order to obtain a motion of the follower approximating to that of the direct lift one on the theoretical cam curve.

Although the subject of cam layouts is outside the scope of the present considerations, it may be mentioned that in cam design it is usual to imagine the cam to be stationary and to consider the follower to rotate about the cam as it moves in and out. The cam base circle is divided into a number of equal parts, e.g. 8 or 12, which are

numbered consecutively, and the corresponding numbered positions of the follower are then obtained by means of arcs about the cam circle centre. Alternatively, if the positions of the follower are given, the shape of the cam can be obtained by the reverse process. Examples of these methods are given in books on mechanism and in some books on geometry.

Experimental Determination of Valve Motion

Although the general characteristics of valve operating mechanisms can be determined by mathematical methods on the lines of those

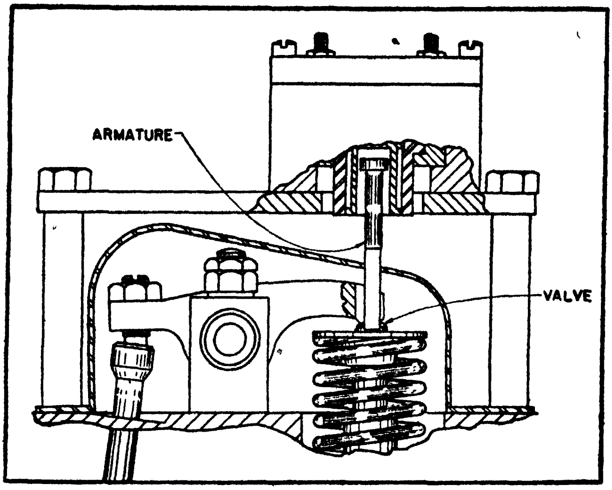


FIG. 57. MAGNETIC TYPE OF PICK-UP FOR VALVE MOTION STUDY

previously considered, the actual results are often found to differ somewhat from the theoretical ones. This is due mainly to the elasticity of the components other than the valve spring under high-speed operating conditions in the engine. In order to check the results of valve train calculations it is desirable to make accurate measurements of the valve's motion in the engine itself when running under its own power. In this way the lift of the valve, plotted on a crankshaft or camshaft angle or a time base, can be compared with the theoretical lift curve obtained from geometrical considerations of the cam itself.

A satisfactory method of studying the problem of valve motion is to employ an electrical pick-up mounted on the valve stem as shown in Fig. 57.* Movements of the pick-up moving member are translated into electric potentials which can be applied across one pair of electrodes

* "Recent Research in Poppet Valve Train Design," C. Voorhies, *Autom. and Aviation Industries*, 1st October, 1944.

of a cathode ray tube so as to produce diagrams on the cathode ray tube or oscillograph screen. The magnetic type of pick-up used consists of a permanent magnet and a coil of fine wire forming the armature or moving member. In moving up and down with the valve's motion between the magnet poles the armature generates an electric current which is proportional to the velocity of the valve. From the velocity diagram both the lift and acceleration diagrams can be obtained. Permanent records of the velocity curves are made photographically.

The strain gauge is another type of pick-up for use with an oscilloscope. It is based on the principle of passing a current through a length of fine steel wire which is fixed to the valve stem or its extension. The actual deflection, or strain of the valve member, is then determined from measurements of the resistance of the wire, whilst changes of length during the valve motion cycle can be studied from changes of potential on the oscillograph. By calibrating the particular set-up with known loads the loads occurring during valve operation can be measured.

Another method employed for the study of valve motion is that of the high-speed cinematograph camera, operating at 2000 pictures or more per second.

An apparatus for measuring the positions of the valve at selected positions of the valve cam, known as a stroborometer, is shown diagrammatically in Fig. 58. It employs a contact breaker on the camshaft or crankshaft which serves to give flashes to the neon light tube at the precise camshaft position selected to measure the valve stem position, this is effected with the aid of the eyepiece, reflecting mirror, *slits in the fixed and moving (valve stem extension "flag")* parts, the neon tube behind the slit and the micrometer.

The neon light is always obstructed except at the instant when the slot in the valve stem extension flag is aligned with the two fixed slots. The position of the light flash is altered by means of the hand crank shown on the left in Fig. 58. When a flash occurs at the disc (on left), a corresponding flash occurs in the neon tube on the valve unit. The slots in the housing are then adjusted up or down by means of the micrometer until the light passes through all three slots. The micrometer reading then gives a measure of the valve position for the corresponding position of the cam as determined from the graduated disc on the left in Fig. 58. Valve position micrometer readings are obtained in this manner from different settings of the graduated disc or cam angle, so that a valve position-cam angle graph can be plotted, with lift readings correct to within one-thousandth of an inch.

Fig. 59* is a reproduction of an actual valve motion curve obtained in this manner. Curve A is the theoretical or slow speed graph

* *Vide* footnote page 72.

obtained at 600 r.p.m., it is identical with the original valve lift design curve. Curve *B*, obtained at a speed of 3000 r.p.m., indicates certain variations of the actual from the theoretical curve. It shows that the valve opening is delayed at the higher speed, owing to deflection effects. Further, the valve attains its full opening about 10° after the theoretical period and closes before the estimated time (as shown at (3)) with a series of "bounces" or deflections (4) instead of smoothly as at low speeds.

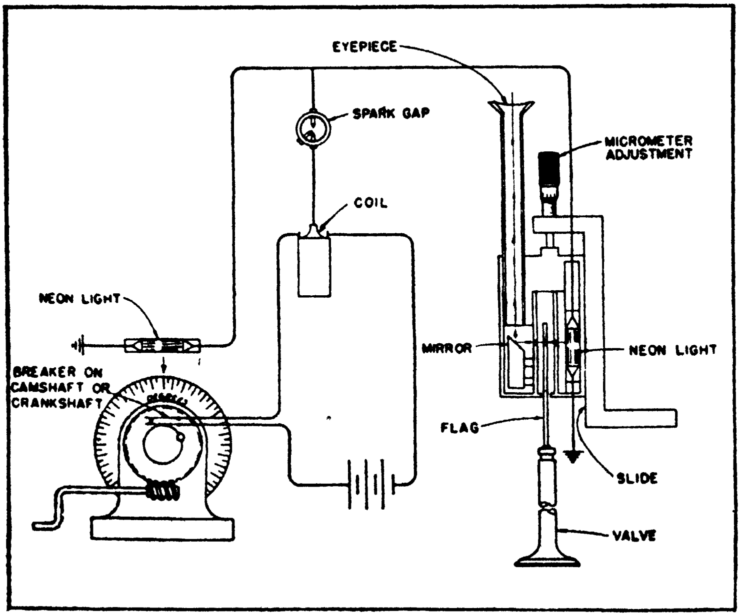


FIG. 58. THE STROBOROMETER METHOD OF VALVE MOTION ANALYSIS

The valve velocity graph for Curve *A*, shown below, indicates the cam positions for maximum and zero valve speeds.

Among the other interesting facts revealed by experimental investigations of valve gear operation under actual working conditions are those relating to *valve accelerations*. Thus, it has been shown that the acceleration may attain a value as high as 18,000 ft. sec.^{2} at the maximum over-speed for a side-valve engine and 8000 ft. sec.^{2} for an overhead valve one.

The *stresses in the valve tappet* or follower, for various shapes of cam nose (maximum lift part of cam), have been measured, and it has been found that unless the radius of the nose exceeds a certain value there are excessive stresses between the cam and follower. Thus, as the nose radius is reduced the stress for a given spring load increases;

many causes of tappet and cam failure have been traced to the cam nose having too small a radius.

Fig. 60 illustrates the manner in which the valve tappet face stress changes with the valve cam nose radius and also the corresponding values of the valve open spring loads. The variation of the valve tappet face stress with the eccentricity of the nose is also indicated in the same diagram.

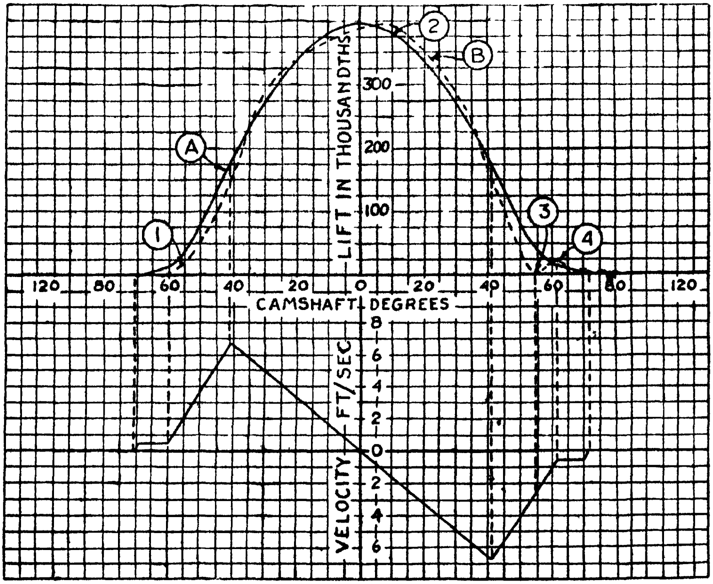


FIG 59. SLOW AND HIGH-SPEED VALVE MOTION CURVES

In regard to the effect of acceleration, the stress at maximum acceleration is usually low, failure occurs on the nose, since the lower the acceleration the higher the stress on the nose for a given lift. In the original article referred to at the foot of page 72, charts for determining the cam nose stresses, spring loads, tappet face stresses, etc., for most design purposes are given.

Another fact revealed by the experimental method is that if the axis of the camshaft (or the face-width of cam) is not exactly perpendicular to the axis of the follower, the actual face stress is increased; a small angular misalignment usually causes the face stress to become doubled or even trebled in value.

In the case of flat-faced tappets errors due to angular misalignment or deflection can be obviated by making the cam contact face of the tappet of spherical instead of flat shape.

Notes on Poppet Valves

In modern high-speed petrol and Diesel engines the performance is governed largely by the area of the valves and their lifts, one function of the valves and their operating cams being to afford the maximum possible breathing capacity to the engine, i.e. the highest value for the volumetric efficiency.

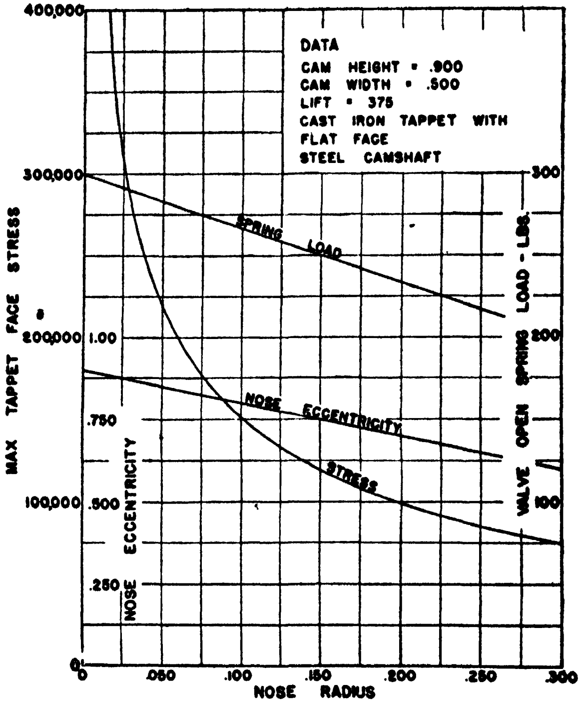


FIG. 60. VALVE TAPPET FACE STRESS VALUES

The *cam design* determines both the period of valve opening during the working cycle and the effective valve lift over this period.

The area of the valve and its lift govern the quantity of gas or charge flowing out of or into the engine during the exhaust and inlet operations.

In practice the *area of the valve* is limited by cylinder dimensions, so that it cannot exceed more than a certain percentage of the piston area, unless special shapes of combustion head are employed.

In the case of overhead valve engines designed for maximum output, e.g. aircraft engines, it is usual to employ two inlet and two exhaust valves. The reason for this will be apparent from Fig. 61, which represents a plan view of a circular cylinder head, of the overhead valve pattern. If a single inlet and single exhaust valve are employed,

the maximum permissible diameter D is less than one-half the cylinder diameter; usually it lies between 0·43 and 0·45 of the cylinder bore.

If, however, instead of a single valve D, two valves of diameter *d*, giving the same marginal clearances be used, then it can readily be shown that the total valve area can be increased by about 33 per cent; the diameter *d* is about 80 per cent of D. Thus, for the same engine speed, the velocity of the gases through the valve ports can be reduced appreciably and the volumetric efficiency increased.

The *period of valve opening* should be as long as possible, relatively to the cycle period, but it is necessary for practical reasons to effect a compromise between several factors, although the ultimate aims are (1) to obtain the maximum weight of fresh charge into the cylinder, and (2) to expel the maximum quantity of burnt gases before the inlet valve opens again.

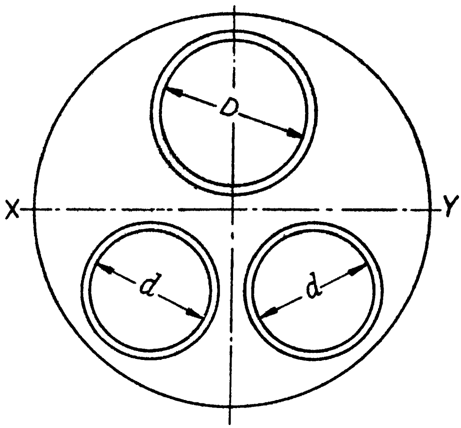


FIG. 61. COMPARISON OF TWO AND FOUR VALVES PER CYLINDER

The *exhaust gases*, by virtue of their momentum, will continue to leave the exhaust port when the piston has reached the end of its scavenging stroke, so that it is advantageous to leave the port open for some appreciable time after the dead centre has been reached.

Again, *when the inlet valve is opened*, in the case of a high-speed engine, it requires a definite small interval before the fresh charge can be given its proper speed of flow, on account of its inertia, so that instead of opening the inlet valve on its top dead centre it is better to commence at some position before the piston reaches its top centre. Thus, for maximum charging and exhausting effect, it is necessary for both the exhaust and inlet valves to be open together; this is termed *valve overlap*. In the case of two-cycle engines, with inlet and exhaust ports at opposite ends of the cylinder, the overlap is often arranged to produce a negative pressure in the exhaust port, thus ensuring improved scavenging of the burnt gases.

Fig. 62 illustrates the valve timing diagram of a British poppet-valve aircraft engine of high efficiency and performance. In this example the exhaust valve closes at 40° of crank angle after top dead centre (T.D.C.), whilst the inlet valve opens at 29° before T.D.C., so

that the overlap period is 69°. It will be noted that the inlet valves, of which there are two per cylinder, have a total opening period of 256°, whilst the two exhaust valves are both open for 296°; this longer period ensures the best possible clearance of the exhaust gases and results in a cooler cylinder and less dilution of the fresh charge.

Valve and Performance Data

In regard to *the valve head dimensions*, these are limited in overhead valve engines by the cylinder bore to a maximum of about 0·45 of

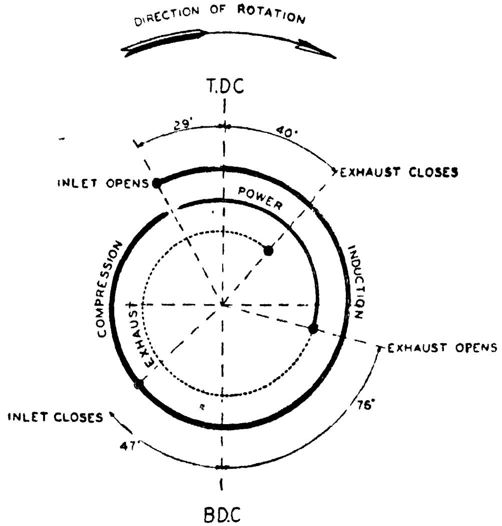


FIG. 62. VALVE TIMING DIAGRAM FOR OVERHEAD VALVE AIRCRAFT ENGINE

the cylinder bore for two valves per cylinder. In the case of side-valve engines an examination of a large number of American car engines —which favour the side valve—of recent date shows that the ratio of valve head diameter to cylinder bore lies between 0·410 and 0·510; as mentioned previously, it is possible to employ valves of larger diameter with special shapes of combustion chamber, of which those used in side-valve engines are examples.

The average ratio for twenty-five different makes of car engine was 0·442.

The mean gas velocity through the valve ports ranged from 250 to 370 ft. per sec. for the maximum engine speeds, namely, 3000 to 4000 r.p.m.

The valve area per square inch of piston area was from 0·100 to 0·130, the average being about 1·20.

The valve area per cubic inch of piston displacement was from 0·029 to 0·037, the average being about 0·033.

The h.p. per cubic inch of piston displacement ranged from 0·385 to 0·505, the average being about 0·450.

It should be pointed out that in comparing engine performances on a basis of valve or piston area, some allowance should be made for the stroke-bore ratio, since for the same piston displacement or cylinder capacity the stroke-bore ratio of an automobile engine may vary considerably. In practice it is found that this ratio, for modern engines, varies from 1·00 to 1·35.

To take this factor into account it has been suggested that the results should be corrected to a standard stroke-bore ratio, that of unity being a convenient one.*

Thus, if d = bore and s = stroke, then the piston displacement, V, is given by—

$$V = 0{\cdot}7854d^2s$$

and if $d = s$

$$V = 0{\cdot}7854d^3$$

or

$$d = \sqrt[3]{1{\cdot}273V}$$

If x is the diameter of any engine part under consideration and d' the bore, then

$$\frac{x}{d'} = \frac{x}{\sqrt[3]{1{\cdot}273V}}$$

Thus, the formula for average (side) valve diameter becomes

$$x = 0{\cdot}442\sqrt[3]{1{\cdot}273V}$$

To ascertain the valve diameter for any new engine it is only necessary to multiply the piston displacement of one cylinder by 1·273 and find the cube root of the product. This value, multiplied by 0·442, gives the valve diameter required. The range of cylinder bores to which this formula applies is from 2·5 in. to 3·5 in.

Effective Valve Area

In the case of poppet valves having flat faces and seatings, it can readily be shown that for the effective area of the lifted valve to be equal to the port area or diameter of the valve head, the lift must be equal to one-quarter of the valve head diameter.

With poppet valves having the usual 45° face angle, the effective valve area, with the inlet valve fully opened, is taken as the area of the truncated conical surface of which a line from the small diameter of the valve to the seat is the generatrix.

* "Engine Comparisons," E. G. Ingram, *Automotive Industries*, 1st August, 1940.

The area in question can readily be shown to be as follows

$$\text{Effective area} = \pi\left(\frac{\mathrm{D}h}{\sqrt{2}} + \frac{h^2}{2\sqrt{2}}\right)$$

$$= 3{\cdot}1416\,(0{\cdot}707\mathrm{D}h + 0{\cdot}3535h^2)\ \text{sq. in.}$$

where D = port diameter in in. and h = valve lift in in.

The effective area ranges from about 0·70 sq. in. for a valve of 1·28 in. head diameter to about 1·43 sq. in. for one of 1·72 in. diameter. A valve of $1\frac{3}{8}$ in. to $1\frac{1}{2}$ in. head diameter would have an effective area of 1·00 to 1·200 sq. in., respectively, in the case of a modern automobile engine.

The following formula represents *the average valve area* for twenty-five different makes of modern car engine—

$$A = 0{\cdot}093\left(\sqrt[3]{1{\cdot}273\mathrm{V}}\right)^2\ \text{sq. in.}$$

The *average valve head diameter* $\mathrm{D} = 0{\cdot}460\sqrt[3]{1{\cdot}273\mathrm{V}}$.

CHAPTER IV

VIBRATIONS IN ENGINES

One of the most important subjects with which the engine designer is concerned is that of vibrations, their causes, types, and magnitudes. With a fairly complete knowledge of the fundamental principles of engine vibrations it is then possible to minimize their effects or to obviate these altogether.

In the case of high-speed internal combustion engines of the reciprocating type, the origin of the vibrations that are experienced is the exploding charges in the cylinders, which in turn produce variations of engine torque; reciprocating forces due to the pistons and connecting rods, valves and their springs; and centrifugal forces of the rotating parts.

Unbalanced forces in an engine may give rise to (1) direct or linear type vibrations (2) those caused by rocking couples or (3) rotating unbalanced forces, e.g. centrifugal forces.

In general, engine vibrations tend to produce stresses (or increased stresses) in the engine components concerned; fatigue effects due to the alternating stresses set up; audible effects, i.e. engine noises; transmitted vibrational effects to the engine supports, and increased bearing loads in the case of shafts experiencing centrifugal effects. Other minor effects of engine vibrations include the tendency of nuts to unscrew, oil and fuel pipes or their connexions to fracture, exhaust pipes to crack, and electrical cables or their connexions to chafe or break.

In the present chapter the subject of vibrations of engines and machines in general is dealt with from the viewpoint of the basic principles involved and the various conclusions obtained from an analysis of the results are considered from the practical application aspect.

Some Considerations on Vibrations

In engineering practice, vibrations are associated with periodic changes of force due to various causes, including unbalance of rotating or reciprocating members. Usually, the changes consist of similar cycles of force fluctuation, repeated regularly. The amplitude of the movement caused by the force variation during a cycle is usually defined as one-half the range, although some authorities consider the whole range of movement.

The commonest type of vibration occurring in engineering is the *simple harmonic* one, and corresponds to the movement of the piston in a petrol-type engine, with connecting rod of infinite length.

Referring to Fig. 63, a particle P is shown rotating about a fixed centre O, with uniform velocity V. If a perpendicular PQ be dropped on to the horizontal line AB, then the motion of the point Q along AB will be a simple harmonic one of amplitude* OA = OB = r, and the frequency will equal that of the rotation of P.

Using the notation given in Fig. 63, and denoting the distance of Q from the centre O by x at any time t after commencement from O, then

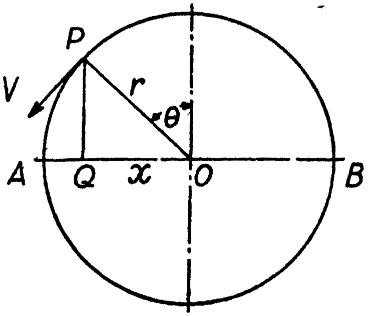


FIG. 63

$$x = r \sin \theta \quad . \quad . \quad (1)$$

and $$t = \frac{\theta}{2\pi N}$$

Hence $x = r \sin 2\pi Nt = r \sin nt$, where $n = 2\pi N$.

Also, since $V = 2\pi N r$, $n = \frac{V}{r}$.

From (1) it follows that x is a maximum when $\theta = 90°$ and equals r.

If the displacements of Q from the centre are plotted on a time, i.e. crank-angle, base, the resulting curve will be as shown in Fig. 64 (*A*), where displacements above the base line are positive and correspond to those to the left of O, whilst negative values are to the right.

The velocity of Q at any time t is given by differentiation of the displacement expression (1).

Thus, $$V_Q = \frac{dx}{dt} = r \cos \theta \, . \frac{d\theta}{dt} = r \cos nt \, . \frac{d\theta}{dt}$$

But $r \, . \frac{d\theta}{dt} = V$, the linear velocity of P, so that

$$V_Q = V \cos nt \quad . \quad . \quad . \quad . \quad (2)$$

This expression shows that the maximum velocity occurs when $\theta = 0°$, i.e. when Q is passing through the centre O, and zero velocity, corresponding to $\theta = 90°$, at the extremes of the path, i.e. at A and B. The velocity-time curve therefore takes the form shown in Fig. 64 (*B*). *The acceleration of* Q is obtained by differentiation of the velocity in respect to t.

Thus, $$f_Q = \frac{dV_Q}{dt} = \frac{d^2x}{dt^2} = -V \sin \theta \, . \frac{d\theta}{dt}$$

* It is now usual to define the amplitude OA as the semi-range and the full range AB as the double-amplitude.

but $r \cdot \frac{d\theta}{dt} = V$, so that

$$f = -\frac{V^2}{r} \cdot \sin \theta \quad . \quad . \quad . \quad . \quad (3)$$

By substituting for V the value $V = nr$, the expression becomes

$$f = -n^2 \cdot r \sin nt \quad . \quad (4)$$

In this expression, $r \sin nt$ represents the distance of the point Q from the centre O, i.e. the distance x, so that the acceleration of the particle moving in simple harmonic motion is proportional to its distance from the centre O, being a maximum at the extremes of its amplitude and zero at the centre of its path. The acceleration-time curve is given in Fig. 64 (*C*).

In general, the vibrations that occur in engineering practice can usually be expressed as a simple harmonic motion as follows—

$$x = r \sin (nt + c)$$

where c is a term that takes account of the phase or moment at which the vibration commences.

Most complex vibration systems* met with by the engineer can be resolved into a series of simple harmonic motions of the following form—

$$\begin{aligned} x = a_0 &+ a_1 \sin (nt + c_1) \\ &+ a_2 \sin (2nt + c_2) \\ &+ a_3 \sin (3nt + c_3) \\ &+ \ldots . \text{ etc.} \end{aligned}$$

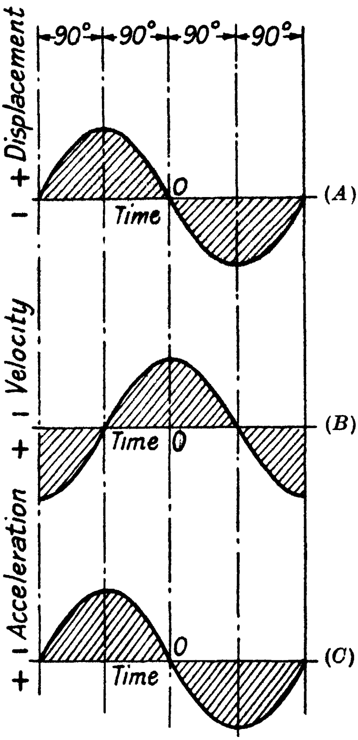


FIG. 64. SIMPLE HARMONIC MOTION DISPLACEMENT, VELOCITY AND ACCELERATION CURVES

where a_0, a_1, a_2, a_3, etc., and c_1, c_2, c_3, etc., are constants and, as before, $n = 2\pi N$, where N is the cycle frequency.

It will be observed that the complete vibration consists of a fundamental one of frequency N, together with a series of vibrations of frequencies 2N, 3N, etc., occurring with different amplitudes and at different phases. The displacement curve ordinates of all these simple

* This subject is dealt with fully in *Waveform Analysis* ("Interpretation of Periodic Waves, including Vibration Records"), R. G. Manley (Chapman & Hall, Ltd., London).

harmonic vibrations, when added algebraically, give the ordinates of the complex vibration displacement curve.

Free Vibration Frequencies

When an engineering part possesses elasticity it is theoretically capable of performing natural vibrations in at least one direction. Thus a bar of steel, suspended at one end and struck a lateral blow at the other will vibrate transversely; or if struck in an axial direction it will tend to vibrate longitudinally.

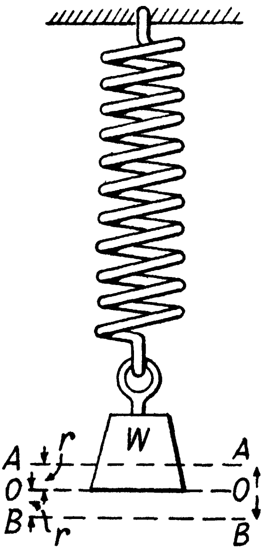


FIG. 65. VIBRATION OF LOADED SPRING

The extreme positions of the bottom of weight are denoted by AA and BB, the amplitude being r.

The simplest example of a freely-vibrating body is that of a helical spring fixed at its upper end and loaded with a weight W (Fig. 65) at its other end. If the weight is depressed a little below its mean stationary position 00 and then released, the spring and weight combination will vibrate in a vertical sense. In this case the restoring force exercised by the spring is proportional to the displacement of the weight, as measured from the mean or stationary position, so that the weight performs simple harmonic vibrations, in the absence of any other influencing factors, e.g. damping effects.

If x denotes the displacement at any time t from the mean position, then

$$x = r \sin nt \quad . \quad . \quad . \quad . \quad (5)$$

The velocity,

$$\frac{dx}{dt} = rn \cos nt$$

and the acceleration

$$\frac{d^2x}{dt^2} = f = -n^2 r \sin nt \quad . \quad . \quad . \quad . \quad (6)$$

Eliminating sin nt from (5) and (6), the following relation is obtained

$$\frac{\text{displacement } (x)}{\text{acceleration } (f)} = -\frac{1}{n^2}$$

The negative sign may here be neglected, and since $n = 2\pi N$, this reduces to the form

$$N = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}} = \frac{1}{2\pi}\sqrt{\frac{f}{x}} \quad . \quad . \quad (7)$$

This expression gives the *free vibration frequency* or *frequency constant* of an elastic member vibrating without any restraint.

Stiffness and Vibration Frequency

In engineering considerations it is usual to define the *stiffness* of an elastic member as the force required to cause unit displacement, or, for angular applications, the value of the torque that will produce unit angular displacement. In general, the stiffness of a part is calculated from its shape, dimensions, and the mechanical properties of the material of which it is composed.

If the stiffness for linear displacements be denoted by k, then the force F necessary to cause a displacement x will be kx.

Since the force F necessary to produce an acceleration f in a mass M is given by

$$F = Mf$$

then the acceleration $$f = \frac{k \, . \, x}{M}.$$

and $$\frac{f}{x} = \frac{k}{M}$$

Substituting this value for $\frac{f}{x}$ in (7), the vibration frequency becomes

$$N = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\text{stiffness}}{\text{mass}}} \quad . \quad . \quad . \quad (8)$$

Here, N = vibrations per second, where the stiffness k is in poundals per foot deflection and the mass M in pounds.

Energy Considerations

The maximum energy of vibration occurs when the velocity is a maximum, and in a simple harmonic motion, this has been shown to occur, when the body is passing through its mean position. Since the velocity at any time t is given by rn cos nt the maximum value, for $nt = 0$, will be rn, and the kinetic energy is then given by

$$K.E = \frac{Mr^2n^2}{2}$$

When the velocity is zero the *strain energy* is a maximum, and when a maximum, namely, at the mean position of vibration, this energy falls to zero.

The velocity is zero when cos nt is zero, i.e. when nt is 90° and $x = r$. The value of the strain energy is then equal to $\frac{r^2k}{2}$, the kinetic energy then being zero.

Since these two energy values must be the same, it follows that

$$\frac{r^2k}{2} = \frac{Mr^2n^2}{2}$$

from which
$$n^2 = \frac{k}{M}$$

and
$$n = \sqrt{\frac{k}{M}}$$

or
$$N = \frac{1}{2\pi}\sqrt{\frac{k}{M}}, \text{ as before.}$$

Static Deflection Formulae

In place of the mass M in the preceding formula (8) can be written $\frac{W}{g}$ where W is the weight in lb. and g the acceleration due to gravity, i.e. 32·2 ft. sec.^{2} or 386·4 in. sec.^{2}

Substituting for M in formula (8)—

$$N = \frac{1}{2\pi}\sqrt{\frac{g \cdot k}{W}} = \frac{1}{2\pi}\sqrt{\frac{386{\cdot}4k}{W}}$$

Since, when a weight of W lb. is suspended from an elastic member it will cause a deflection of $d = \frac{W}{k}$, then

$$N = \frac{1}{2\pi}\sqrt{\frac{386{\cdot}4}{d}} \text{ (inch-second units)}$$

A convenient practical formula derived from the above is as follows—

$$N' = \frac{188}{\sqrt{d}}$$ vibrations per min., where d = static deflection in in

Effect of Damping

In the previous considerations it has been assumed that the body was free to vibrate without any restraining influences; it will therefore perform simple harmonic motions indefinitely.

In practice, however, the movements are resisted by internal and external effects, such as mechanical hysteresis, friction, air, or other fluid resistance, etc. Thus, during each vibration from the initial one, some of the energy of vibration is absorbed, with the result that the amplitudes diminish progressively and, eventually, the vibration dies out in the manner indicated in Fig. 66. By introducing artificial

damping effects, e.g. fluid dashpots, the vibrations can be reduced in amplitude very quickly or may be prevented altogether, if desired.

The *critical degree of damping*, by artificial or natural means, occurs when the initially displaced elastic body just returns to its zero or mean position, without passing beyond it, as indicated by time-displacement line ABC in Fig. 66. This type of damping is of particular importance in certain types of needle-indicating instruments. It should, however, be pointed out that the subject of critical damping

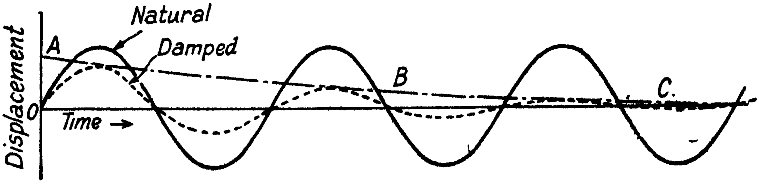


FIG. 66. UNDAMPED AND DAMPED VIBRATIONS

is a somewhat controversial one, for the elastic body may just return to the mean position without passing beyond it if the damping is far greater than the critical degree. Further, if the damping is critical the body may, according to the initial conditions, i.e. displacement and velocity at $t =$ zero, pass beyond the mean position. When metallic members are set into vibration by external influences the principal damping factor is that due to the internal friction of the metal itself.

In the case of undamped simple harmonic vibrations, for the example of the vertically oscillating spiral spring shown in Fig. 65, the force causing the acceleration at any displacement x is $\frac{W}{g} \cdot \frac{d^2x}{dt^2}$, and if, as before, the stiffness is k, i.e. a force of k lb. extends the spring by 1 in., then the force at x will be kx, and since these two forces are equal and opposite *the equation of motion* may be written as

$$\frac{W}{g} \cdot \frac{d^2x}{dt^2} + kx = 0 \qquad . \quad . \quad . \quad . \quad (9)$$

In most common instances of damping the restraining or damping force is proportional to the velocity of the vibrating body, i.e. the damping force may be represented by $b \cdot \frac{dx}{dt}$ where b is a constant. *The equation of motion for damped oscillations* is therefore of the following form—

$$\frac{W}{g} \cdot \frac{d^2x}{dt^2} + b \cdot \frac{dx}{dt} + kx = 0 \qquad . \quad . \quad . \quad (10)$$

If the motion is an angular one, it takes the corresponding form of

$$I \cdot \frac{d^2\theta}{dt^2} + c \cdot \frac{d\theta}{dt} + d \cdot \theta = 0 \quad . \quad . \quad . \quad (11)$$

where I = moment of inertia, θ = angular displacement, and c and d are constants.

The general solution of equation (10) is of the following form—

$$x = A\varepsilon^{-\alpha t} + B\varepsilon^{-\beta t} \quad . \quad . \quad . \quad . \quad (12)$$

where $f = \frac{bg}{2W}$, $n^2 = \frac{gk}{W}$, and α and β are the roots of the following auxiliary equation—

$$m^2 + 2fm + n^2 = 0, \text{ the roots being as follows—}$$

$$m = -f \pm \sqrt{f^2 - n^2} \quad . \quad . \quad . \quad (13)$$

If f is greater than n, damping is greater than *critical* and there is therefore no vibration.

If f is less than n, then the quantity under the root sign is negative and the values of m are complex. The exponentials are then expressible in terms of real sines and cosines.

If f is equal to n, the condition of *critical damping* occurs.

In the case of a damped simple harmonic motion, if $x = 0$ when $t = 0$, it can be shown that the solution of the equation takes the following form—

$$x = A\varepsilon^{-ft} \sin \sqrt{n^2 - f^2} \cdot t \quad . \quad (14)$$

The effect of damping has therefore been to introduce the multiplier ε^{-ft} into the expression for simple harmonic vibration displacement.

If the undamped frequency is N_1 cycles per sec. and the damped frequency is N, the following relation holds—

$$N^2 = N_1^2 - \frac{f^2}{4\pi^2} \quad . \quad . \quad . \quad . \quad (15)$$

For small degrees of damping the value of f is less than unity and the damped and undamped frequencies are very nearly equal, but for higher degrees of damping, such as those of oil dashpots, f is appreciably higher than unity and the undamped frequency N_1 is greater than N, i.e. the damped period of vibration is greater than the undamped one.

It should be pointed out that the actual solution of the equation (12) depends upon the values and signs of the roots of the auxiliary equation (13), so that it is necessary to consider each example of damped vibration on its own merits.

Forced Vibrations

In connexion with the study of engines and machines that are subject to vibrations the general problem to be considered involves:

(1) The natural vibrational frequency and amplitude of the vibrating member; (2) the existing damping effects upon this frequency and amplitude; and (3) the effect of superposed, or forced, vibrations due to operational causes, upon the behaviour of the vibrating member.

A typical instance of this problem is that of a petrol engine crankshaft which has its own natural period of angular vibration, the latter being subjected to damping influences due principally to internal friction or mechanical hysteresis effects, but also to coupling members and other causes. The crankshaft is, in addition, given a series of periodic torque impulses due to the firing of the charges in the individual cylinders, so that it may be regarded as an elastic member subjected to forced vibrations of an angular kind about the axis of the shaft.

The analogous linear example to this is that of a vertical spiral spring having a weight at its lower end, but instead of being fixed at its upper end it is connected to a vibrating member. Fig. 67 shows a spring having a weight W in the form of a disc (for air or fluid damping purposes) at its lower end B and with its upper end A connected to a rotating crank Q which can be operated at various speeds, corresponding to different forced vibration frequencies. By varying the speed of the shaft S the effects of the relative frequencies of the impulses due to S and of the spring itself can be studied.

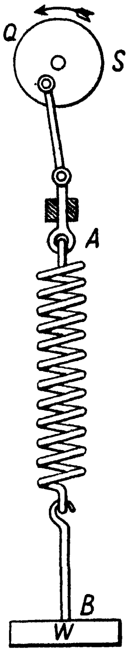


FIG. 67
FORCED LINEAR VIBRATIONS

The equation of motion for a vertical spring with damping is deducible from equation (10), by considering that when the weight is displaced downwards by the distance x the point of support is also displaced downwards by a distance y, where y is a function of the time. Thus, if the point of support is given a simple harmonic motion of frequency Q, then

$$y = a \sin qt, \text{ where } q = 2\pi Q$$

The equation of motion (10) then becomes modified as follows—

$$\frac{W}{g} \cdot \frac{d^2x}{dt^2} + b \cdot \frac{dx}{dt} + kx = ky \quad . \quad . \quad . \quad (16)$$

Thus, if $y = 0$ the original damped simple harmonic motion remains.

If the effects of damping are disregarded, the term $b \cdot \frac{dx}{dt}$ can be omitted, and this will enable the results of forced vibrations to be

studied in a more simple manner, without affecting the general conclusions arrived at, and Equation (16) then becomes

$$\frac{d^2x}{dt^2} + \frac{g}{\mathrm{W}} \cdot kx = \frac{g}{\mathrm{W}} \cdot ky \quad . \quad . \quad . \quad (17)$$

If the upper spring support be given a simple harmonic motion in phase with that of the loaded spring, such that $y = a \sin qt$, where a and q are constants, then (17) can be written in the following manner.

$$\frac{d^2x}{dt^2} + n^2 \cdot x = n^2 a \sin qt \quad . \quad . \quad . \quad (18)$$

where $n = \sqrt{\frac{gk}{\mathrm{W}}}$. But the natural vibration period of the spring has been shown to be given by the relation $\mathrm{N} = \frac{1}{2\pi}\sqrt{\frac{gk}{\mathrm{W}}}$, from which

$$k = \frac{4\pi^2 \cdot \mathrm{N}^2 \cdot \mathrm{W}}{g} \quad . \quad . \quad . \quad . \quad (19)$$

and since $n^2 = \frac{gk}{\mathrm{W}}$,

$$k = \frac{\mathrm{W}n^2}{g} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

From (19) and (20) it follows that $n = 2\pi\mathrm{N}$.

The solution of Equation (18) is as follows—

$$x = \frac{n^2 a}{n^2 - q^2} \cdot \sin (qt + \alpha) \quad . \quad . \quad . \quad (21)$$

This result shows that the effect of imposing a simple harmonic vibration upon the upper end of the spring is *to produce a forced vibration* of the weight W having a frequency equal to that of the imposed vibration and an amplitude c which is given by

$$c = \frac{n^2 a}{n^2 - q^2} = \frac{1}{1 - \frac{q^2}{n^2}} \cdot a \quad . \quad . \quad . \quad (22)$$

that is, the amplitude is $\frac{1}{1 - \frac{q^2}{n^2}}$ times that of the imposed vibration.

It will be observed that the forced vibration amplitude depends upon the ratio of q to n or, since $n = 2\pi$ times the natural frequency N of the spring, i.e. upon the ratio of the imposed vibration frequency to the natural frequency.

Fig. 68 illustrates the manner in which the amplitude of the forced vibration depends upon the ratio of the impressed frequency $\frac{q}{2\pi}$ to the natural frequency $N = \frac{n}{2\pi}$, i.e. $\frac{q}{n}$. It follows from Equation (22) that when $q = n$ the value of the amplitude a is infinitely great and we have *a natural resonance effect*, whereby a small imposed

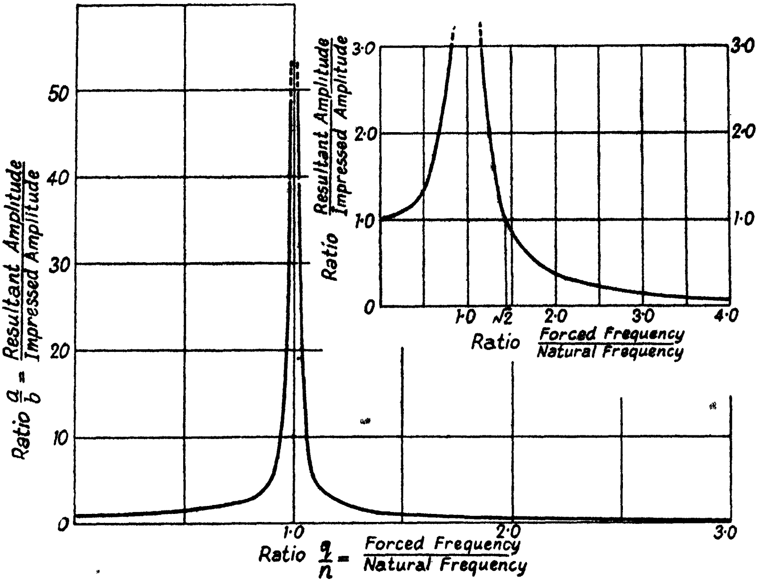


FIG. 68. ILLUSTRATING RESONANCE EFFECT DUE TO FORCED VIBRATIONS ON AN ELASTIC MEMBER

vibration of a frequency equal to the natural frequency of a vibrating member can, theoretically, produce an amplitude of infinite amount.

In practice, the effect of damping influences limits the maximum value of the resonant amplitude, but, nevertheless, the resonant amplitudes in certain engineering parts or structures are apt to produce stresses well above the safe working values. Typical instances are those of steel bridges (more particularly the suspension type) where periodic vibrations due to traffic or troops on the march may cause serious resonant movements. Another example is that of the crankshafts of multi-cylinder engines, in which the resonance effects of torsional vibrations have caused fracture; further reference to this subject is made in Chapter V. The effect of sea waves of certain frequencies upon the rolling motion of ships may become serious if

resonance between the wave and rolling frequencies occurs. Numerous other instances of possible resonance effects in engineering applications might be cited.

In every practical case of forced and natural vibrations, the general rule for engineering structures and components is, by a suitable choice of stiffness, to arrange the natural frequency so that the working or impressed frequency is remote from either the natural frequency or a harmonic of the natural frequency. If the engineering part has to pass through resonant conditions in attaining its normal working speed or frequency, the passage through the resonant frequencies should be as brief as possible.

Some General Considerations on Forced Vibrations

From the results obtained by the particular solution of Equation (18) for the case of the resultant vibrations produced by an external force having a simple harmonic motion, certain general conclusions may be deduced.

Referring to Fig. 68, which is drawn to scale for the example considered, it will be observed that when the forced or impressed frequency is a small fraction of the natural frequency, i.e. when the ratio $\frac{q}{n}$ is less than about 0·4, the forced vibration of the weight W is very nearly the same as that of the point of support A. Thus, for ratios of $\frac{q}{n}$ increasing from zero to 0·5 the amplitude of the forced vibration increases from 1·0 to 1·3.

The most rapid increase in the forced vibration amplitude occurs close to the value of $\frac{q}{n} = 1$, when resonance conditions result. Beyond the resonant frequency $\left(\frac{q}{n} = 1\right)$ the amplitude magnification curve is not symmetrical with the portion before the resonant frequency as shown by the inset graph in Fig. 68, in which the amplitude amplification scale is enlarged ten times.

When $q^2 = 2n^2$, the expression (22) shows that the resultant and impressed amplitudes are equal, i.e. $\frac{q}{n} = \sqrt{2}$, as shown in Fig. 68. As the impressed frequency increases beyond this value the amplitude of the forced vibration diminishes rapidly, until when the impressed frequency is many times the natural frequency the motion of the weight W (Fig. 67) is very small. If damping occurs the weight W remains practically undisturbed.

The effect of damping in the case of forced vibrations is to reduce the

amplitude of the forced vibration over the whole range of impressed vibration frequencies as indicated by the dotted line graph in Fig. 69. As mentioned previously, the presence of damping effects results in a definite limitation of the resonant amplitude; this is indicated by the peaked portion of the dotted curve in Fig. 69.

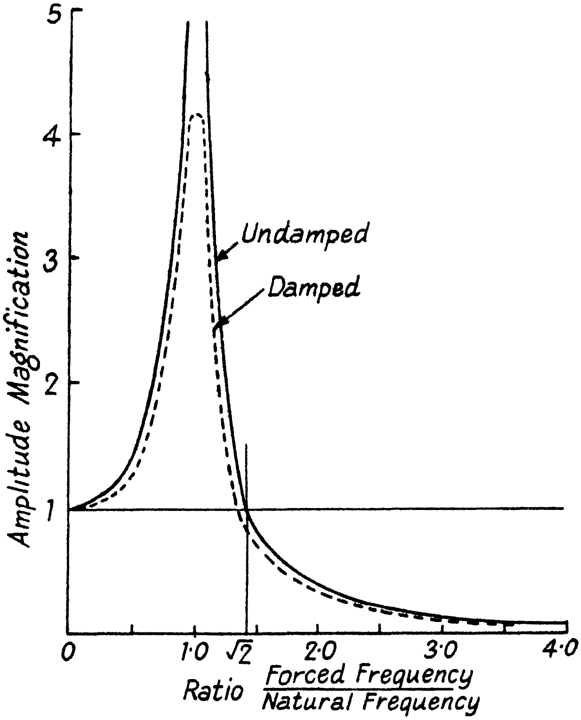


FIG. 69. EFFECT OF DAMPING IN FORCED VIBRATIONS

Phase of Forced Vibrations

As the impressed frequency given to the support A of the spring (Fig. 67) increases, progressively, in value from zero the forced vibration is at first in phase with it, but for values of $\frac{q}{n}$ up to unity the forced vibration gradually tends to lag behind the impressed one, until when $\frac{q}{n}$ equals unity, i.e. for resonant conditions, it is 90° behind, i.e. *in quadrature*. There is a fairly rapid increase in the phase lag on either side of the resonant position, after which the lag slowly increases, until for high multiples of the natural frequency, the forced

vibrations lag 180° behind the impressed ones. It follows that the forced vibration can be represented by the equation

$$x = A \sin (qt - \alpha) \quad . \quad . \quad . \quad . \quad (23)$$

where α is the phase lag; or if the external force is denoted by $f = B \sin qt$, then the value of the force in the spring is

$$f_1 = B \,.\, m \,.\, \sin (qt - \alpha) \quad . \quad . \quad . \quad (24)$$

where m is the magnification factor, i.e. the ratios of the amplitude of the force in the spring to that of the external force.

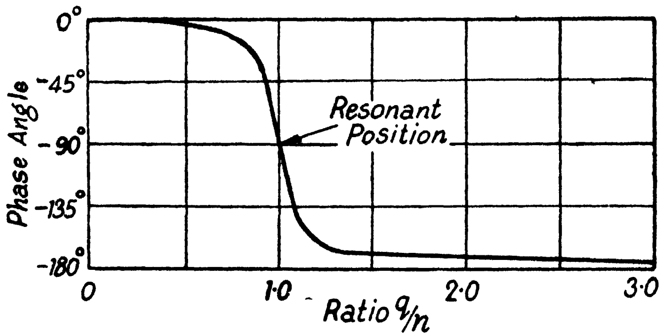


FIG. 70. PHASE RELATIONSHIP BETWEEN NATURAL AND FORCED VIBRATIONS OF A BODY

(See also Fig 68)

It can be shown that the value of the phase lag angle α is given by

$$\tan \alpha = \frac{1}{m_0} \cdot \frac{n^2}{n^2 - q^2} \cdot \frac{q}{n} \quad . \quad . \quad . \quad (25)$$

where m_0 is the magnification when $q = n$ for damped vibrations and is taken as a measure of the damping effect in the system. In this expression n is 2π times the natural frequency for the undamped system and q is 2π times the frequency of the impressed vibration.

In these considerations it has been assumed that the equation of motion is similar to that given in (16), for a damping force proportional to the velocity as represented by the term $b \,.\, \frac{dx}{dt}$ in (16).

If the damping loss is known for any given amplitude of vibration, the value of m_0 is deducible from the following relation

$$m_0 = 2\pi \,.\, \frac{E_s}{E_d} \quad . \quad . \quad . \quad . \quad (26)$$

where E_s is the maximum strain energy of the system and E_d is the energy loss by damping for a cycle.

Application of Results to Vibrating Spring

If the results of this analytical study be applied to the vibrating system shown in Fig. 67 it will be seen that for small impressed frequencies at the point of support A of the spring, the movements of W are very nearly the same as A, but as the frequency of A is slowly increased the weight W tends to lag a little behind A until at the resonant frequency, i.e. when $q = n$, it is 90° behind, so that the weight W is in its middle position when the external force is a maximum. As the frequency of A is still further increased the weight W lags further behind until at higher frequencies it is nearly at its top position when the external force is at its maximum value in the downward sense.

The manner in which the phase of the forced vibration of W changes as the frequency of A is increased is illustrated in Fig. 70, from which it will be noticed that the greatest change in phase occurs in the vicinity of the resonant position. At low frequencies the displacement and force are in phase, whilst for high frequencies they are out of phase.

CHAPTER V

TORSIONAL OSCILLATIONS IN ENGINES

THE fact that there are fluctuations in the torque output of the engine, and indeed of each cylinder, is responsible for several distinct vibration effects to which designers must perforce give consideration, in the interests of safety and efficiency.

The most important of these effects are those due to torsional oscillations of the crankshaft and connected systems. In these oscillations the various parts of the shaft system suffer periodic displacements relative to each other. As a result, the crankshaft, transmission, etc., are subjected to alternating stresses; the propeller (in the case of aircraft engines) has fluctuations imposed upon its otherwise uniform rotation, these fluctuations setting up alternating bending stresses in the blades, and in reduction gears there is the possibility of severe stressing due to the combined action of actual torque reversal and backlash.

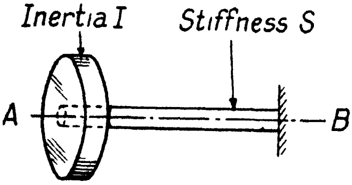


FIG. 71. TORSIONAL PENDULUM

Of recent years the theory of vibration, and the practical technique of vibration engineering, have been developed so extensively that it is not possible here to treat the subject in very great detail. A brief account of the salient features has been included, however, and references to the literature have been given so that the reader may pursue the subject in greater detail.

The Torsional Pendulum

The diagram, Fig. 71, depicts one of the simplest types of torsional vibrating system. A body, capable of rotation about the fixed line AB, is carried at one end of a light circular shaft, the other end of which is fixed rigidly.

If the body is rotated through a small angle about AB, and then released, it performs a twisting or torsional vibration about the original position, in just the same way as the mass in Fig. 67, page 89, vibrates when it is pulled downwards and then released.

The periodic time of this vibration is given by

$$t = 2\pi\sqrt{\frac{I}{S}}$$

where I is the moment of inertia of the body about AB, and S is the

torsional stiffness of the shaft, which is the torque required to be applied at the free end to produce a deflection of one radian there.

The periodic time can be expressed in terms of the constants of the shaft; in practical units the formula is then

$$t = \sqrt{\frac{1{\cdot}040 Il}{d^4 C}}$$

where t = periodic time in seconds,

d = diameter of shaft in inches,

C = modulus of rigidity of the material of the shaft, in lb./in.^{2},

l = length of shaft in inches,

and I = moment of inertia of body, in lb. in.^{2}

The reciprocal of the periodic time is the natural frequency of torsional vibration.

If an alternating torque T sin qt, i.e. a torque with maximum magnitude T and frequency $q/2\pi$ vibrations per second, is applied to the body, it is found that the amplitude of vibration depends upon the ratio of the impressed frequency and the natural frequency, just as in the case of the linear vibration discussed on page 89.

If the impressed frequency is small in comparison with the natural frequency, the amplitude of vibration is practically the same as that which would be obtained if the heavy body were absent and the torque acted solely on the shaft, and the displacement is in phase with the applied torque.

For impressed frequencies which are very great in comparison with the natural frequency, the resulting vibration is very small, and is anti-phased with respect to the torque, so that it is a maximum in one direction when the torque is a maximum in the other direction.

As the frequency of the applied torque is gradually increased from a small value, the amplitude of vibration increases, at first slowly and then more rapidly, until as the natural frequency is approached the amplitude increases very rapidly and the resonance effect is seen (Fig. 68, page 91). The amplitude diminishes as the frequency is increased beyond the natural frequency, as can be seen from the diagram.

The variation of the phase-lag of the resulting displacement behind the applied torque is exactly the same as in Fig. 70, page 94.

It will be seen that there is a precise analogy between the behaviour of this simple torsional pendulum under the action of an applied alternating torque and that of the system in Fig. 67 under the action of an applied alternating force.

Extension of System

The actual torsional systems encountered in practice, and consisting of the engine crankshaft and transmission or propeller, are more complicated than the torsional pendulum, and they are acted upon by a more complicated set of torques.

Disregarding for the moment the possible existence of reduction gears, flexible propellers, and similar special units, the torsional system may be idealized into a set of rigid inertias (usually visualized as

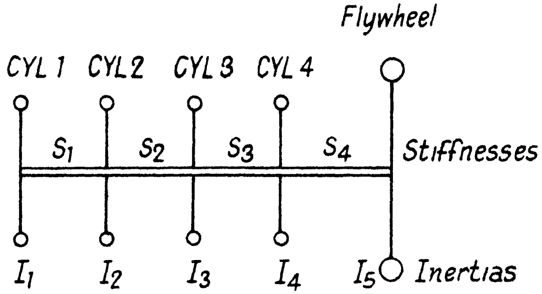


FIG 72. EQUIVALENT SYSTEM REPRESENTING ENGINE

flywheels) connected by sections of light, flexible shafting. Thus a normal four-cylinder automobile engine with flywheel is represented as shown in the diagram, Fig. 72. Each inertia represents the polar inertia of a crank or of the flywheel, the crank inertias being calculated to allow for the effects of accelerations in the connecting rods and pistons, and the stiffnesses of the connecting shafts are calculated from their physical dimensions. (Detailed accounts of the methods of calculating these quantities are given by Ker Wilson, see reference 1 on page 138.) In place of one inertia mounted on one shaft, the other end of which is rigidly fixed, there are five inertias joined by four connecting shafts.

Acting upon this system there are a number of alternating torques at various frequencies; these are the fundamental components and harmonics of the torque output of each cylinder, the frequencies of which are multiples of the firing frequency, i.e. multiples of half the rotational speed of the crankshaft in the case of four-stroke engines. Certain of these harmonics, as will be seen later, are particularly important.

Instead of one natural frequency, the shaft system has many, and if the relation between the operating speed of the engine and the natural frequencies is such that the frequency of one of the important harmonics is the same, or nearly the same, as one of the natural frequencies, the magnification effects of resonance are experienced, and severe torsional vibrations are set up.

The avoidance of resonance is the keynote of vibration engineering. While it is true that serious vibrations can and do occur in off-resonance conditions, the great majority of dangerous vibrations are rendered dangerous by the resonant magnification effect.

In order to be able to avoid resonance, it is clearly necessary to know the natural frequencies of the system, and also the frequencies of any large alternating torques acting upon it.

Determination of Natural Frequencies

Unfortunately, simple formulae, such as that given on page 96 for the torsional pendulum, cannot be given for the more complicated systems corresponding to actual engines. An entirely different method of approach, which is still however completely practical in conception, has been adopted by the leading manufacturers.

Another reason for the adoption of this method, in the case of aircraft engines, is the fact that the flexibility of the propeller blades has a very profound effect upon the natural frequencies. This matter is discussed later, here it is sufficient to note that it is extremely desirable to have a method of calculating a certain characteristic function for the engine, and of calculating or determining experimentally a similar function for the propeller, and to be able to determine the natural frequencies *of the combination* by a study of the relation between these two functions.

Dynamic Stiffness and Effective Inertia

Neglecting the effects of hysteresis in the shaft and other damping agencies, the amplitude of the alternating torque at any frequency that must be applied to the outer end of the shaft in the diagram, Fig. 71, page 96, in order to produce a torsional oscillation of unit amplitude at that frequency, is S. That is to say, the dynamic stiffness of the shaft is the same as its static stiffness.

Considering now the effect of the inertia of the body, let its angular displacement from the mean position be θ. For a vibration at the frequency of $q/2\pi$ cycles per second, this displacement can be expressed in the form

$$\theta = a \sin qt$$

where a is the amplitude. The acceleration of the body is obtained by differentiating this expression twice, and is found to be

$$\frac{d^2\theta}{dt^2} = -aq^2 \sin qt$$

Consequently, the amplitude of the torque that must be applied in order to maintain unit amplitude of vibration of the body is $-Iq^2$,

the minus sign indicating that the vibration is anti-phased with respect to the applied torque.

Since the stiffness of the shaft and the inertia of the body operate simultaneously, in order to maintain the complete system in a state of torsional oscillation of unit amplitude it is necessary to apply a torque of amplitude $S - Iq^2$. Thus if the applied torque has an amplitude T and a frequency $\frac{q}{2\pi}$, the amplitude of the resulting vibration of the body is given by

$$a = \frac{T}{S - Iq^2} = \frac{T/S}{1 - q^2 I/S} \quad . \quad . \quad . \quad (1)$$

(Compare with Equation 22, page 90.)

When q equals $\sqrt{S/I}$ the formula indicates an infinite amplitude of response, which is, however, limited in practice by the action of damping forces, as in the case of linear vibrations.

The quantity

$$Z = S - Iq^2 \quad . \quad . \quad . \quad . \quad (2)$$

is termed the "dynamic stiffness" of the torsional pendulum, with respect to torques applied at the outer end. It is precisely analogous to a static stiffness, since it is the amplitude of applied torque required to produce unit amplitude of torsional displacement.

Two important properties of the dynamic stiffness must be noted. First, it is a function of the frequency of the applied torque, since the expression contains q. Secondly, it refers to torques applied at a definite part of the system, and to displacements at that point.

The natural frequency of the system is determined by the value of q, which gives zero value to Z: this corresponds to an infinite response (neglecting damping) to a finite applied torque.

The "effective inertia" of the pendulum is the amplitude of alternating torque that must be applied in order to maintain a vibration with unit amplitude of *acceleration*, since *Torque* is *Inertia* × *Acceleration*. Since for the frequency $\frac{q}{2\pi}$ vibrations per second the amplitude of acceleration is the product of the amplitude of displacement and the factor $(-q^2)$, it follows that the effective inertia I_e is given by

$$I_e = -\frac{Z}{q^2} = I - \frac{S}{q^2} \quad . \quad . \quad . \quad (3)$$

The effective inertia of the pendulum may be interpreted as the inertia of a flywheel (regarded as mounted on a short shaft which runs freely in bearings), the response of which to an applied alternating torque is the same as the response of the pendulum to the same torque.

The curves in the diagram, Fig. 73, show how these quantities vary with the frequency, for the particular case in which $I = 386$ lb. in.^{2} or 1 lb. in. sec.^{2}, and $S = 1 \times 10^6$ lb. in./radian. It can be seen that at very low frequencies the dynamic stiffness Z is nearly the same as the stiffness of the shaft, and that at very high frequencies the effective inertia I_e is nearly the same as the inertia of the body. In other words, at very low frequencies the stiffness of the shaft is the main controlling

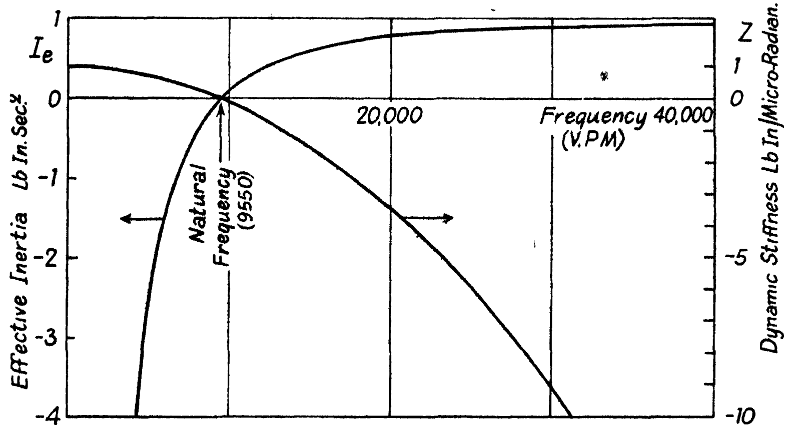


FIG. 73. DYNAMIC STIFFNESS AND EFFECTIVE INERTIA CURVES FOR TORSIONAL PENDULUM

influence on the magnitude of vibration, while at very high frequencies this function is taken over by the inertia of the body.

Tuning Inertia and Stiffness

For a frequency of 5730 vibrations per minute the effective inertia in this particular case is $-$ 686 lb. in.^{2} Thus, if the inertia of the body is increased by 686 lb. in.^{2}, the effective inertia of the modified pendulum would be zero at this frequency; in other words, 686 lb. in.^{2} is the value of the inertia which must be added in order to *tune* the system to resonance at 5730 vibrations per minute. This value is known as the "tuning inertia" for that frequency, and in general the tuning inertia is equal in magnitude, but opposite in sign, to the effective inertia. For frequencies greater than the natural frequency (9550 v.p.m.) the tuning inertia will evidently be negative, so that the inertia of the body must be decreased in order to increase the natural frequency.

Similarly, the dynamic stiffness at 5730 v.p.m. is $0{\cdot}64 \times 10^6$ lb. in./radian. If the stiffness of the shaft is decreased by this amount, the modified pendulum will have zero dynamic stiffness at 5730 v.p.m.,

so that this will be the new natural frequency. The amount by which the stiffness must be altered in order to tune the system to resonance at any frequency is termed the "tuning stiffness" for that frequency.

From graphs such as those in the diagram it is very easy to ascertain the effects of varying either the inertia of the body or the stiffness of the shaft, or both.

Owing to the close relation between dynamic stiffness and effective inertia, expressed in Equation (3), it is only necessary in any particular calculation to consider one of these quantities The choice depends largely upon the nature of the problem.

Application to Practical Cases

The general methods outlined above can easily be extended to the more complicated systems. Consider the four-cylinder automobile engine and flywheel represented in Fig. 72, page 98. The natural frequencies are the frequencies for which (neglecting the action of damping forces) the dynamic stiffness is zero, that is to say, if the system is caused to vibrate torsionally by the application of an alternating torque at any point, the resulting amplitudes of vibration will be infinitely great (neglecting damping) when the frequency of the applied torque equals any one of the natural frequencies.

It can be shown* that, provided the damping forces are comparatively small, as they normally are in engines, their influence upon the natural frequencies can be ignored, and also that their influence upon the amplitudes of vibration is negligible except in the immediate vicinity of a resonant condition, where the infinite amplitudes are limited to finite values. For practical purposes it is therefore permissible to neglect damping forces in the calculation of natural frequencies and vibration amplitudes, a procedure which greatly simplifies the calculations.

Denoting the inertias and shaft stiffnesses as in the diagram, suppose that a torque $T_5 \sin qt$ is applied to the flywheel I_5. The calculation aims at determining the resulting amplitude a_5 of torsional vibration at the flywheel; thence the ratio $\frac{T_5}{a_5}$, or the dynamic stiffness at the flywheel, is obtained, and the natural frequencies are determined by the values of q for which the value of this ratio is zero.

Assume that the amplitude at I_1 is a_1. Then the amplitude of torque, T_1, that must be applied there to maintain this vibration of the inertia is $-I_1q^2a_1$ (see page 99). Since this torque is in fact applied via the shaft S_1, the amplitude a_2 of the inertia I_2 can be found in terms of a_1;

* R. G. Manley, *Fundamentals of Vibration Study* (Chapman & Hall, 1942), p. 54.

for the torque transmitted by the shaft is the product of the twist in the shaft and its stiffness, i.e.

$$T_1 = -I_1q^2a_1' = S_1(a_2 - a_1)$$

whence
$$a_2 = a_1\left(1 - \frac{I_1q^2}{S_1}\right) \quad . \quad . \quad . \quad . \quad (4)$$

The amplitude of torque, T_2, which must be applied at cylinder 2 to maintain the vibration of amplitude a_2 there, is given by

$$T_2 = T_1 - I_2q^2a_2 = S_2(a_3 - a_2)$$

whence
$$a_3 = a_2 - (I_1a_1 + I_2a_2)\frac{q^2}{S_2} \quad . \quad . \quad . \quad (5)$$

Proceeding in this manner, the relations are obtained—

$$\left.\begin{aligned} T_3 &= T_2 - I_3q^2a_3 = S_3(a_4 - a_3) \\ T_4 &= T_3 - I_4q^2a_4 = S_4(a_5 - a_4) \\ T_5 &= T_4 - I_5q^2a_5 \\ a_4 &= a_3 - (I_1a_1 + I_2a_2 + I_3a_3)\frac{q^2}{S_3} \\ a_5 &= a_4 - (I_1a_1 + I_2a_2 + I_3a_3 + I_4a_4)\frac{q^2}{S_4} \end{aligned}\right\} \quad . \quad (6)$$

The calculations are conveniently performed by means of a tabulation. For any frequency, a table is constructed as follows—

Column (1): the inertias I_1–I_5 are listed in order.
,, (2): each inertia is multiplied by the value of q^2 corresponding to the frequency and the product entered in this column.
,, (3): the amplitudes of vibration of each inertia are entered as calculated; in the first line an arbitrary value of unity is entered.
,, (4): the entries in columns (2) and (3) are multiplied together, and the product entered here.
,, (5): the entry in each line is the sum of the entry in column (4) and the preceding entry in column (5); thus, entries here form a running total of the entries in column (4).
,, (6): the stiffnesses S_1–S_4 are listed in order.
,, (7): the entry in column (5) is divided by the stiffness in column (6), and the quotient entered here.

In all lines after the first, the amplitude in column (3) is found by subtracting the entry in column (7), in the preceding line, from the preceding amplitude.

Column (4) gives the inertia torques* corresponding to the amplitudes in column (3). Column (5) gives for each inertia the amplitude* of the applied torque necessary to maintain that inertia, and all parts of the system to the left of it, in vibration with the amplitudes specified. Column (7) gives the twists in the respective shafts.

Table VII shows a specimen calculation for an actual engine, in which the constants are—

$$\text{Inertias: } \left.\begin{matrix}\text{Cyl. 1–Cyl. 4} & 0{\cdot}1249 \\ \text{Flywheel} & 1{\cdot}943\end{matrix}\right\} \text{lb. in. sec.}^2$$

$$\text{Stiffnesses: } \left.\begin{matrix}\text{Inter-cylinder} & 4{\cdot}7 \\ \text{Engine/Flywheel} & 3{\cdot}3\end{matrix}\right\} \times 10^6 \text{ lb. in./radian}$$

The table gives the working for a frequency of 20,200 v.p.m.

TABLE VII

TORQUE SUMMATION FOR 20,200 V.P.M. ($q^2 = 4\ 5 \times 10^6$)

	1	2	3	4	5	6	7
Cyl. 1 .	0·1249	0·5621	1·0000	0·5621	0 5621	4·7	0·1196
Cyl. 2 .	0·1249	0·5621	0 8804	0 4949	1·0570	4·7	0 2248
Cyl. 3 .	0·1249	0·5621	0·6556	0·3685	1·4255	4·7	0 3033
Cyl. 4 .	0·1249	0·5621	0·3523	0 1980	1·6235	3·3	0·4920
Flywheel .	1·9430	8 7435	- 0·1397	— 1·2215	0·4020	—	—

$$Z = -\left(\frac{0\ 4020}{-\ 0{\cdot}1397}\right) = 2{\cdot}878 \times 10^6 \text{ lb. in./radian}$$

$$I_e = -\ Z/q^2 = -\ 0{\cdot}640 \text{ lb. in. sec.}^2$$

There are several important points to be noted. First, it is essential to express the inertias and stiffnesses in the correct units. Inertias must be in lb. in. sec.^{2} units, the value being obtained by dividing the value in lb. in.^{2} by g = 386 in./sec.^{2} Stiffnesses must be expressed as lb. in./radian. It is convenient, however, to make a simplification in entering values in the table: the factor 10^6 is omitted from columns (2), (4), (5), and (6). For this purpose some authorities recommend expressing the stiffnesses as lb.-in./micro-radian, and mentally "dropping" the factor 10^6 from the value of q^2.

Secondly, signs must be taken into account throughout. Thus, a negative amplitude in column (3) results in a negative entry in column (4), so that the total in column (5) is diminished instead of increased. Similarly, if at any stage the total in column (5) is negative, so is the corresponding twist in column (7), so that the amplitude in the next line is greater instead of less.

* With a reversal of sign for convenience.

The dynamic stiffness Z is obtained by dividing the final torque total (column (5)) by the final amplitude (column (3)), and *reversing the sign.*

The effective inertia I_e is obtained by dividing the dynamic stiffness

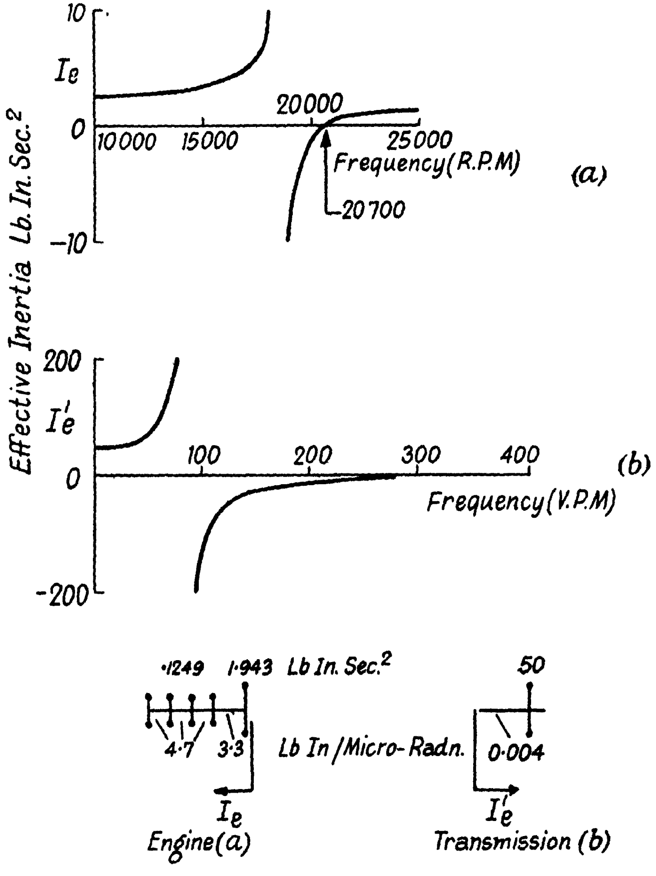


FIG. 74. EFFECTIVE INERTIA CURVES FOR AUTOMOBILE SYSTEM: (*a*) ENGINE (*b*) TRANSMISSION

Z by the value of q^2, and reversing the sign again; or more directly by dividing the final torque by the product of the final amplitude and the value of q^2.

The diagram, Fig. 74, (*a*) shows the variation in effective inertia I_e of this system over the frequency range 10,000–25,000 v.p.m. From zero frequency to 10,000 v.p.m. the effective inertia increases very

slowly, from 2·443 (at zero) to 2·608 (9550 v.p.m.). It will be seen that the effective inertia is zero at about 20,700 v.p.m., and this is the lowest natural frequency of the system. Natural frequencies higher than 25,000 v.p.m. can normally be disregarded.

Effect of Coupled Transmission

In actual fact, of course, the system does not end at the flywheel. Coupled to the flywheel is another system, comprising the gearbox, propeller shaft, rear axle, and road wheels. Of these, the most important item is the propeller shaft. The inertias of the gears are small in comparison with that of the flywheel and they are practically rigidly connected to this much larger inertia; and, as will shortly be seen, the actual value assumed for the inertia of the road wheels does not affect the result to any appreciable extent.

This coupled system can be taken into account by adding a further line to the torque summation table, and this procedure would enable a curve to be plotted showing the variation of the effective inertia at the road wheels. To demonstrate the peculiar advantages of the general method (dynamic stiffnesses and effective inertias), however, and also to illustrate more easily the effect of the gear ratios in the gearbox, another procedure is here adopted.

Taking a value of $0{\cdot}004 \times 10^6$ lb. in./radian for the stiffness of the transmission shaft, and assuming a value of 50 lb. in. sec.^{2} for the equivalent inertia of the road wheels, with an allowance for the gear reduction at the rear axle (see page 108), the graph in Fig. 74 (*b*) shows the variation in effective inertia of this coupled system, the reference point being the forward end of the shaft.

Given the curves for the engine system and for the transmission, the natural frequencies of the complete system can easily be obtained. Let the effective inertias, referred to the coupling point (i.e. the forward end of the transmission shaft, to which the curves illustrated refer) be I_e and I_e' respectively. Suppose that at some frequency these inertias are equal in magnitude, but opposite in sign, so that $I_e + I_e' = 0$. Suppose further that the system is performing torsional vibrations at this frequency, with an amplitude a at the coupling point. The amplitude of torque that must be applied at this point to maintain this amplitude is $I_e a + I_e' a$, which sum is zero. The effective inertia of the combination is therefore zero at this frequency; so this is a natural frequency.

The general condition for natural frequencies is therefore

$$I_e + I_e' = 0 \quad . \quad . \quad . \quad . \quad (7)$$

In practical applications, the equation is rewritten in the form

$$I_e' = -I_e$$

By plotting the two curves on the same axes, first reversing the sign of one of them, the natural frequencies are easily obtained as the values corresponding to the intersections of the two curves.

This procedure is illustrated in Fig. 75. In this diagram, the sign of the curve for the engine has been reversed. It has been necessary to alter the scales somewhat to show up the intersections. It will be

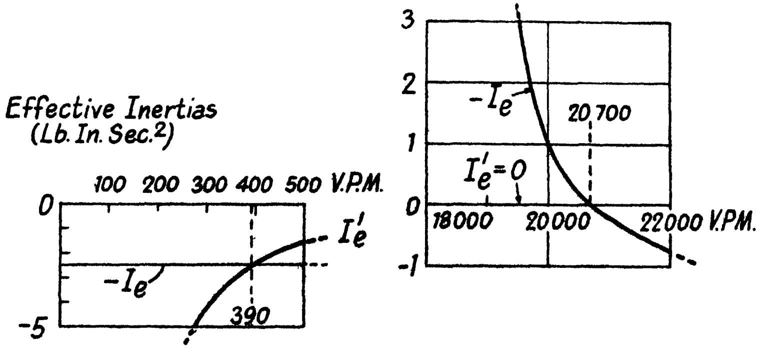


FIG. 75. EFFECTIVE INERTIA CURVES FOR AUTOMOBILE SYSTEM OF FIG. 74

seen that within the frequency range plotted there are two natural frequencies, one at about 390 v.p.m. and the other at 20,700 v.p.m.

These two frequencies deserve further consideration.

Natural Frequencies of Automobile System

As will be seen later, when the forcing torques have been considered, the lower natural frequency is too small to be of importance. Since, however, developments in automobile engineering may possibly bring about considerable changes in the transmission system, the existence of this frequency should not be disregarded.

The formula on page 96 for the periodic time for the torsional pendulum can be rewritten in the form

$$\text{Natural frequency} = 9{\cdot}55\sqrt{\frac{S}{I}} \text{ v.p.m.} \qquad (8a)$$

where the stiffness is in lb. in./radian and the inertia in lb. in. sec.^{2}; or if the stiffness is expressed in lb. in./micro-radian,

$$\text{Natural frequency} = 9550\sqrt{\frac{S}{I}} \text{ v.p.m.} \qquad (8b)$$

If therefore the inertia of the road wheels, etc., is supposed to be infinite, and the natural frequency is calculated for the system comprising the flywheel and crankshaft inertias, all lumped together into

one inertia, mounted on the transmission shaft (thus forming a torsional pendulum), this frequency is found to be

$$9550\sqrt{\frac{0{\cdot}004}{2{\cdot}443}} = 386 \text{ v.p.m.}$$

This is, for all practical purposes, the same as the true value found above for the lowest natural frequency of the complete engine/transmission system. This frequency can, in fact, always be found approximately in this fashion, provided the transmission shaft is very flexible in comparison with the crankshaft.

As calculated, this frequency refers to the case where the transmission shaft runs at crankshaft speed, i.e. in top gear. To allow for the different gear-ratios in the gearbox, the value of the transmission dynamic stiffness must be multiplied by the square of the ratio (transmission r.p.m./crankshaft r.p m.). The value of the frequency can be obtained by the method of intersections, the ordinates of the curve for I_e' being divided by $6^2 = 36$, if the reduction is 1 : 6, in accordance with the general rule, which may be stated thus—

> "To calculate the dynamic stiffness or effective inertia of a shaft system running at N r.p.m., referred to a gear-connected shaft running at NR r.p.m., divide the value referred to the first shaft by R^2."

(See also page 111.)

The lowest natural frequency in direct drive, then, can usually be determined quite accurately by considering a torsional pendulum and using formulae (8), in which S is the stiffness of the transmission shaft, and I is the sum of the flywheel and crank inertias.

In order to appreciate the reason for the validity of this simplification, the curves for dynamic stiffness may be studied. Relevant parts of these curves are shown in the diagram, Fig. 76. It will be seen that for frequencies above about 200 v.p.m. the dynamic stiffness of the transmission hardly varies, and is approximately the same as the stiffness S of the shaft; while for the frequency range plotted (and, in fact, for frequencies below about 6000 v.p.m.) the dynamic stiffness of the engine system is very nearly $- 2{\cdot}443q^2$, i.e. the same as the dynamic stiffness of a plain inertia equal to the sum of the flywheel and crank inertias. From these facts the conclusions set out above follow.

It will now be apparent that the effective inertia of the transmission is extremely small at high frequencies. At 20,000 v.p.m. the value is less than 0·001. Consequently, the second intersection in the diagram, Fig. 75, occurs at practically the same frequency as that for which the effective inertia of the engine system is zero. In other words, the

transmission has no appreciable effect on the second natural frequency. Implicit in this result is the conclusion that this natural frequency is not affected by the gear-ratio.

An approximate value for this second natural frequency can be found as follows. Let I_f be the inertia of the flywheel, and I_c *half* the sum of the crank inertias. Let S′ be the stiffness of the complete crankshaft, found from the formula

$$\frac{1}{S'} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \frac{1}{S_4}$$

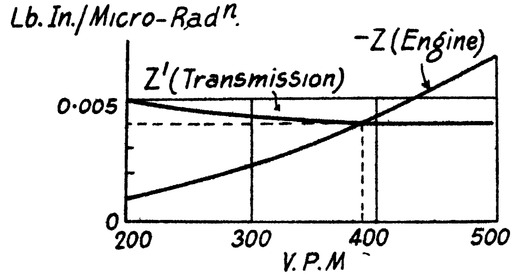


FIG. 76. LOWEST NATURAL FREQUENCY OF AUTOMOBILE SYSTEM

where $S_1 - S_4$ are the stiffnesses of the separate lengths of the crank-shaft. Then

$$\text{Second natural frequency} = 9550\sqrt{S'\left(\frac{1}{I_f} + \frac{1}{I_c}\right)} \quad . \quad . \quad (9)$$

if the inertias are in lb. in. sec.^{2} and the stiffness is in lb. in./micro-radian.

In the example quoted, $I_f = 1{\cdot}943$, $I_c = 0{\cdot}250$, and $S' = 1{\cdot}062$, and the frequency is found to be 20,900 v.p.m. This is, in this case, a remarkably good approximation to the true value (20,700 v.p.m.). A result correct to within five per cent may be expected, provided that the flywheel inertia is large in comparison to the crank inertias, and that the stiffnesses of all four parts of the crankshaft are very approximately equal.

Summary—Frequencies in Automobile Systems

In the range of frequencies which are of practical importance, there is only one natural frequency of torsional vibration of the crankshaft/transmission system likely to require serious consideration.

Ker Wilson gives typical values of this frequency as follows—

Four-cylinder engines	17,000–24,000 v.p.m.
Six-cylinder engines	12,000–17,000 v.p.m.
Eight-cylinder engines	9,000–13,000 v.p.m.

Alterations in any part of the system to the rear of the flywheel have no appreciable effect on the value of the frequency.

There is a lower natural frequency, in the region of 400 v.p.m., which is, however, too small to be of significance.

It must be emphasized that these general conclusions refer to the orthodox type of installation, involving a long and relatively flexible transmission shaft. The general method of analysis described in the foregoing sections can, however, be used to determine the natural

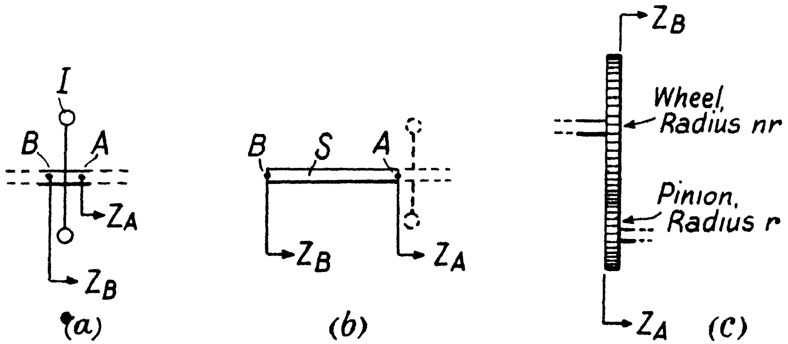


FIG. 77. TRANSFERENCE OF REFERENCE POINT

frequency of any torsional system, and can therefore be applied to the study of the vibration characteristics of new developments.

Other Types of Coupled Systems

Equation (7) can be used to determine the natural frequencies of coupled systems of various types, including those linked by flexible units and gearing.

It is in this connexion that it is important to emphasize the point mentioned previously, that dynamic stiffnesses and effective inertias are quantities which refer to some definite part of the system. Thus, in calculating the natural frequencies of the complete automobile engine/transmission system by means of superimposed graphs relating each to one part, it was necessary to draw the graphs of dynamic stiffness or effective inertia at the point common to both parts.

It is often convenient to make use of formulae for *transferring* the reference point; these are formulae expressing the relation between the dynamic stiffness of a system to one side of a unit (inertia, flexible shaft, or gear-step) and that of the same system as viewed from the other side of the unit, and similar formulae for effective inertias.

Referring to the diagram, Fig. 77 (*a*), let Z_A and Z_B be the dynamic stiffnesses of the systems to the right of the points A and B immediately

each side of the inertia I; that is, Z_A refers to the system to the right of I, and Z_B refers to the combination of I and this system. Then

$$Z_B = Z_A - Iq^2 \qquad . \quad . \quad . \quad . \quad (10a)$$

Similarly, for transference of the reference point through the shaft in Fig. 77 (*b*),

$$\frac{1}{Z_B} = \frac{1}{S} + \frac{1}{Z_A} \qquad . \quad . \quad . \quad . \quad (10b)$$

Finally, for the gear-step illustrated at (*c*), let Z_A refer to the system to the right of, and *including*, the pinion, and let Z_B refer to the system to the right of, but *excluding*, the wheel. Then

$$Z_B = n^2 Z_A \qquad . \quad . \quad . \quad . \quad (10c)$$

(For proofs of these formulae, see references (2) and (3) on page 138.)

Suppose now that Z_B' is the dynamic stiffness of a system coupled directly to the left-hand side of the unit in each case. The equation for determining the natural frequencies is

$$Z_B' + Z_B = 0$$

and on substituting from Equations (10), the following formulae are obtained—

$$\left.\begin{aligned} Z_A + Z_B' &= Iq^2 \\ \frac{1}{Z_A} + \frac{1}{Z'_B} &= -\frac{1}{S} \\ Z_B' &= -n^2 Z_A \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (11)$$

In terms of effective inertias instead of dynamic stiffnesses, the formulae become—

$$\left.\begin{aligned} I_A + I_B' &= -I \\ \frac{1}{I_A} + \frac{1}{I'_B} &= \frac{q^2}{S} \\ I'_B &= -n^2 I_A \end{aligned}\right\} \qquad . \quad . \quad . \quad . \quad (12)$$

By suitable manipulation of these formulae it is possible to see at a glance what will be the effect of altering the inertia, stiffness, or gear-ratio of the linking unit. Thus, for changes in the inertia I, the quantity $-(I_A + I_B')$ is plotted against frequency; for any chosen value of I, a line corresponding to this value intersects the curve at the natural frequencies. For changes in the shaft stiffness S, the quantity $-\left(\frac{1}{Z_A} + \frac{1}{Z_B'}\right)$ is plotted, and the frequencies for which this quantity has the value $\frac{1}{S}$ are the natural frequencies. Finally,

in the case of the gear-step, the quantity plotted is the ratio $\frac{-I_B'}{I_A}$, and for any gear-ratio $n : 1$ the natural frequencies are those for which this ratio equals n.

Gear-steps in Torque Summation Tables

The presence of reduction gears in the system is easily allowed for in the tabulation method of calculating dynamic stiffnesses and

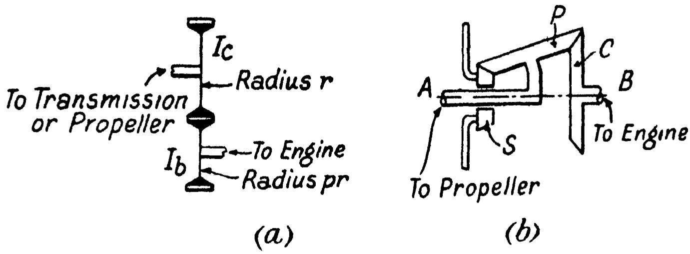


FIG. 78. (a) SPUR REDUCTION GEAR; (b) EPICYCLIC REDUCTION GEAR

effective inertias. Referring to the diagram, Fig. 78 (a), which illustrates the case of a simple spur reduction gear, I_b is the inertia of the pinion on the crankshaft and I_c is the inertia of the meshing gearwheel; the gear-ratio is $p : 1$, so that when the crankshaft speed is N r.p.m. the speed of the driven shaft is pN.

The tabulation is carried through in the usual way as far as the line for I_b. In this line there is no stiffness S entered, as the pinion is directly coupled to the wheel. The amplitude of the wheel is the product of the amplitude of the pinion and the gear-ratio p, the torque total at the pinion (column (5)) must, however, be divided by p before the inertia torque due to the wheel is added. The tabulation is then continued. It is convenient to insert an extra line for the purposes of recording the modified total torque.

As an example, suppose that the diagram refers to the case of a single-row radial aircraft engine installation, in which the numerical constants are as follows—

Engine inertia	= 5·0
Pinion inertia I_b	= 0·4
$p = \dfrac{\text{propeller r.p.m.}}{\text{engine r.p.m.}}$	= 0·6
Gearwheel inertia I_c	= 0·6
Crankshaft stiffness	= 2·5
Propeller shaft stiffness	= 6·0

(inertias in lb. in. sec.^{2}, stiffnesses in lb. in./micro-radian). Then, at a frequency of 2865 v.p.m., the table is as follows—

TABLE VIII

TORQUE SUMMATION FOR 2865 V.P.M. ($q^2 = 90{,}000$)

	1	2	3	4	5	6	7
Engine . . .	5·0	0·45	1·0000	0·4500	0·4500	2·5	0·1800
Pinion . . .	0·4	0·036	0·8200	0·0295	0 4795		
					0·7992		
Wheel . . .	0·6	0·054	0 4920	0·0266	0·8258	6·0	0·1376
Propeller end . .			0·3544				

The total torque at the pinion, 0·4795, is divided by $p = 0{\cdot}6$, the quotient 0·7992 being entered in the extra line. The amplitude 0·8200 at the pinion is multiplied by p, and the product 0·4920 is the amplitude at the wheel. The dynamic stiffness at the propeller end of the propeller shaft is thus found to be

$$-\,0{\cdot}8258/0{\cdot}3544 = -\,2{\cdot}330 \text{ lb. in./micro-radian}$$

and the effective inertia is

$$2{\cdot}330 \times 10^6/q^2 = 25{\cdot}9 \text{ lb. in. sec.}^2$$

The procedure is of course applicable to any type of torsional system involving gear-steps.

Reduction gears other than of the simple spur type can be treated by first deriving an equivalent spur gear which is dynamically similar. A common type of gear in aircraft installations is illustrated in Fig. 78 (*b*). A bevel wheel C attached to the crankshaft meshes with a set of planet bevels P, carried on a spider mounted on the propeller shaft. The planets also mesh with a sun wheel S fixed rigidly to the gear-housing. The crankshaft and propeller shaft are collinear, the axis of rotation being AB.

Let u = inertia of crankshaft bevel about AB,

v = inertia of each planet about its bearing axis,

w = inertia of spider about AB,

m = mass of each planet,

n = number of planets.

Then it can be shown* that the inertias of the pinion and gearwheel

* R. G. Manley, "Torsional Vibration Analysis of Systems connected by Flexibly-mounted Epicyclic Gearing," *Journal of the R.Ae.S.*, July, 1944.

in the equivalent simple spur gear are—

$$I_b = u + \tfrac{1}{2}nr\left(\frac{C}{P}\right)^2$$

$$I_c = w + \tfrac{1}{4}n\left(m - \frac{v}{P^2}\right)(C + S)^2$$

where C, S, and P denote the numbers of teeth in the bevels similarly lettered.

Aircraft Propellers

Although, as has been seen, the transmission in automobile systems does not greatly influence the important natural frequencies of the crankshaft and associated masses, in the case of aircraft engines the torsional vibration characteristics of the engine are profoundly modified by the presence of the propeller.

Originally the propeller was regarded in the same manner as a marine propeller, viz. as a rigid inertia appended to the engine system. Calculations based on this conception were not very successful, and in 1936 it was realized that the flexibility of the propeller blades must be taken into account.

The shape of the blades is such that the *calculation* of the vibration characteristics of the propeller is an exceedingly complicated and tedious matter. Fortunately, however, it is possible to obtain the necessary data by experimental means, the details of which fall outside the scope of this book. Here it is sufficient to note that by such methods can be derived a graph showing the variation with frequency of the torsional dynamic stiffness or effective inertia of the propeller; superposition of this curve upon a similar curve for the engine, the sign of one curve being reversed, will indicate the natural frequencies of the combination.

Fig. 79 shows very approximately the type of diagram that is obtained. It will be noted that the propeller curve has many more branches than has the engine curve in the same range of frequencies, and consequently there are likely to be several natural frequencies within the operating range. In the case illustrated the engine is a single-row radial, and there are seven intersections in the range of the graph (from zero to 25,000 v.p.m.); if the propeller were a rigid body there would be only one intersection as marked at P. The magnitude of the errors involved in the assumption of a rigid propeller will be evident.

A further complication which must be taken into account is the effect of centrifugal forces upon the propeller curve. These forces tend to stiffen the blades, so increasing their natural frequencies and

shifting the curve somewhat to the right (i.e. in the direction of increasing frequency), the greater the rotational speed the greater is the stiffening effect. As a result, for extreme accuracy a separate curve should be drawn for each rotational speed, but in practice it is found to be sufficient merely to indicate a series of bands, the bounds of which are the branches of the curve for the lowest and for the highest rotational speeds In place of each simple intersection in Fig. 79 there is therefore a comparatively small range of frequencies,

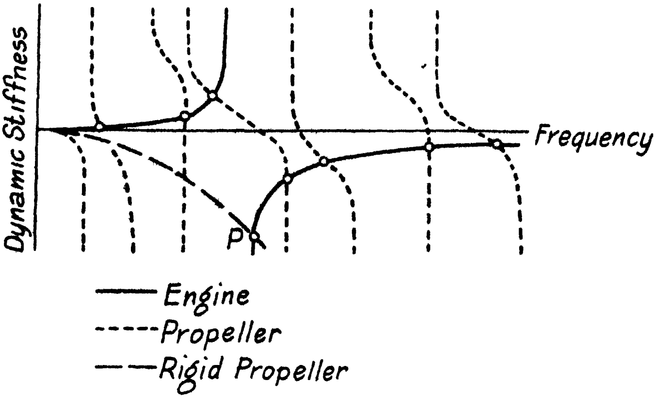


FIG 79. DYNAMIC STIFFNESS CURVES FOR AIRCRAFT ENGINE AND PROPELLER SYSTEM

and the natural frequencies for any particular rotational speed can be estimated with sufficient accuracy for practical purposes. The method of calculating the effects of centrifugal forces has been described by Morris (see list of references on page 138).

Helical Gearing

Helical gearing, the use of which in transport power plant is being developed (particularly in multi-crankshaft aircraft engines), introduces another complication. If one shaft of a single helical gear unit is fixed rigidly, torsional vibrations can take place in the other shaft if a small relative motion between the two gear wheels in the axial direction is possible. Consequently, the extent to which axial motion is restrained has an important influence upon the torsional vibrations of the shaft systems. Even if axial motion is supposed to be prevented by the provision of adequate thrust bearings, it must be remembered that the housing to which the bearings are fixed cannot be infinitely rigid, and may indeed be comparatively flexible. The subject is too complex to be given a detailed treatment here; the application of the dynamic stiffness method to this problem has been described elsewhere (see reference (3) on page 138).

Nodes

The torque summation table, besides being used for determining natural frequencies, enables the "swinging forms" of the shaft system to be obtained. If a table is completed for one of the natural frequencies of the system, the entries in column (3) are proportional to the amplitudes of vibration at each inertia, for a free vibration of the system at that frequency.

It will be found that the various natural frequencies are distinguished by the number of points in the shafting whereat there is zero amplitude. These points are called *nodes*. At the lowest natural frequency, there is one node: at the second lowest, two nodes, and so on.

The position of the nodes has an important effect on the distribution of stresses due to torsional vibration. Briefly it may be said that portions of shaft containing nodes are more highly stressed than those not containing nodes. For further details, the reader should consult the standard textbooks. Furthermore, the position of the nodes has a controlling influence on the response of the shaft system to the various harmonics of the applied torques (see page 119).

Table IX shows the distribution of amplitudes through the engine/transmission system of Fig. 74, at the first two natural frequencies. It will be seen that at the lowest frequency there is a node in the transmission, near the road-wheels; and that at the second frequency the two nodes are near the road-wheels and flywheel.

TABLE IX

AMPLITUDE DISTRIBUTIONS AT FIRST TWO NATURAL FREQUENCIES
(System of Fig. 74)

Frequency	Relative Amplitudes					
	Cyl. 1	Cyl. 2	Cyl. 3	Cyl. 4	Flywheel	Road-wheels
390 v.p.m. .	1·00	1·00	1·00	1 00	1·00	− 0·049
20,700 v.p.m. .	1·00	0·88	0·64	0·33	− 0·18	less than + 1/300,000

The actual amplitudes of vibration at any frequency except a natural frequency can be found by means of the torque summation table, modified by the inclusion of terms to represent the forcing torques at the various cylinders.

Forcing Torques

The periodic torques responsible for the initiation and maintenance of torsional vibrations in the engine system are the harmonic components of the torque output from each cylinder. As has already been

described (pages 39–49), this torque output is by no means uniform. It fluctuates periodically, the duration of the cycle of variation being the same as the firing interval (i.e. the time taken for the crankshaft to rotate once, in the case of two-stroke engines, and twice in the case of four-stroke engines).

Torsional vibrations with corresponding periods may therefore be expected; but the situation is also complicated by the fact that the torque output from each cylinder does not vary in the manner of a sine-wave, but contains higher harmonics.

The output can be regarded as the sum of a number of terms, of which one represents the steady (or mean) torque, while the others are sine-wave variations at frequencies which are multiples of the firing frequency.

In a single-cylinder engine all these harmonics are present, although it is true that the magnitudes of the higher harmonics are negligible. In a multi-cylinder engine, advantage is taken of the possibility of suitably disposing the cranks and the firing sequence of the cylinders, so as to minimize the effects of the harmonics.

Ker Wilson gives a very comprehensive treatment of this subject (see references on page 138). A simple example will here suffice to illustrate the method of determining which of the harmonics are important.

It is customary to refer to the various harmonics by their "order numbers." The order number of a harmonic is the number of cycles which occur during one revolution of the crankshaft. In two-stroke engines, the order numbers are necessarily all integers, the first harmonic (or "fundamental") being referred to as the 1st or 1X order.

The notation typified by "1X" is useful, as it indicates the frequency of the harmonic as a multiple of crankshaft speed Thus, in the case of the 3X order, with a crankshaft speed of 3500 r.p.m., the frequency is $3 \times 3500 = 10{,}500$ v.p.m. In four-stroke engines, since the firing cycle extends over two engine revolutions, order numbers such as ½X, 1½X, 2½X, etc., occur, as well as the integers.

Vector Summation

It can be shown that the severity of the effects due to any harmonic in the torque output from each cylinder is directly proportional to a certain vector sum, which is calculated in the following manner.

This vector sum is, in effect, a measure of the vibrational energy input, and is determined by considering the work done by the harmonics of each cylinder during the vibration, account being taken of the fact that, since the cylinders fire at different times, there are phase-differences between the harmonics from different cylinders.

A diagram termed a "½X order phase diagram" is first prepared. Fig. 80 (*a*) shows such a diagram for the case of a six-cylinder, four-stroke engine with conventional crankshaft arrangement, and with the firing order 1–5–3–6–2–4. The cylinders fire at even intervals over the 360° of camshaft revolution, and this disposition is shown in the diagram In engines which have uneven firing, the vectors will be

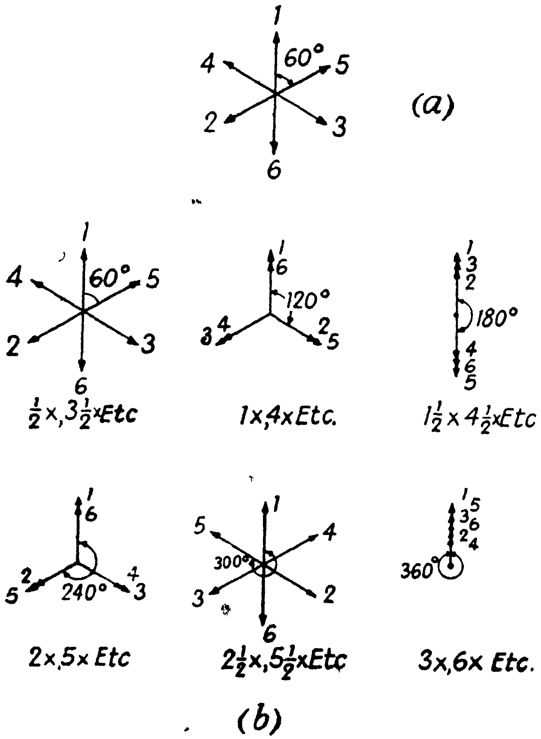


Fig 80. Vector Diagrams

similarly spaced unevenly round the circle. In all cases, cylinder 1 is represented by a vector pointing upwards, and the other cylinders by vectors spaced at angles corresponding to the camshaft angles of the firing sequence.

Phase diagrams for the other orders are obtained by doubling, trebling, etc., the angular intervals. Thus, the angular intervals for the 1X order are double those for the ½X order; those for the 1½X order are treble those for the ½X order, and so on. The diagram, Fig. 80 (*b*) shows the result for the six-cylinder engine which is being considered.

It will be observed that there are only six possible diagrams, several orders having the same diagram, as follows—

(*a*) $\frac{1}{2}$, $3\frac{1}{2}$, $6\frac{1}{2}$, etc.	(*c*) $1\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, etc.	(*e*) $2\frac{1}{2}$, $5\frac{1}{2}$, $8\frac{1}{2}$, etc.
(*b*) 1, 4, 7, etc.	(*d*) 2, 5, 8, etc.	(*f*) 3, 6, 9, etc.

and also that the types (*d*) and (*e*) are mirror images of the types (*b*) and (*a*) respectively.

Assuming that (as is normally the case) the indicator diagram for each engine cylinder is the same, vectors proportional to the amplitudes of vibration at the various cylinders are plotted in the directions indicated. The relative amplitudes are obtained by means of the torque summation tables, calculated for the particular natural frequency considered.

The severity of any harmonic at that frequency is indicated by the length of the *vector sum* of these vectors in the corresponding phase diagram.

If the amplitude of any cylinder is negative, indicating that its vibration is anti-phased with respect to that of cylinder 1, the corresponding vector is of course reversed in direction.

Significance of Nodes

The position of the nodes, and generally the shape of the "swinging form," profoundly affect this vector summation, since they determine the lengths of the various vectors plotted.

The following amplitude distributions are typical cases for a six-cylinder engine—

(*a*) *Node remote from crankshaft* (e.g. lowest natural frequency in automobile system).

Cylinders . .	1	2	3	4	5	6
Amplitudes .	1·00	0·99	0·97	0·94	0·90	0·85

(*b*) *Node just beyond cylinder* 6 (e.g. second natural frequency in automobile system).

Cylinders . .	1	2	3	4	5	6
Amplitudes . .	1·00	0·95	0·84	0·69	0·51	0·29

(*c*) *Node between cylinders* 3 *and* 4 (e.g. one of the higher natural frequencies in aircraft system).

Cylinders . .	1	2	3	4	5	6
Amplitudes . .	1·00	0·75	0·40	− 0·20	− 0·60	− 0·90

If the vector summations are performed, it will be found that the lengths of the resultant vectors are as follows—

Orders:	$\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, etc.	1, 2, 4, 5, 7, etc.	$1\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, etc.	3, 6, 9, etc.
Case:				
(*a*)	0·11	0·06	0·27	5·65
(*b*)	0·48	0·22	1·30	4·28
(*c*)	1·17	0·09	3·85	0·45

The orders $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, etc., give appreciable vector sums in the various cases. With the node remote from the crankshaft, Case (*a*), the only orders giving large vector sums are the 3, 6, etc., orders. These are sometimes termed the "major orders," such orders being readily distinguished by the fact that their order numbers are multiples of half the number of cylinders (in four-stroke engines). In two-stroke engines the corresponding major orders are those whose order numbers are multiples of the number of cylinders.

Severity of Resonances

The vector summation described in the preceding section indicates the degree in which the system is susceptible to vibrations caused by the various harmonics, at the natural frequency for which the summation is calculated. Clearly, the severity of the resulting vibration depends also upon the magnitude of the torque harmonics at each cylinder. These magnitudes are determined by a harmonic analysis of the torque output curve, which can be calculated from the indicator diagram and then corrected to allow for inertia effects, as described on pages 22 to 27.

The standard textbooks give curves showing the typical relative magnitudes of the various torque harmonics. It is found that these vary somewhat with I.M.E.P Since in general this pressure is not constant over the full operating speed range of the engine, the engine speed corresponding to the particular resonance involved must be determined, and the corresponding I.M.E.P. used for calculating the torque harmonics.

In Case (*b*), if the system is that of an automobile engine/transmission and the frequency is 12,600 v.p.m., the engine speeds corresponding to the important orders 3, 6, 9, etc., are respectively 4200, 2100, 1400, etc., r.p.m. The first of these is likely to require serious attention, as it might correspond to, say, cruising at 40 m.p.h. in top gear. These speeds, whereat the major orders give resonance with a natural frequency of the system, are termed "major critical speeds."

Single-row Radial Engines

In single-row radial aircraft engines, since all the pistons are connected to the same crank the amplitudes of vibration for all cylinders are the same, except for the effect of articulating all but one of the connecting rods on to the master rod. Disregarding this effect for the moment, it will be seen that the vector summations are all zero except those for the major orders, i.e. for the orders $3\frac{1}{2}$, 7, $10\frac{1}{2}$, etc., in a seven-cylinder four-stroke engine, or 7, 14, etc., in a seven-cylinder two-stroke engine.

At first it appears, therefore, as if the problem of avoiding resonances

is far easier to solve in the case of radial engines than with in-line engines. In actual fact, however, the existence of a large number of natural frequencies within the operating range, occasioned by the flexibility of the propeller blades (see page 114), has the effect of rendering this simplification less significant, as the number of critical speeds is still large.

The principal effect of articulation of the connecting rods is to introduce a 1X order surge into the torque output from the engine.

In general, therefore, the critical speeds of single-row radial engine installations are determined by dividing each natural frequency of the engine/propeller system by the order numbers 1 and $\frac{1}{2}kn$, where k is any integer and n is the number of cylinders.

Other Types of Engine

Two- and three-row radial engines, and the more complicated in-line engines (Vee, X, H, Fan, etc.) can be studied by an extension of the methods described. In all cases it is possible to derive phase diagrams for the various orders, and to determine the important orders by vector summation. It is not possible to give general expression to the results to be expected, as each case must be treated on its merits, and with these engines there are many different possibilities for firing-order.

Avoidance of Vibration Troubles

There are three general methods of decreasing the unwanted effects of torque fluctuations—

(1) Avoidance of resonance, by adjustment either of the natural frequencies or of the forcing frequencies, or of both.

(2) Suppression of the excitation, as for example by altering the firing order suitably.

(3) Absorption of the vibrational energy.

In practice, some or all of these methods may be utilized at the same time, and the first and last are frequently combined together in the technique of applying vibration "dampers" and "absorbers."

Avoidance of Resonance

Any alteration made to the dynamic constants of the system (i.e. to the inertias or the stiffnesses of the shafts) will affect the natural frequencies. Thus, it is frequently possible to avoid the destructive effects of vibration merely by so adjusting the stiffness of the principal shaft, or the inertia of a flywheel, as to bring all the natural frequencies outside the operating range of frequencies. The critical speeds are then outside the operating speed range.

From time to time, various proposals have been made to provide

an easier method of adjusting the natural frequencies by means of added masses. The rubber/metal bonded damper, which is now undergoing extensive development, acts partly in this manner. For the purposes of this section, such a damper can be regarded as an additional flexible shaft attached to the engine system, and carrying at its end an additional inertia. In practice the flexible element is a rubber ring which is bonded to the metal of the inertia.

The action of a damper of this type can be partly understood by a study of the dynamic stiffness or effective inertia characteristics, from which the effect on natural frequencies of fitting the damper can readily be ascertained. This is not the only effect, however. The damper can also influence the vibration characteristics by absorption of the vibrational energy, as will be discussed below.

Alterations in the firing order may have virtually the effect of avoiding resonance by changing the forcing frequencies, if the alterations are such that the vector summations for the previously important orders become very small, while the vector summations for some previously unimportant orders become appreciable. It is indeed seldom that it is possible to avoid all resonances within the running range by this means; usually a compromise is necessary, one or two (or even more) comparatively unimportant resonances being tolerated for the sake of avoiding a very serious one.

Suppression of Excitation

In general, the effect of changing the firing order is to increase the severity of some orders (as judged by their vector summations for each relevant natural frequency), to decrease that of others, and to leave the remainder practically unaltered. It may be possible to reduce a troublesome order to such an extent that the resulting stresses are tolerable, without eliminating it completely.

In the more complicated types of aircraft engines, particularly those with more than one crankshaft, there is a great number of possible firing orders from which a choice can be made. There are, of course, other factors besides torsional vibrations to be considered in making the selection. Bearing pressures are greatly influenced by the firing order; and a firing order which is excellent from the point of view of torsional vibration may be exceedingly awkward to accommodate owing to difficulties in the lay-out of the induction manifold.

Considering only those arrangements in which the firing impulses are evenly spaced over the firing cycle, a six-cylinder in-line, four-stroke engine has only one balanced crankshaft arrangement, and four possible firing orders. An eight-cylinder in-line engine has three crankshaft arrangements and a total of 24 firing orders; a ten-cylinder engine would have 12 crankshaft arrangements and a total of 192 firing orders.

With more than six cylinders, the tendency is to arrange them in more than one bank; and when it is recollected that there are many different Vee-angles in use, besides other more complicated dispositions, it will be appreciated that there is practically no limit to the number of different firing orders and cylinder arrangements.

Absorption of Energy

Vibrational energy can be absorbed in two ways: by dissipation, and by what may be termed "forcing of nodes."

The conversion, under closely controlled conditions, of vibrational energy into heat is utilized in the rubber damper. From a knowledge of the dimensions and physical properties of the rubber element it is possible to calculate the amount of energy dissipated per second for any given amplitude of vibration in the damper at any given frequency. The details of the calculation are given by Zdanowich and Moyal (see reference (6) page 138). It is, however, exceedingly difficult to incorporate such calculations in a general survey of the vibration properties of an engine, as the nature and extent of the damping already present in the engine are largely indeterminate.

Various other types of dissipative devices have been used successfully, including those relying on solid friction and hydraulic turbulence. The basic principle of this method of damping is that arrangements are made whereby torsional vibration in the shaft system causes relative motion between two members of the damper. This relative motion is resisted by frictional forces in such a way that a constant input of energy is required to maintain the vibration. The only source of such energy is the vibrating crankshaft, and consequently the vibrations in the shaft system are diminished by the withdrawal of part of the available energy.

The normal rubber damper, as already stated, acts both as a dissipative device and as a re-tuning device. In both capacities it is most effective at a particular frequency determined by the design of the damper. This frequency is the natural frequency of the damper as a torsional pendulum when the point of attachment to the crankshaft is held rigidly. The damper tends to vibrate most readily at this frequency, and so tends to absorb most energy when the point of attachment is itself vibrating at this frequency. Furthermore, it is found that the re-tuning effect (i.e. the alteration of natural frequencies of the shaft system) is greatest in the neighbourhood of this natural frequency of the damper as a one-mass torsional pendulum.

While such a damper often has a useful application to automobile engines, in cases where there is only one important critical speed, corresponding to a large vector summation for one order in particular at one natural frequency, the normal aircraft installation has a so

much more complicated series of natural frequencies that an entirely different type of absorber has had to be developed. This is the "pendulum absorber," which has of recent years been applied to many different types of aircraft installation.

Examples of Vibration Dampers

The vibration dampers employed upon earlier model automobile engines were of the fluid-friction or hydraulic type, a typical example

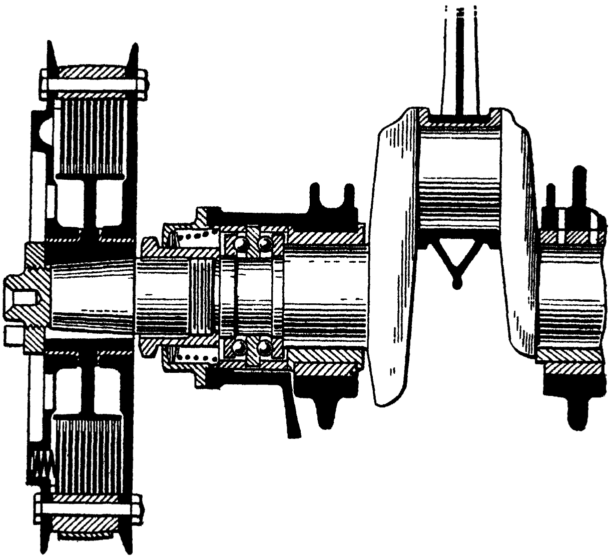


FIG. 81. THE LANCHESTER VIBRATION DAMPER

of which is the Lanchester design shown in Fig. 81. It consists of two principal members, namely, a set of metal discs or plates attached to a unit keyed to the end of the crankshaft and another set of discs secured to the belt pulley for the radiator fan drive. The two sets of discs were separated by small spaces and the interior of the pulley was filled with a somewhat viscous grade of oil. Normally, the whole damper would rotate as a more or less rigid unit but at any critical speeds the fluid friction between the oil and the plates would damp out the crankshaft vibrations.

In connexion with the fitting of a damper of this and analogous types to the crankshaft of any engine care must be exercised to ensure that there is not a node (zero amplitude and maximum stress) at the fixing plane, or the damper will be rendered ineffective.

The hydraulic type of damper has been to a large extent supplanted by the bonded rubber type, for the latter possesses certain advantages.

Thus, it contains no rubbing parts, so that the question of wear does not arise. It has fewer parts and is therefore cheaper to manufacture. For this reason it is preferable to the hydraulic and also the mechanical friction type of damper. In the latter form the initial friction or "sticktion" is much higher than the "moving condition" friction, so that the damper is rendered somewhat insensitive. The exceptionally high energy absorption per unit weight of rubber renders it particularly suitable for use in light designs of vibration damper.

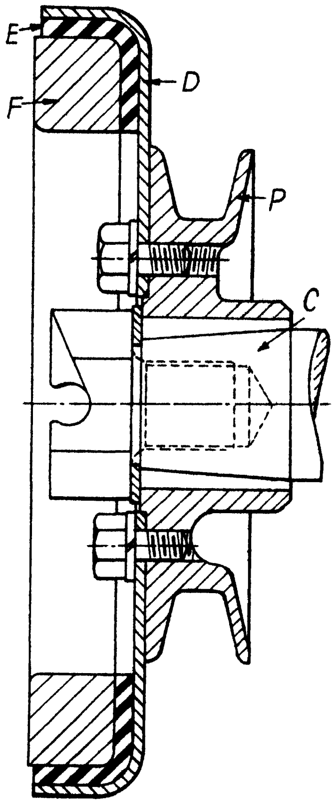


FIG. 82. THE METALASTIK TORSIONAL VIBRATION DAMPER

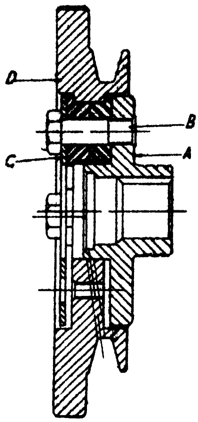


FIG. 83. ANOTHER RUBBER-TYPE TORSIONAL VIBRATION DAMPER

Fig. 82 illustrates the Metalastik torsional vibration damper, consisting of the fan drive Vee-belt pulley P, which is keyed to the front end of the crankshaft C. Attached to P is a flanged steel disc D, which is bonded to a rubber unit E; the latter is also bonded to the metal inertia disc F. This neat and effective system is available in a range of different designs for various types of petrol and Diesel engine.

Another type of rubber damper, providing complete insulation between the crankshaft and belt pulley, is shown in Fig. 83. The crankshaft member A has a flange into which a series of bolts B are screwed to secure the steel disc C compressing the two rubber discs shown supporting the relatively heavy belt pulley and disc D. In this example the drive to the belt pulley is taken through the rubber, thereby causing torsional stress, whereas in the design shown in Fig. 82

the rubber is passive in so far as the fan drive is concerned and acts purely for the purpose of damping-out engine torsional vibrations.

Pendulum Absorbers

The diagram, Fig. 84 (*a*), shows a simple "two-mass" system of masses and springs. It can be shown that if a vertical vibratory force is applied to the upper mass A, there is one particular frequency of vibration for which there will be a node at A, all the motion taking place in the lower spring/mass system. It is found further that this frequency is the natural frequency of the lower mass on its spring, when the upper end of the spring is held rigidly.

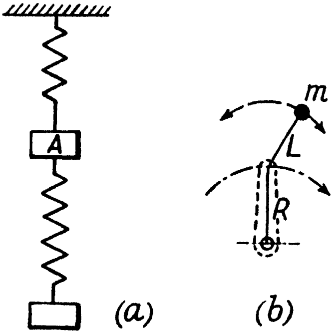


FIG. 84. (*a*) TWO-MASS LINEAR SYSTEM; (*b*) SIMPLE PENDULUM ABSORBER

It will be noted that this result is analogous to the action of an attached torsional damper, as described in the preceding section.

If the effective inertia of the whole system is calculated, taking A as the reference point, it will be found that this effective inertia is infinite at this particular frequency.

A series of torsional dampers, applied one to each of the cranks of the crankshaft, would have the effect of preventing the crankshaft from vibrating at one particular frequency, this frequency being the natural frequency of each damper as a torsional pendulum swinging about its point of attachment. Thus, vibration would be prevented by "forcing nodes" at all cranks.

The pendulum absorber, however, utilizes a true pendulum instead of a torsional pendulum. A simple representation is given at (*b*), Fig. 84. A mass m carried on a light arm of length L forms a pendulum, which is attached to the crankshaft at a point distant R from the axis of rotation.

The constancy of the natural frequency of an ordinary pendulum depends upon the constancy of the gravitational field in which it is suspended. In this case, however, the significant field of force is not the gravitational field but the centrifugal field due to the mean rotational speed of the crankshaft, and the natural frequency of the pendulum is proportional to the rotational speed.

By a rather involved but straightforward mechanical analysis the effective inertia of the pendulum can be calculated. Terming that part of the crankshaft to which the pendulum is attached the "carrier," this effective inertia is the amplitude of alternating torque that must

be applied to the carrier in order to maintain a torsional vibration of unit amplitude at the carrier, the effect of the pendulum only being taken into account, and it is found to be

$$I_e = \frac{m(R + L)^2}{1 - n^2 \frac{L}{R}} \qquad . \quad . \quad . \quad . \quad (13)$$

where n is the order number of the vibration frequency, i.e (frequency of vibration/crankshaft speed).

When the order number n is equal to $\sqrt{\frac{R}{L}}$, the effective inertia of the pendulum is infinite. In consequence, the carrier is prevented from vibrating at a frequency equal to the product of $\sqrt{\frac{R}{L}}$ and the crankshaft speed; a node is forced at that part of the system.

If similar pendulums are fitted to all the cranks in an in-line engine, the vibrational energy of the order corresponding to the value of $\sqrt{\frac{R}{L}}$ is absorbed in the pendulums, and the crankshaft as a whole is free from the influence of that order.

In practice, a simple pendulum cannot be employed, and various equivalent assemblies have been devised. For in-line engines, a common design based on Salomon patents has a pendular mass consisting of two steel rings. These rings roll on a steel pin, which is itself free to roll in a bush fitted to a balance-arm opposite the crank-web. The effective length of the pendulum, i.e. the length of the equivalent simple pendulum, is a function of the various diameters of rings, pin, and bush. Such pendulums have been fitted to a number of different engines, including the de Havilland Gipsy Six.

It is not essential to make all the pendulums of the same dimensions, in fact, in some cases it is more convenient to fit pendulums designed to eliminate two or three orders.

Another type of pendulum which has been used on radial engines consists merely of a heavy steel ball rolling in a groove.

In the simple statement of the theory given above, account has not been taken of several limiting factors. Most important of these is the fact that for true rolling action, such that the suspended mass acts correctly as a pendulum, the amplitude of vibration of the pendulum must not exceed a value dependent upon the dimensions. Since the amount of energy absorbed is proportional to this amplitude, it is evident that the absorbing power of the pendulum is restricted. For this reason it is advisable to make the pendular mass as substantial as possible, since the energy absorbed is proportional also to this mass.

The design illustrated in Fig. 87 is excellent in this respect, particularly as there is no actual added mass, part of the balance-weight being utilized for the pendulum.

It is not possible here to describe in greater detail the theory and application of pendulum absorbers. Very complete accounts are given in the works of Ker Wilson, and of Zdanowich and Wilson (see list of references at end of this chapter). It must, however, be stated that modern practice favours tuning the absorber so that it has a large but

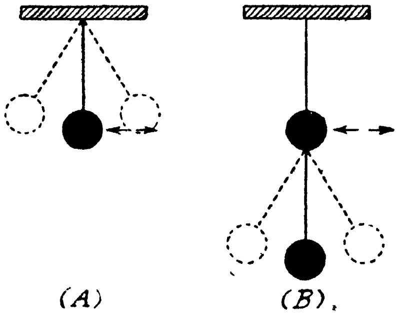


FIG. 85. PRINCIPLE OF PENDULUM-TYPE VIBRATION DAMPER

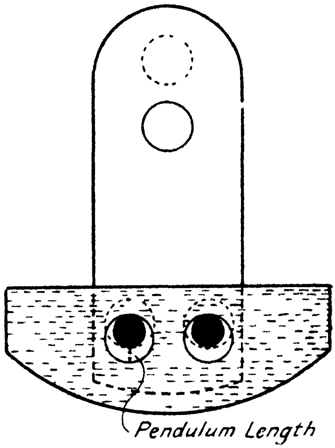


FIG. 86. PENDULUM DAMPER APPLIED TO RADIAL ENGINE

not infinite inertia for the order it is designed to eliminate. Tuning for infinite inertia requires adherence to very close manufacturing limits, which are liable to be upset by wear. The modern technique enables much wider limits to be used.

The Wright Pendulum Damper

The principle of the pendulum damper used on the Wright radial engines can be understood more clearly, by reference to Fig. 85, in which diagram (A) represents a simple pendulum with its own natural period of oscillation, whilst diagram (B) illustrates how the oscillations of (A) can be brought to rest by providing a second pendulum of equal weight and length, i.e. of equal oscillation frequency. If it is assumed that the pendulum in (A) is set into vibration as the result of a resonance effect, analogous to the torsional vibrations set up in a crankshaft in tune with the natural torsional vibrations, then the additional pendulum shown in (B) may be regarded as a vibration damper.

Referring to Fig. 86, which shows the principle applied to a radial engine crankshaft, a short pendulum is hung on to the crankshaft

and tuned to the normal speed power impulses so that the vibrations are damped out in an analogous manner to that shown in diagram (*B*), Fig. 85.

In the nine-cylinder Wright radial engine, torsional vibration impulses due to the firing strokes occur $4\frac{1}{2}$ times per revolution, and to absorb the resulting vibrations the rear crankshaft counterweight is suspended on the crank cheek as described previously. The restoring force of this pendulum is the centrifugal force due to the crankshaft rotation. As the centrifugal force and also the power impulses causing the torsional vibrations are both functions of the engine speed, the damping action of the pendulum is effective over the whole range of engine speeds.

In regard to the construction of the Wright pendulum damper (Fig. 87) the slotted steel counterweight is movably suspended from the crank cheek which extends down to the slot, by means of two spool-shaped steel pins passing through oversized holes in both the counterweight and crank cheek. The pendulum length is determined by the difference in diameter between the pins and holes, whilst the two-point suspension causes the weight to swing through the same degree of arc without rotation about its mass centre. A special feature of this construction is that it utilizes part of the actual balance weight and thus adds no more to the weight of the engine. This form of damper is practically frictionless, so that there is no energy dissipated through friction as with other mechanical and hydraulic friction dampers. The movement of the weight is so small that there is no measurable wear.

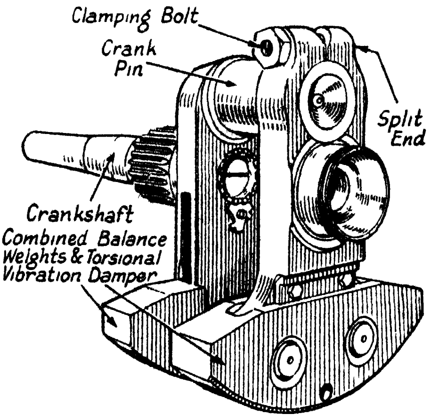


FIG. 87. CRANKSHAFT OF WRIGHT RADIAL ENGINE WITH PENDULUM-TYPE TORSIONAL VIBRATION DAMPER

The Double Pendulum Damper

In the case of certain engines, of which the Lycoming four-cylinder opposed is an example, trouble is sometimes experienced on account of torsional vibrations occurring at critical speeds within the working speed range. Thus, in the example mentioned it was found that there were two marked critical speeds due to harmonics of two and four times crankshaft speed frequency. As the ordinary friction damper did not prove effective it was decided to use a pair of double-pendulum

dampers, namely, a pair of 4th order pendulum dampers on the second crank arm and a pair of 2nd order dampers on the fifth arm. Without these dampers the reduction gear was very noisy in operation, and in one instance pinion failure occurred, but after fitting the dampers

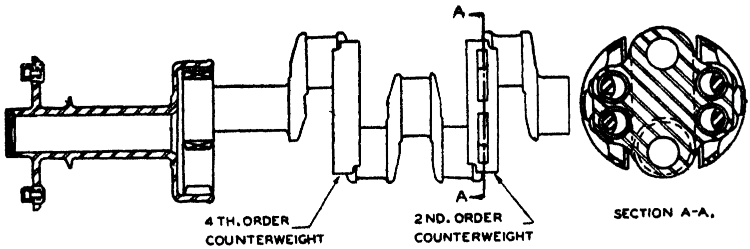


FIG. 88. LYCOMING FOUR-CYLINDER CRANKSHAFT DOUBLE PENDULUM DAMPER

the gears ran quietly and no failures occurred. The manner in which the double damper reduces the maximum stress in the case of an aircraft propeller over that associated with a single damper is illustrated in the examples shown in Fig. 89. It will be noted that whilst

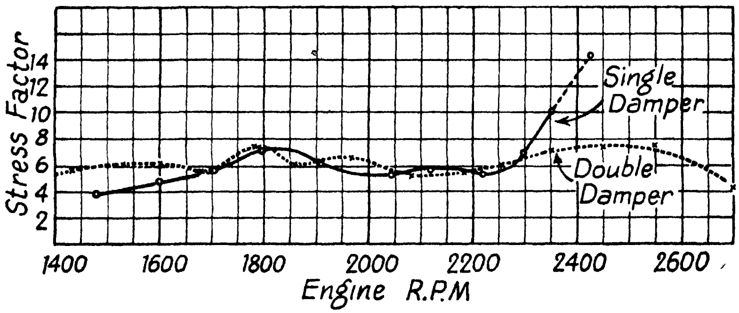


FIG. 89. EFFECT OF DOUBLE DAMPER IN REDUCING MAXIMUM STRESS OVER THE SPEED RANGE

reducing the value of the stress factor it also greatly extends the effective range of engine speeds over which it is effective.

Torsiographs

For various reasons which will be apparent, it is convenient for designers and manufacturers of engines to have at hand reliable experimental means of determining the severity of torsional vibrations actually present in their products.

Torsiographs, i.e. instruments for recording torsional vibrations, are

available in several distinct patterns. They may be divided into two fundamentally different classes: (*a*) those which measure the alternating twist in a section of the shafting of the engine system, and (*b*) those which measure the alternating deviation from steady rotation at some point in the system.

The effect of this difference in technique will be most readily comprehended by reference to a simple example. If a system comprising two similar flywheels coupled by a shaft is acted upon by two equal in-phase alternating torques applied to the flywheels, so that the whole system rocks bodily backwards and forwards without any twist in the shaft, a torsiograph of type (*a*) recording the twist in the shaft will not register, while a torsiograph of type (*b*) applied at either flywheel will indicate the movement at the point of attachment.

One of the best-known types of twist-recording torsiograph is that developed by the Royal Aircraft Establishment at Farnborough, and known as the R.A.E. Torsiograph. A very full description of this instrument is given by Ker Wilson (reference 1). Briefly, the relative motion between the two ends of the reference shaft (which in the case of aircraft engines is the propeller shaft) is used to deflect a small mirror, as a result of which deflection a photographic record is obtained by means of a reflected ray of light. The instrument is so arranged that when there is no alternating twist in the shaft the photographic record consists of a circular trace. Torsional vibrations twisting the shaft register as radial deviations from this base circle, and the order-number of the vibration is easily determined by counting the number of lobes in the figure. Thus, a polar diagram is obtained. The amplitude of twist is proportional to the deviation from the base circle, so that the instrument supplies a good deal of useful information

Such an instrument is, however, fairly bulky and cumbersome, and necessitates the provision of a special hollow shaft for its installation. Small torsiographs of the seismic type are much more convenient from this point of view. A typical and well-known seismic instrument is the Sperry-M.I.T. Torsiograph, which also is described in detail by Ker Wilson. An inertia member is driven from the crankshaft by means of very light springs (or, in a modification described by Stansfield, reference 11, by means of grease packing), and rotates steadily at the mean speed of the crankshaft. Relative movement between the crankshaft and the steadily rotating inertia member is converted into alternating electric potential by means of the relative movement between a magnet and a coil, one of which is attached to the crankshaft and the other to the inertia member. The voltage developed is proportional to the velocity of the relative motion, and after amplification can be recorded by means of either an electromagnetic or a cathode-ray tube oscillograph. If desired, the electrical apparatus can incorporate

an integrating unit, which has the effect of rendering the final record indicative directly of displacement instead of velocity.

From careful study of torsiograph records much information regarding the torsional vibration of the crankshaft system can be obtained. Unfortunately, however, recent work has shown that the results require extremely careful interpretation if the instrument is of the seismic pattern. A recent paper by Ker Wilson (reference 12) draws attention to the difficulties, which are due to the fact that both twisting and rolling motions may be present. Measurements of displacements at certain parts of the system, as for example at the free end of the crankshaft, are not necessarily truly indicative of twisting stresses.

The type of experimental technique most favourable from the theoretical point of view involves the measurements of torsional stresses directly by means of strain gauges. Practical difficulties include the problems of affixing strain gauges to important bearing surfaces, such as those of the crank pins, without too greatly modifying the system by the machining of recesses to accommodate the strain gauges, and the provision of slip-rings to enable the necessary electrical connexions to be made to the gauges. A very useful introduction to the strain gauge technique, with particular reference to the measurement of torsional stresses, is given in the paper referred to.

Fig. 90 illustrates some typical torsiograph polar records taken from an eight-cylinder in-line Diesel engine at speeds of 450, 750, 850, and 950 r.p.m. with its coupling. These records show that there is a marked resonance effect in regard to the torsional vibrations at the highest speed, namely, 950 r.p.m. The magnitude is such that there is a variation of plus and minus 30 times the mean torque. The usual order of the torque variation at speeds other than resonant ones was about 0·75 times the mean torque. As the resonant speed, in the present example, was close to the normal operating speed of the engine, it was necessary to fit a spring coupling of special design to increase the resonant speed well above the normal speed of this engine.

Calculation of Crankshaft Stiffness

It is necessary to know the torsional stiffness of a crankshaft in order to determine its torsional synchronous speeds, but at the time of writing the analytical method of estimating this quantity is rather a complex one. There are several important factors, many of a practical nature, which have to be taken into account, making it a difficult matter to arrive at any reliable formulae by purely analytical methods.

Most of the data now employed in crankshaft calculations have been derived from experimental results, the formulae in consequence being empirical in nature.

It is not possible, owing to space and other considerations, to give a fuller account of the methods employed, but references are given on pages 138 and 139 for those wishing to pursue this subject more fully.

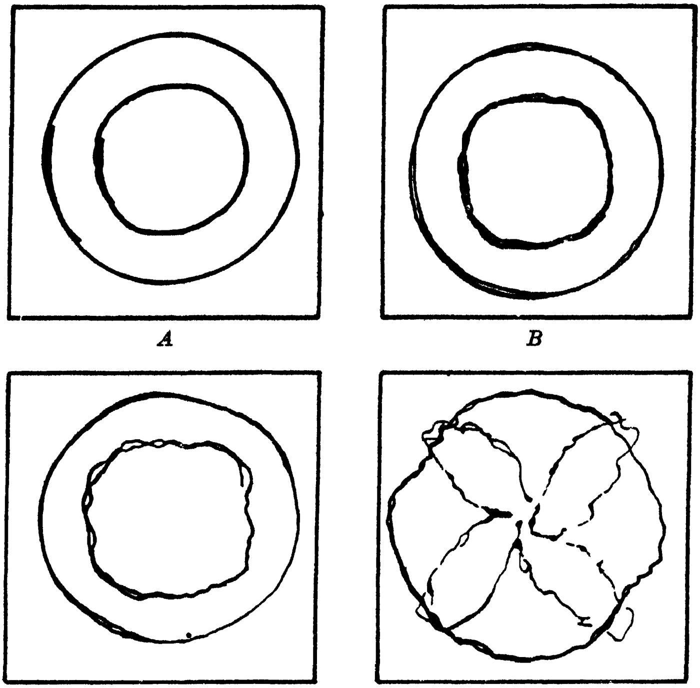


FIG 90 TORSIOGRAPH RECORDS SHOWING CRANKSHAFT VIBRATIONAL AMPLITUDES. EIGHT-CYLINDER DIESEL ENGINE

A—450 r p m , B—750 r p m , C—850 r p m , D—950 r p m

Carter's Formula

An empirical formula partly based upon rational assumptions, and partly evolved from the results of stiffness tests in regard to the values of the constants which appears to give fairly accurate results when applied to small petrol engines, aircraft engines, and marine engines, is that due to Maj. B. C. Carter. It expresses the length l of shaft, having the journal cross-section, that has the equivalent stiffness of the crankshaft under consideration.

Referring to Fig. 91 the formula for the equivalent length consists of three terms as follows—

$$l = (2b + 0{\cdot}8h) + \frac{3}{4}\left(\frac{d_1^4 - \delta_1^4}{d_2^4 - \delta_2^4}\right)a + \frac{3r}{4}\left(\frac{d_1^4 - \delta_1^4}{h \, . \, w^3}\right)$$

The first term gives the equivalent length of the journals themselves, with the addition of a term to allow for the effects of junctions with the webs.

The second term represents the equivalent length of the crankpin portion of the shaft.

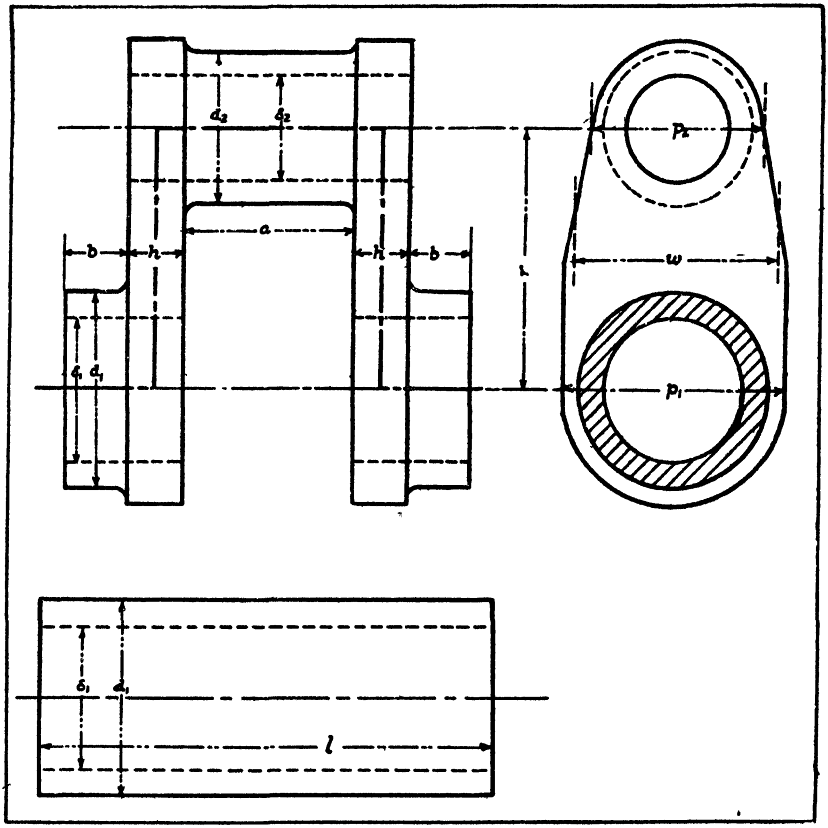


FIG. 91. ILLUSTRATING NOTATION EMPLOYED IN FORMULA FOR CRANKSHAFT STIFFNESS

The third term gives the allowance to be made for the bending effects of the webs in their own planes.

The coefficients preceding these three bracketed terms were obtained by trial and error, to give the best average agreement with the test results for shafts subject to torque in their own bearings.

Application of this formula to practical results has shown that it gives the stiffness of aero engine crankshafts to within 10 per cent

on either side; it also gives results within the same degree of error in the case of a number of marine shafts.

Other formulae that have been proposed for expressing the equivalent stiffness length l include those of Geiger and Evans; references to the original Papers are given on pages 138 and 139.

Constant's Method for Crankshaft Stiffnesses

Constant* has developed a formula which, while it is based on the results of torsion tests, yet gives an insight into the factors affecting the stiffness of a shaft and its stress distribution.

He obtains first a formula for the stiffness of a shaft when unrestrained by bearings; then the ratio of the stiffness in its bearings to its stiffness out of its bearings—known as the *stiffening ratio*—is considered. Finally these two quantities are combined together to give a semi-empirical formula for the stiffness of the shaft in its bearings.

For this purpose a number of crankshafts were subjected to static torsional tests out of bearings, and a formula evolved. Then, from the results of torsion tests on crankshafts in their bearings, the above mentioned ratio was obtained. The conditions affecting this stiffening ratio were then investigated, and a formula giving this ratio in terms of crankshaft dimensions was developed. Finally, the application of this formula to the determination of torisonal resonance speeds was briefly considered.

The formula for in-bearings stiffness gives results which do not diverge from experimental results by more than 7 per cent for shafts of reasonable dimensions.

Reference should be made to the original paper for the various formulae evolved from static tests, and to the example of a typical calculation of stiffness in and out of bearings.

The formula for in-bearing stiffness of a multi-throw crankshaft under the conditions of static test is given by Constant as follows—

$$\text{Stiffness} = \frac{\left(\dfrac{\mathrm{A}wt^3}{d_2{}^4 - \delta_2{}^4} + \mathrm{B}\right)(1 - k)}{(\mathrm{Z} + 100)\left(\dfrac{2b}{\mathrm{C}_s} + \dfrac{a}{\mathrm{C}_c} + \dfrac{2r}{\mathrm{B}_w}\right)}$$

Where C_s = Torsional Rigidity of Journal

C_c = Torsional Rigidity of Pin

B_w = Flexural Rigidity of Web

Z is a factor, expressed as a percentage, for taking into

* "On the Stiffness of Crankshafts," H. Constant. *Aer. Res. Comm. R. and M. No.* 1201 (Oct., 1928).

account the increase in twist in estimating the out-of-bearing stiffness.

A is a constant = 0·29 for marine shafts
= 1·00 for car and aircraft engine shafts

B is another constant = 91·0 for marine shafts
= 83·6 for car and aircraft engine shafts

The other quantities in the above formula are shown in the standard notation diagram in Fig. 91.

Fig. 92 shows the stiffening ratio $\left(\frac{At^3w}{d_2{}^4 - \delta_2{}^4} + B\right)$ in the above formula, expressed as a percentage, for marine, car, and aircraft engines. The agreement between the theoretical and actual test results as shown by the plotted points is very good.

In applying the results of static torsional tests to actual cases of crankshafts in engines, allowances must be made for the following factors—

(1) Bearing Clearances. If there is any slackness the stiffness will be reduced. On the other hand, if too tight it will be increased.

(2) Dynamic Stiffness. When the engine is running under its own power it has a certain mean effective, or dynamic stiffness which, together with the mass system, determines the resonance speed of the crankshaft. The dynamic stiffness is not always the same as the static stiffness due to various influences which occur, namely, the fluctuating torque reaction at the different crankpins, the torque reaction of the airscrew or flywheel and driven members, and the effect of the pressure-supplied lubricating oil.

It is interesting to note, however, that in a few cases where a direct check has been obtained by experimental observation of the synchronous speed, it has been found that the static stiffness in bearings (using pin thrust reaction) and the dynamical stiffness are identical, in the sense that the experimental resonance speed agreed closely with the value calculated on the basis of this static stiffness.

Theory of Thin Rods Method

Another method of estimating the stresses and deflections of crankshafts, in connexion with torsional vibrations is that employed by Prof. Timoshenko and later modified and extended by Southwell.

The theory treats the crank throw as in all respects equivalent to a bar *mnpqst* (Fig. 93) of which the cross-sectional dimensions are small in comparison with the lengths *mn*, *np*, *pq*, *qs*, and *st*.

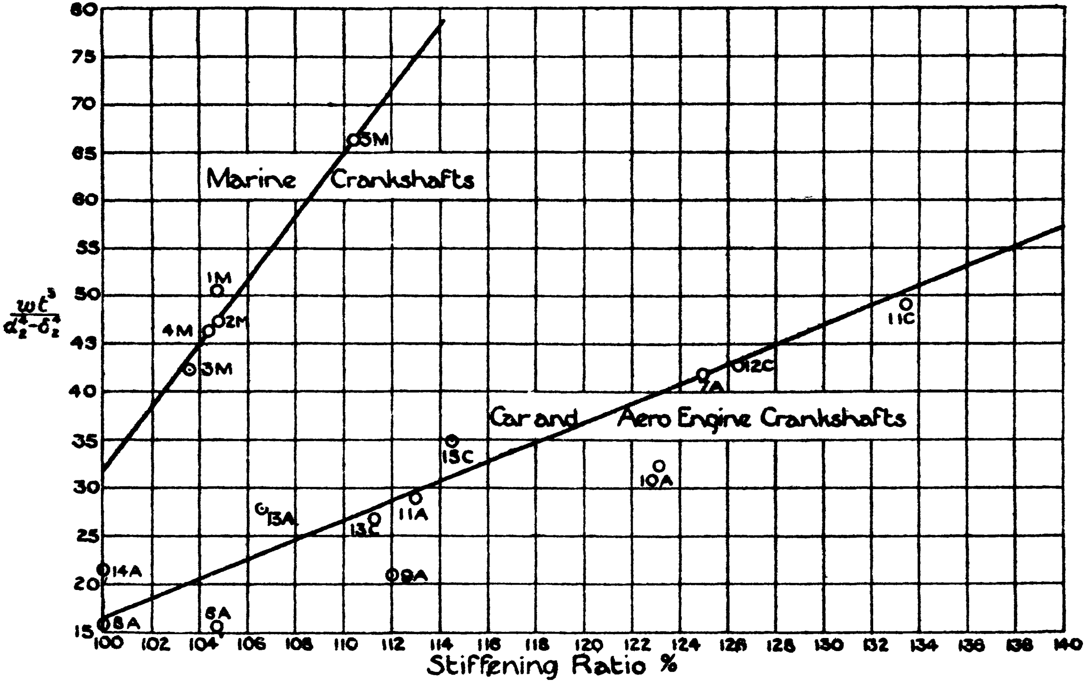


FIG. 92. RESULTS OF CRANKSHAFT STIFFNESS FORMULA APPLIED TO VARIOUS TYPES OF INTERNAL COMBUSTION ENGINE

Southwell has shown that by suitable allowances, by means of empirical constants, it is possible to obtain an expression for the effective stiffness of an encastré shaft, in terms of known quantities.

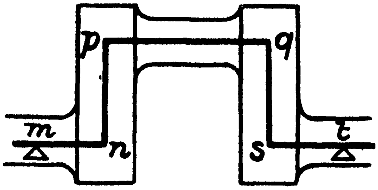


FIG. 93. ILLUSTRATING THEORY OF THIN RODS

REFERENCES

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CHAPTER VI

ENGINE MOUNTINGS

It is frequently impossible in commercial engines to avoid certain imperfections in engine balance or torque fluctuation effects so that the resultant vibrations will, in the case of solid engine mountings, be transmitted to the chassis frame or, in the case of aircraft engines, to the machine framework. In order to reduce or nullify the effects of these vibrations flexible engine mountings utilizing the energy-absorption or damping properties of steel springs or rubber are now widely used. In considerations of the design of flexible mountings the primary object is to obtain a natural vibration frequency of the engine and its mounting well below the engine vibration frequency.

It has been shown in Chapter IV that for a magnification factor of unity the natural frequency must be $\frac{1}{\sqrt{2}}$ (= 0·707) times the impressed, or engine vibration, frequency. The more the natural frequency can be reduced below this value the better will be the elastic mounting in preventing the transmission of engine vibrations to the supporting frame. Thus, if a particular engine when operating at its normal speed has a vibration, due to unbalance, of 2000 per sec., then the natural frequency of the engine and its support should be less than 1400 per sec., but for reducing the amplitudes of the vibrations sufficiently for most practical purposes a much lower natural frequency, namely, of the order of 300 to 400 per sec., would be aimed at.

It will be evident that as the engine is accelerated from its idling speed to the normal one, the unbalanced forces will increase in frequency through the natural value of 300 to 400 per sec., and at the latter frequency resonance will occur. If, however, the engine is accelerated at the usual rate it will pass through the resonance frequency speed with sufficient rapidity to prevent the vibrations from becoming appreciable. It is important also, that the engine vibration frequency at its normal running speed should not be a harmonic of the natural frequency of the engine and its mounting. In connexion with the transmission of force due to engine vibration through the flexible mounting to the engine frame or, in the case of stationary engines, to the ground or foundations, it can be shown from the formula on page 90 that the ratio of the transmitted to the disturbing or engine force is the same as the ratio of the (natural frequency)^2 to (engine vibration frequency)^2, so that if the natural frequency is one-third of the engine frequency only one-ninth of the force causing the engine vibrations will be transmitted to the frame or ground.

Engine Mounting Requirements

The general requirements of a petrol or Diesel engine mounting on any type of framework or foundation are, briefly, as follows—

(1) It must carry the static or dead weight W (Fig. 94) of the engine unit.

(2) It must withstand the torque-reaction T under maximum torque conditions and variations. This torque gives rise to an upward and a downward force F on the engine mounting, tending to place it in tension and compression, respectively.

(3) It must be "stiff" enough to lower the natural frequency of the engine and mounting to well below 0·707 times the engine frequency in whatever direction this occurs. Since the vibrations due to the unbalance of reciprocating and rotating engine components may, according to the engine type, occur in various directions, the mounting must be designed accordingly. Thus, there may be unbalanced vertical forces f occurring at engine frequency or some multiple of it; or horizontal forces f'; or rocking couples t, tending to oscillate the engine in a fore-and-aft sense.

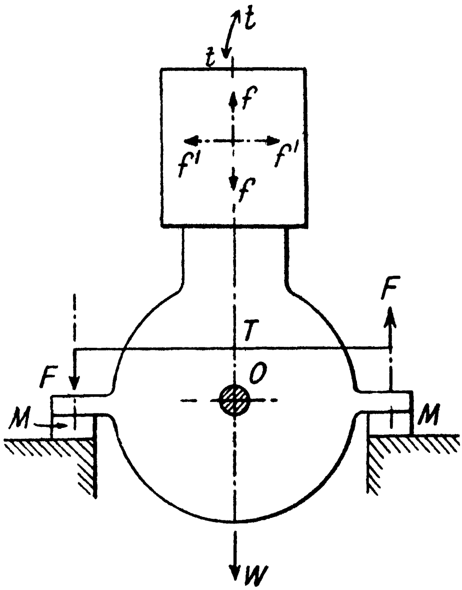


FIG. 94. FORCES ON ENGINE MOUNTING

If the engine were bolted down rigidly to its framework or foundation unit, it would greatly simplify the problem, but owing to the necessity in automobiles and aircraft to insulate the framework from the engine vibrations, the use of flexible mountings is essential, and it follows from the preceding considerations that such mountings should be carefully designed for their special purposes.

Materials for Flexible Mountings

The most suitable materials for the flexible mountings of engines are those which combine the necessary mechanical strength properties with internal friction damping qualities. In addition, it is desirable that the material employed should have good sound insulating properties, and be strongly resistant to the effects of corrosive agents

and abrasion. Thus, in the case of an automobile engine mounting, sound insulation is necessary from the viewpoint of the occupants. Moreover, since the engine mounting material is liable to be exposed to the action of petrol, oil, water, and the atmosphere, it must offer satisfactory resistance to deterioration under these influences; further, it must be unaffected by engine mounting temperature effects.

For some requirements the mounting material is required to afford good electrical insulation, although in the case of the modern automobile, with its single-pole (earth-return) electrical system, this is apt to be a drawback, for rubber-mounted engines must then have a separate flexible earthing conductor to the chassis frame. Among the available engine mounting materials are *plain carbon* and *alloy spring steels*, *rubber*, *felt*, *cork*, and certain *flexible plastics*.

Although compression springs have been used in the past for stationary engine and machine mountings, they have not been found suitable for aircraft and automobile engines, since they do not act as sound insulators, are less effective than the organic materials mentioned and are, in general, more complicated in design and costly.

Of the other available materials, whilst both cork and felt are excellent for absorbing vibrational effects of certain components and are much used for instrument mountings and lighter units subject to vibrational effects, they are ruled out for petrol engine purposes chiefly by mechanical strength considerations.

From all points of view, rubber appears to be the best of the available materials, since not only has it the desirable strength and damping qualities, but it can now be bonded very firmly to metals, such as steels.

Rubber for Engine Mountings

Rubber, of the natural or artificial grades, possesses satisfactory tensile, shear, and compressive strength, combined with exceptional elastic strain and a high hysteresis. The mechanical properties of rubber vary considerably with the grade, i.e. the proportions of rubber, fillers, accelerators, and other ingredients, but for a good quality automobile black rubber, a tensile strength of 3500 to 4200 lb. per sq. in., with an elongation of 500 to 700 per cent, i.e. an ultimate stretch before fracture of five to seven times the original length, is obtained. The shear stress is about the same as the tensile strength, whilst the compressive strength is several times greater than its tensile strength.

The *elastic modulus* of rubber is a variable quantity, since there is no straight portion in the stress-strain curve, but for ordinary engineering purposes it is usual to take the mean slope of the stress-strain curve for a limited extension of about 200 to 300 per cent and to estimate the elastic modulus from this.

The value of the elastic modulus, obtained in this way, for high-grade rubbers, is from 300 to 400 lb. per sq. in.

The *hardness* of rubber on the Shore scale varies from 25° for the softest rubber to 100° for the hardest, e.g ebonite. The engineering rubbers have hardnesses between about 35° and 75° Shore.

In regard to the *hysteresis factor*, from the point of view of engine mountings and also automobile springing in general, rubber absorbs a far greater amount of energy for its weight than other common materials, on account of its greater stretching properties. In other words, the area of the loop on the stress-strain loading and unloading diagram is, relatively, very large. Thus, a high grade of rubber can absorb from 500 to 1000 ft.-lb. of energy per pound weight, whereas spring steel is capable of absorbing from 10 to 20 ft.-lb. per pound, only. The work lost in hysteresis in low-grade rubber may be as much as 70 per cent of the work done in the first extension; for high-grade rubbers the hysteresis loss varies from 35 to 40 per cent.

It has been possible to give a very brief account only of the more important properties of natural rubbers, insofar as they concern the subject of engine mountings. For a much fuller account the reader should consult the reference given in the footnote.*

It may here be mentioned that some of the synthetic rubbers, such as Neoprene, Buna, and Perbunan, possess good mechanical properties and are much more resistant to the action of sunlight, petrol, oil, and grease, and to oxidizing and temperature effects. Usually, however, they have appreciably higher permanent sets and are not so good as natural rubbers in regard to torsional loads.

Bonded Rubber Mountings

Rubber can be used in tension, shear, or compression in flexible mountings. Of these alternatives, the former is seldom employed, since the effect of surface cuts, more readily made in stretched rubber, is to cause a rapid breakdown throughout the section under stress. The most favoured applications are those of shear and compression, and in order to apply these methods it is necessary to unite or bond the rubber to the metal stress-transmitting supports of the engine and also to the framework mounting member.

It has been demonstrated that if the surfaces of the metal are brass-plated and then coated with a protective layer of rubber paint and a thin covering of specially-prepared rubber, the main bulk of the rubber compound can be vulcanized to these prepared surfaces. Usually the rubber mountings are heated to 120° to 150° C. between steam-heated platens, and pressure of ¼ to 2 tons per sq. in. is applied in special

* "Rubber and Its Compounds," A. W. Judge, *Engineering Materials*, Vol. II, Chapter XII. Pitman, London.

hydraulic presses for a period of about 30 min., when the rubber becomes so firmly bonded to the metal that it will give ultimate bond strengths, in tension, of 800 to 1300 lb. per sq. in. Under compression loading the bond strength is much higher, and loadings of about 10,000 lb. per sq. in. can be applied to a high-grade rubber-metal unit without ill effect; in compression loading the limiting factor appears to be the degree of permanent set rather than bond failure.

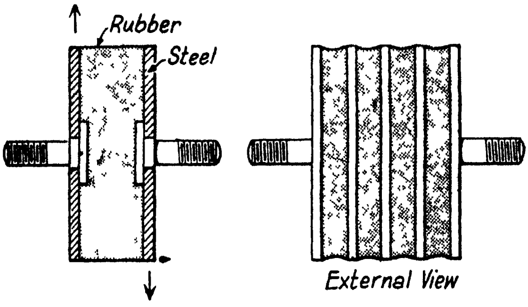


FIG. 95. (A) SANDWICH MOUNTING. (B) MULTI-PLY TYPE MOUNTING

In torsion a relatively large angle of twist can be given before the bond is affected. Thus, in the case of one coupling* of about 3 in. internal diameter and 6 in. external diameter, with an average length of 3 in., the rubber was repeatedly stressed under a torque of over 3000 lb. ft., and an angular deflection of over 300° was obtained.

In regard to temperature effects, natural rubber bonds offer a good resistance, and up to 150° C. the type of failure is the same as that obtained at normal air temperatures, above this temperature the properties of the rubber are affected, e.g. the tensile strength and tearing resistance are reduced, progressively. With certain synthetic rubbers, appreciably higher temperatures are attained before ultimate failure.

Types of Bonded Rubber Mountings

There is now a wide range of bonded rubber mountings for various purposes from those for light delicate instruments to others applicable to heavy machinery and large engines. In the lighter applications of such mountings the rubber is used in shear, as they have a greater resilience for the same load-carrying capacity than the compression type.

Perhaps the simplest design of shear mounting is the *sandwich type*, consisting of two metal plates with one or more studs rigidly affixed

* "Bonding of Rubber to Metals," S. Buchan, *Trans. Inst. Rub. Industry*, Vol. 19, No. 1.

to each, with the rubber element between, as shown in Fig. 95. One plate is bolted to the vibrating member and the other to the frame; alternatively, when it is desired to insulate a member such as an instrument against panel vibrations, the same type of mounting could be used, although there are now improved designs for this purpose.

The sandwich type mounting can also be used as a compression unit, but where greater stiffness is required the multi-ply form is employed. Another type of shear mounting is the concentric cylinder-disc pattern shown in Fig. 96. This has been used for aircraft engines, special trunnions A being bolted to the sides of the engines to take the elastic mounting units, whilst the outer members of the latter are held in split bearings in the aircraft engine frame; usually, there are four of these mountings, namely, two on either side of the engine. Variations of the engine mounting shown in Fig. 96 are obtained by dishing the bonding discs in order to obtain progressive deflection or to restrict longitudinal vibrations. (Fig. 97.)

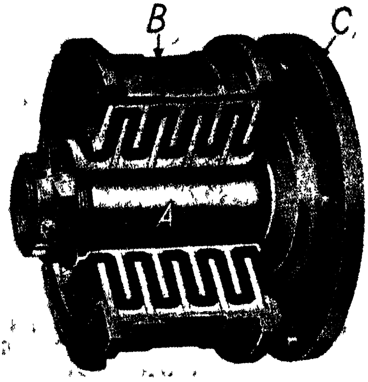


FIG. 96. AIRCRAFT ENGINE TRUNNION TYPE MOUNTING

A—trunnion, B—frame mounting, C—engine flange coupling (Metalastik)

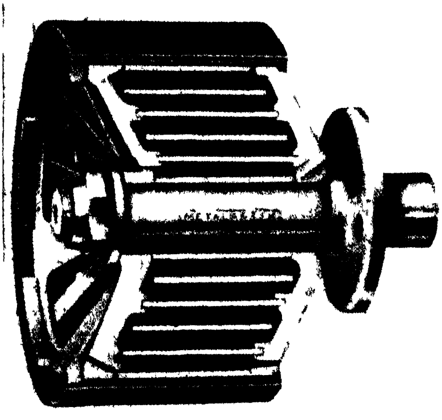


FIG. 97. PROGRESSIVE DEFLECTION TYPE ENGINE MOUNTING

For compression-type mountings, which usually give a higher natural frequency to the unit and mounting than shear mountings, the single or multiple plate sandwich patters are also applicable. The cone-form shown in Fig. 98* has almost equal resilience in all directions and is comparatively simple to apply to light engines and machines.

The designs of bonded rubber mountings used for automobile engines vary considerably, but the more widely employed types are of the

* By courtesy of *The Oil Engine* and André Rubber Co., Ltd.

shear, compression, or shear-compression classes, and are often more complicated than the simple compression and shear types shown in Figs. 95 (A) and (B). The angle-bracket type of mounting shown in Fig 99* has been much used for mounting vertical petrol and Diesel

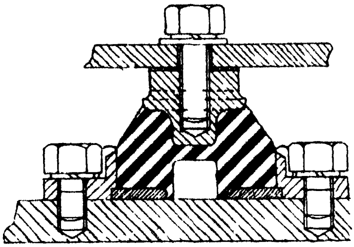


FIG. 98. CONE COMPRESSION TYPE ENGINE MOUNTING

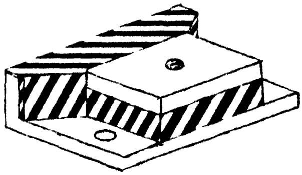


FIG 99. ANGLE BRACKET TYPE OF MOUNTING

engines. There, the engine bearers or base plate are bolted down to the upper plate member, whilst the angle unit is secured to the framework or foundation.

Another type of flexible mounting, known as the *horse-shoe type*,

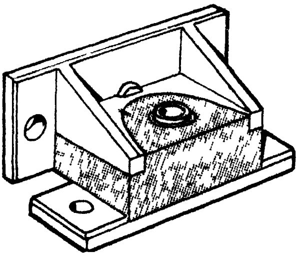


FIG. 100 HORSE-SHOE TYPE MOUNTING

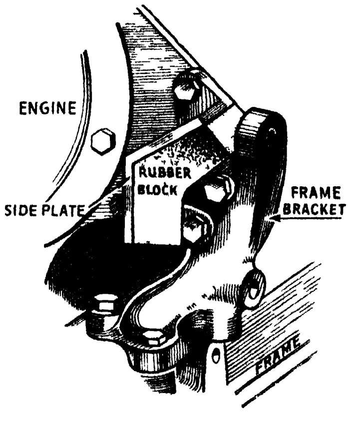


FIG. 101 AUSTIN ENGINE MOUNTING

is depicted in Fig. 100. In this application the upper metal bracket is bolted to the side of the engine, whilst the lower one is secured to the framework or foundation. Several of these units—usually four—are employed for the ordinary "in-line" engine.

The Austin car engine has a four-point flexible mounting consisting of two angle blocks similar to those shown in Fig. 101 at the front and two

* *Vide* footnote, page 145.

at the rear of the engine-gearbox unit in front of the propeller shaft. The more recent engines have a rear mounting consisting of a short metal frame at each side, with rubber blocks at both the gearbox unit and chassis frame ends, so that the earlier single flat block is now replaced

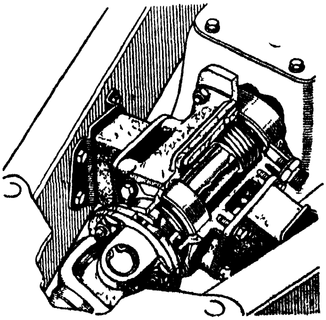


FIG. 102. REAR DOUBLE MOUNTING OF AUSTIN ENGINE

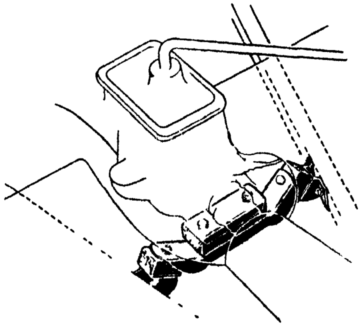


FIG. 103. AUSTIN GEARBOX MOUNTING

by two separate blocks, one at either end of the metal frame connecting member. Fig. 103 illustrates the rear double-mounting of another Austin engine and gearbox unit, this mounting consists of two sandwich shear members between the gearbox and chassis frame. (Fig. 104.)

A method of flexible mounting used on Vauxhall cars shown in Fig. 105 employs rubber blocks with "Z"-shaped slots. Normally, the block has the shape shown at A, but under torque or inclined loads the slot tends to close up, as shown at B, and the resistance then increases with the side load.

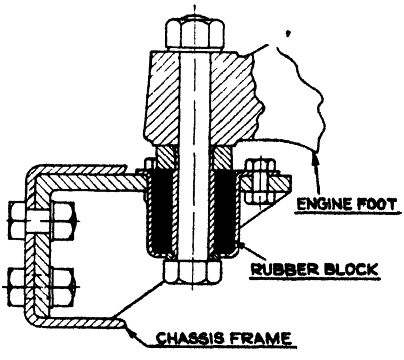


FIG. 104. CONCENTRIC TYPE CAR ENGINE MOUNTING

The Chrysler " Floating Power " Method

A method known as the "centre-of-gravity" support, that has been employed with very satisfactory results for the Chrysler car engines, aims at arranging the supports so that the effect is that of a centre-of-gravity support. The supports are so positioned and the vertical and horizontal stiffnesses selected to give the desired result under the combined weight, torque, and any unbalanced engine-derived vibrations. The method of attaining the results, illustrated in Fig. 106, is to provide suitable flexible supports at A, B, and C, to ensure that the

rocking axis passes through the centre of gravity and universal joint centre. The front mounting A (Fig. 107) consists of a Metalastik rubber-bonded metal unit of the compression type, attached to a channel-shaped engine cross-bearer which is in turn affixed to the chassis frame. The mounting B consists of two pairs of rubber compression members, each pair arranged on either side of a radial metal lug on the gearbox. Only one of the two mountings is shown in Fig. 107, the other similar symmetrical unit is on the other side of

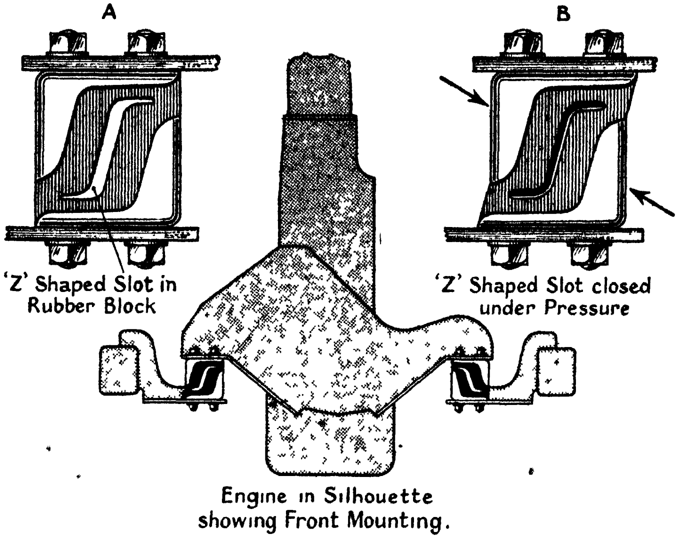


FIG 105. VAUXHALL ENGINE MOUNTING

the gearbox. The rear mounting C is a curved compression-shear unit mounted to a channel-section cross member of the chassis frame.

Three-point Engine Mounting

Although both three- and four-point engine flexible mountings are employed for automobiles, the former method has certain advantages In this method the engine torque is taken by the two rear members and a trunnion mounting is used at the front end of the engine. With the three-pointed method of mounting it is impossible to set up a strain in the engine unit due to chassis frame distortion, whilst the arrangement gives a low torsional natural frequency and therefore an efficient means of insulating the chassis from engine vibrations.

Flexible Connexions to Engine

An important point to remember when installing engines on flexible mountings is that engine connexions to the fixed framework must

also be made flexible in order to prevent fracture and to avoid any restraint or interference in the vibration damping system. Thus, in the case of car engines, the exhaust pipe must be flexibly supported to the chassis frame, whilst the petrol piping from the fuel tank to the

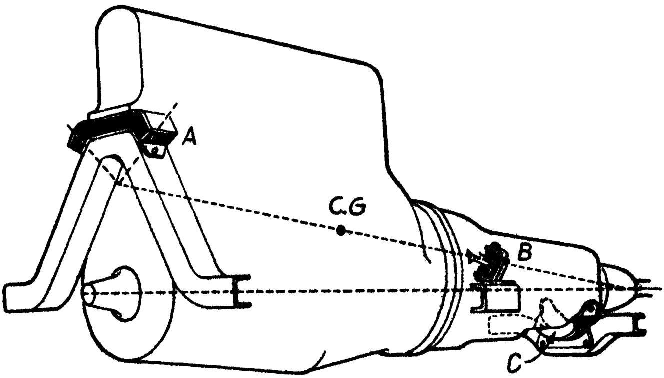


FIG. 106. CHRYSLER "FLOATING POWER" METHOD OF ENGINE MOUNTING

carburettor must have a flexible section. Usually, this is effected by mounting the fuel pump on the engine side of the dashboard and using a flexible braided-rubber connexion to the carburettor. Mention has already been made of the necessity for earthing the engine unit to the

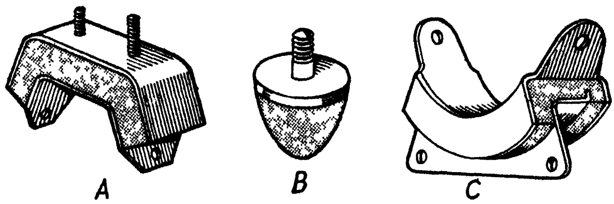


FIG. 107. BONDED RUBBER SUPPORTS FOR CHRYSLER ENGINE MOUNTING

chassis frame in "earth-return" electrical systems with flexible rubber mountings.

Aircraft Engine Flexible Mountings

As mentioned previously, the trunnion type flexible mounting shown in Figs. 96 and 97 is employed for "in-line" (and also Vee-type) engines, and has the merits of effective vibration insulation combined with ease of installation.

In the case of radial engines, an analysis of the unbalanced forces and vibrations caused is necessary in order to determine the best method of engine mounting. In this connexion the reader is referred

to the paper mentioned in the footnote,* which gives an analysis of the problem of radial engine vibrations and an account of a dynamic suspension method using rubber mountings of the Metalastik type.

The flexible mounting employed for certain Bristol radial engines is shown in Fig. 108†. The mounting consists of nine rubber shear

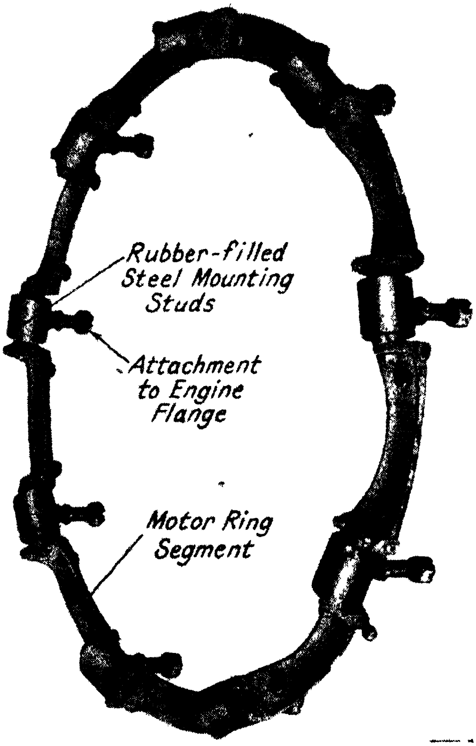


FIG. 108. BRISTOL RADIAL ENGINE FLEXIBLE MOUNTING

mountings spaced equidistantly between light alloy support brackets of long channel section. This ring is attached to the front face of a conical pressed metal mounting which is secured to the fuselage frame. The nine studs from the rubber mounting units protrude rearwards through clearance holes provided in the cone mounting flange for this purpose. These studs are used for the attachment of the engine to the flexible mounting ring. The rubber blocks effectively damp out

* "Dynamic Suspension—A Method of Aircraft Engine Mounting," K. A. Brown, *S.A.E. Journal*, May, 1939 (Reprint by Messrs. Metalastik, Ltd., Leicester, available).

† *Aircraft Engines*, Vol. 2, A. W. Judge (Chapman & Hall. Ltd.).

engine torque vibrations, whilst ensuring the requisite stiffness against the inertia effects due to the engine's weight; the mounting is not affected by gyroscopic torque.

Typical Aircraft Engine Vibration Investigation

The following outline of the method employed successfully to ascertain the nature of the engine vibrations* and to effect a cure may serve as an example of the application of experimental methods in solving vibration problems.

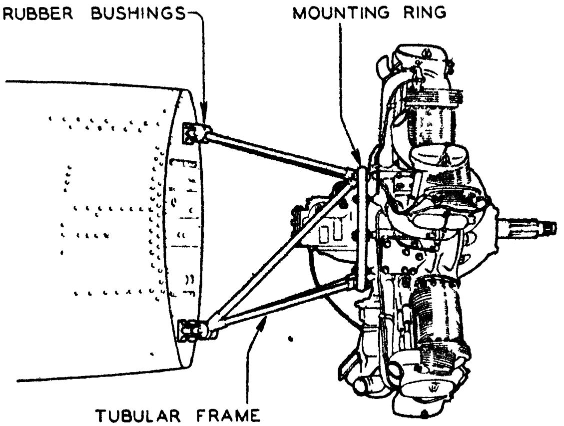


FIG. 109. RADIAL ENGINE MOUNTING INVESTIGATION

The engine in question, a five-cylinder air-cooled radial, was prone to excessive vibrations at its normal operating speed range, causing structural failure of the airframe, cracking of the fuselage skin, and shearing of rivet heads. In a few instances crankshaft failure occurred during flight.

The original lay-out of the engine mounting is shown in Fig. 109. The engine was supported in a cantilever manner by three metal tubes from the mounting ring of the engine to three rubber bushings on the fuselage firewall bulkhead. The bushings were arranged radially at 120° apart.

To determine the source of the trouble electrical pick-up units (to measure the vibrations) were mounted on the engine in such a way that they could detect and record vibrations in any of the six degrees of freedom which are existent in the case of a body suspended in space (Fig. 110). The pick-ups were located, as shown, in pairs, equidistant from the centre of gravity of the engine. The outputs of

* "Combating Vibration in Aircraft Power Plants and Their Mountings," T. D. Copeland and G. Getline, *Autom. and Aviation Industr.*, 1st July, 1944.

these pick-ups were fed through an integrating amplifier to obtain a measure of the amplitude and then to a vibration wave separator which added and subtracted the alternating voltages from the pick-ups. These were then transmitted to a recording oscillograph, which gave a permanent record of the vibrations. Readings were obtained from

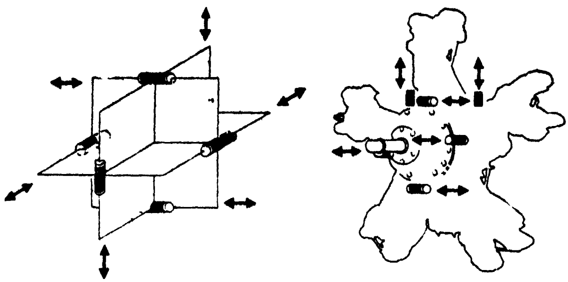


FIG 110 DEGREES OF FREEDOM IN RADIAL ENGINE

each pair of pick-ups to determine the following movements in the six degrees of freedom, namely, (1) roll, about the thrust axis, (2) pitch, about the lateral axis, (3) yaw, about the vertical axis (4) lateral translation; (5) vertical translation.

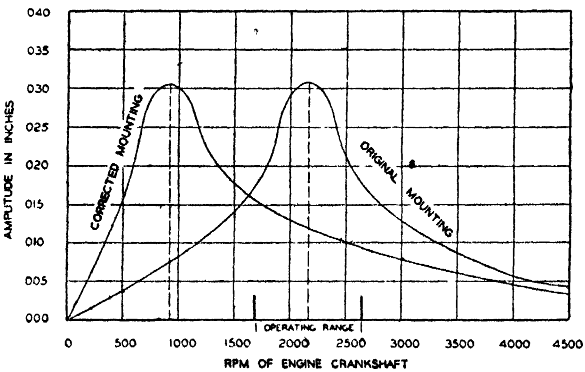


FIG. 111. RESULTS OF RADIAL ENGINE VIBRATION TESTS

In addition, a torsiograph was attached to the rear end of the crankshaft to study its vibration characteristics, and these movements were also recorded on the oscillograph chart.

The aircraft was then flown horizontally, at various speeds, and a record was made on the oscillograph at each different engine speed. The vibration records were subsequently analysed harmonically and graphs of harmonic motion and engine speed were then made; one of these is shown in Fig. 111. It will be observed that with the original

mounting engine resonance at crankshaft speed frequency occurred within the operating range, thus indicating the cause of the vibration trouble. To remedy this, the bushings were removed from the firewall bulkhead and placed at the mounting ring as the original was a highly-coupled system. The bushings were arranged to *support effectively the power plant at its centre of gravity*, and at the same time the stiffness of the bushings was reduced in order to make the natural frequency of the engine plus mounting occur at a lower speed, as shown by the left-hand graph in Fig. 111. The crankshaft was fitted with a pendulum-type damper, tuned to the $7\frac{1}{2}$th order, so that the vibration forces previously occurring in the crankshaft were absorbed by the damper. As a result of these changes the earlier vibration troubles were eliminated and the engines proved quite satisfactory in their aircraft.

In connexion with the five-cylinder radial engine it is known that this produces forcing functions of $\frac{1}{2}$, 1, 2, and $2\frac{1}{2}$ times engine speed. Further, the propeller generates forces due to its own static, dynamic and aerodynamic unbalance as well as the forces that result from the resistance of the air about points of obstruction of the engine, cowling, and other items. As a four-cycle five-cylinder engine fires every second revolution for each cylinder, the forces due to the gas pressure torque cycle are $\frac{1}{2}$ order functions. The inertia forces and couples due to unbalance of the rotating parts are of the 1st and 2nd order. It was from a consideration of these forcing functions that the records obtained, in the example quoted, were analysed, and from the results it was shown that 1st order engine resonance occurred within the operating range of the engine and also that the crankshaft had a critical speed within the same range due to $7\frac{1}{2}$th order excitation.

CHAPTER VII

THE BALANCING OF ROTATING PARTS

THE balance of the working parts of an engine is a very important factor in engine design. A perfectly-balanced engine is one in which the relative motions of the component parts have no tendency to make the engine vibrate as a whole.

The elimination of vibration, especially in automobile and aeronautical practice, should be one of the chief objects of every designer of high-speed internal combustion engines.

An engine, if properly balanced, would, if suspended perfectly freely in space, exhibit no vibratory or other movements. In connexion with this method of regarding engine balance, mention might here be made of an experimental apparatus due to the late Prof. Perry, consisting of four discs keyed to a shaft driven by means of a belt from an electric motor.

The discs can be made to be out-of-balance to any extent; and since the whole frame carrying the motor and shaft bearings is suspended by wires from an external support, any out-of-balance effect of the discs when rotating causes the whole frame to vibrate or move.

If an engine be perfectly in balance, it will, theoretically, require no foundation bolts or means of holding it to the ground, but if unbalanced, the reactions of the movements of the unbalanced parts will be transmitted to the foundations or frame, through the bedplate or supporting arms.

Contributory Causes of Engine Vibrations

There are three principal sources of vibrations that may occur in high-speed petrol and compression engines, namely—

(1) The cyclical changes of torque upon the crankshaft assembly due to varying piston loads and connecting-rod obliquity. The effects of these torque fluctuations have already been considered.

(2) The unbalanced effects of centrifugal forces on rotating members, such as the crankshaft and camshaft.

(3) The unbalanced effects caused by the reciprocating members of the engine, such as the pistons and upper parts of the connecting rods, the valves, and springs.

It is proposed in the present chapter to deal exclusively with the second contributory cause, namely, the centrifugal force effects on unbalanced rotating members, and methods of balancing these. The effects of the forces due to the reciprocating masses is considered in the chapter that follows.

Effects of Vibrations

In the case of vibrations caused by unbalanced centrifugal forces, the magnitudes of the latter increase as the square of the speed of rotation, so that at the high operating speeds of engines their effects

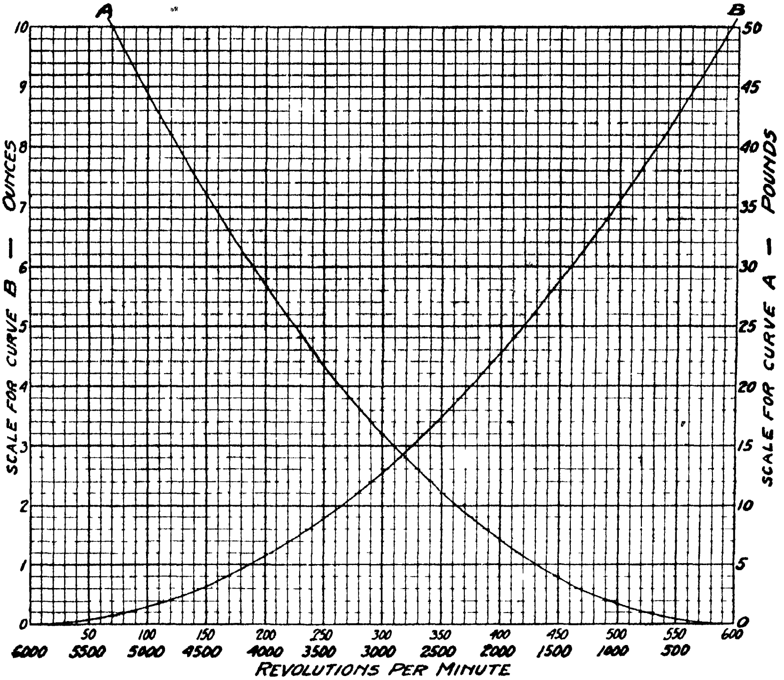


FIG. 112 CENTRIFUGAL FORCES AT VARIOUS SPEEDS FOR 1 OZ WEIGHT AT RADIUS OF 1 IN

are apt to become serious. Thus the increased amplitudes of vibration, apart from causing noisy operation, tend to loosen the connexions of components, nuts, and screws. In addition, the high frequency alternating loads associated with such vibrations may result in fatigue stresses in some of the members. Further, in the case of unbalanced rotating members, the loads on the bearings are increased, with greater wear effects and resultant power loss. Moreover, should the period of vibration happen to coincide with the natural vibration period of any component—or of the engine unit as a whole—excessive and possibly dangerous vibrations will be set up.

Masking Effects of Mass

In the case of slow speed machines the external effects of unbalance are not usually serious, more especially if relatively massive foundations

are employed, but the absence of any appreciable vibrations must not be taken as an indication of satisfactory balance of the machine components, nor does a massive foundation and heavy construction of a machine remove the harmful effects of internal vibrations upon the bearings and other related parts. It follows, therefore, that the method of testing the balance of a machine or engine at slow speeds by noting the absence of vibration is unreliable. The only satisfactory method is to check the balance of the rotating or reciprocating members independently and, if necessary, at the maximum operating speeds.

Centrifugal Force

If a mass M be rotated about a fixed centre, with an angular velocity ω, and if R be the radius of the circular path described, then the body will experience a radial force F acting outwards given by $F = M \cdot \omega^2 \cdot R$, or writing $\omega = 2\pi N$, where $N =$ the number of revolutions per second,

$$F = M \cdot 4\pi^2 N^2 \cdot R$$

If R is in feet and M in pounds, then the value of the force in pounds weight is given by

$$F' = \frac{M}{g} \cdot 4\pi^2 N^2 \cdot R$$

Taking the value of $\pi = 3{\cdot}141593$ and $g = 32{\cdot}19$ ft. per sec. per sec.

$$F = 1{\cdot}2264 MN^2 \cdot R \text{ lb.}$$

If the velocity of the rotating body be V ft. per sec. then the centrifugal force may be written as

$$F = \frac{MV^2}{gR}$$

In regard to quantitative values, it is useful for balancing computations to be able to express the centrifugal force in terms of ounce-inches for various speeds expressed in terms of revolutions per minute.

Thus, if $m =$ weight in ounces and $r =$ radius in inches, the centrifugal force is given by:

$$F^1 = \frac{m\, 4\pi^2 N^2 \cdot r}{12g} \text{ oz.}$$

this reduces to the following form, namely,

$$F^1 = 0{\cdot}1022\, mN^2 r \text{ . oz.}$$

The values of F and N are shown plotted in Fig. 112,* in which the

* *Balancing Machines*, T. Olsen.

Curve A shows the centrifugal force in pounds for speeds up to 5300 r.p.m. and Curve B the centrifugal force in ounces for speeds up to 600 r.p.m., for a weight of 1 oz. rotating at a radius of 1 in.

General Case

As a general case, consider a body of mass m, whose centre of gravity is moving in any path whatever Let r be the radius of curvature, Fig. 113, of the path of the C.G. at any instant. Then the centrifugal force acting upon the body is

$$f = m\omega^2 r$$

and f will be a function of both ω and r.

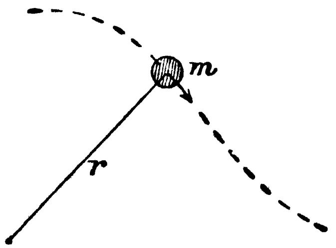


FIG. 113

Proceeding from this general case, imagine a number of particles of masses, m_1, m_2, m_3, . . . revolving about a common centre in the same plane at different radii r_1, r_2, r_3, . . . and each moving with an angular velocity ω, as if attached to the same rotating disc. Then the axis of rotation of the disc will be subjected to a number of pulls of magnitude $m_1\omega^2 r_1$, $m_2\omega^2 r_2$, $m_3 . w^2 . r_3$, . . . in the respective directions of the radii r_1, r_2, r_3, ...

The resultant pull upon the axis will be represented by the resultant of all the component pulls, that is, by vector sum of

$$\omega^2 \quad (m_1r_1 + m_2r_2 + m_3r_3 + . . .)$$

The resultant, as given by the force diagram, will represent the single centrifugal force equivalent to the component forces, and an equal and opposite force to this will completely balance the system of component pulls.

Thus, if the vector sum of the forces be a single force of magnitude X, the forces may be balanced by a mass M, at a distance R from the axis, and in a direction diametrically opposite to the single resultant, provided that M . R = X.

It is obvious that any mass M, or any radius R, can be chosen to suit other conditions, provided that the product fulfils the above condition.

As an example, consider the case of two masses M_1 and M_2 at radii R_1 and R_2 respectively (Fig. 114).

It is required to balance these by a single mass placed at a radius R.

Draw AB parallel to the radius R_1 and proportional to M_1R_1, and AC parallel to the radius R_2 and proportional to M_2R_2. Then BC will

represent in magnitude and direction the resultant single centrifugal force, say, F.

Hence the two masses M_1 and M_2 may be balanced by a mass $= \frac{F}{R}$ placed as shown.

This procedure may be followed in a simple application. The weight

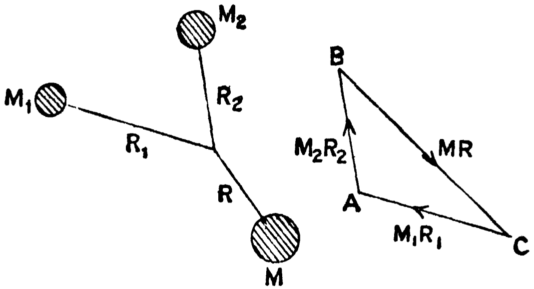


FIG 114. VECTOR METHOD FOR RESULTANT FORCES

of the webs and crank pin of a single crank are assumed to be of value M, and it is required to balance this by two webs, such as is diagrammatically shown in Fig. 115, upon the other side of the crankshaft (i.e. extensions of the crank webs).

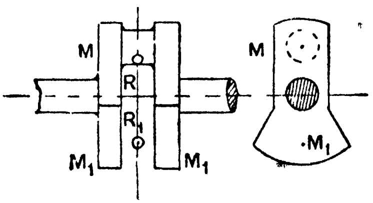


FIG. 115

Let R — distance of C.G. of main crank from the axis of rotation and M_1 = mass of each web balance weight, the C.G. of which is at R_1 from centre.

Then $M\omega^2 r = 2M_1\omega^2 R_1$

or $$M_1 = \frac{Mr}{2R_1}$$

Centrifugal Couples

If a shaft AB of length a be provided with arms of equal length r at its ends, and equal weights M be attached to these arms (Fig. 116), then the centrifugal force upon each arm when the shaft is rotating will be $M \cdot \omega^2 \cdot r$, and the shaft will experience a couple of moment $M \cdot \omega^2 \cdot r \cdot a$ tending to rotate or twist it in the manner indicated by the arrow, that is, about an axis perpendicular to the direction of rotation and of the centrifugal forces themselves. It is important to notice that this couple will be of constant magnitude, but of varying direction—the axis of the couple rotating in a plane perpendicular to the axis of the shaft and at the same angular velocity.

Such a couple would occur in the case of a two-cylinder engine with cranks at 180°, and would remain unbalanced.

The Balancing of Crank Webs

It has been shown that each crank pin and its webs, which give rise to radially acting centrifugal forces, can be balanced by means of a pair of balance weights upon the opposite side of the crankshaft. In practice, however, this is not always done—in fact, it is the exception rather than the rule. In multi-cylinder car engines the difficulty of obtaining uniformity in manufacture and the cost of providing such weights, which is considered out of proportion to the results obtained, are the main reasons given for dispensing with balance weights, except in the more expensive cars.

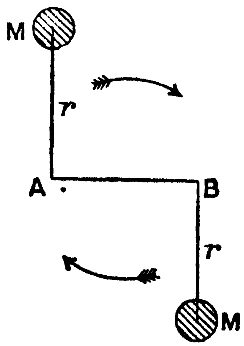


FIG. 116

The result of leaving these cranks unbalanced is to increase the bearing pressures upon the main bearings, and to cause severe bending actions to occur upon the crank case.

Taking the case of a four-cylinder crankshaft (Fig. 117) of the three-bearing type, with a cylinder bore and stroke of 4 in. and 6 in. respectively, at a speed of 1500 r.p.m., there is a resultant centrifugal force of 1700 lb. at each crank pin. These unbalanced forces cause additional pressures upon the main bearings of 700 lb. each for the end and 1400 lb. for the central bearings.

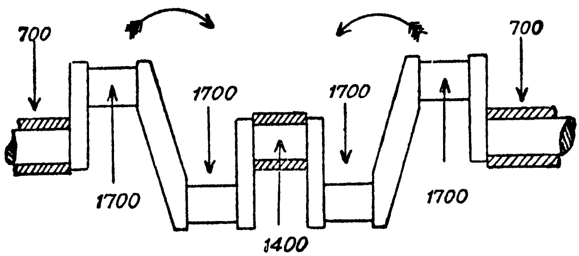


FIG. 117. BALANCE OF FOUR-CYLINDER ENGINE

These forces rotate with the direction of the plane of the webs, that is, at crankshaft speed, and give rise to an alternating bending action upon the crank case, which may cause appreciable distortion, the magnitude of this bending moment in the example given being 8500 lb.-in., which is equivalent to half a ton acting at the centre of the crank case, if it is taken as supported at the ends.

Balance of a Single Mass

The next consideration involves the balancing of both forces and couples due to a single weight of mass M. The possible solution by

using two balance weights is represented in Fig. 118, in which the conditions for balance are—

(1) That the resultant centrifugal force must be zero. For this,

$$M \cdot R = M_1R_1 + M_2R_2$$

(2) That the resultant centrifugal couple must be zero. For this,

$$M_1R_1x = M_2R_2y$$

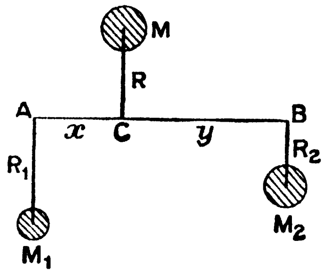


FIG. 118

(3) The weights M, M_1, and M_2 must be in the same plane, containing AB.

It will thus be evident that the rotating parts can be perfectly balanced by suitably disposed weights, and that the balance of the cranks of any type of engine is only a matter of the application of the foregoing methods.

General Case of Rotary Parts Balance

A number of rotating masses in different transverse planes are to be balanced by two masses in two given transverse planes, the magnitudes of these masses to be found.

(1) *Graphical Method.* Consider the crankshaft OZ as shown in Fig. 119. Let A and B be the given transverse planes of the balance weights whose masses are to be found. We are given the mass m, radius r, and angular direction α, and also the distance x, from reference plane A for each of the rotating masses, so that the position of any mass can be determined by xmr_{α}.

First consider the centrifugal forces acting, then every centrifugal force like $m \cdot r$ (unit angular velocity of 1 radian per second being assumed throughout) can be replaced by a force mr in the reference plane A and a centrifugal couple of moment mrx perpendicular to the direction of the force in the plane A. The direction of the axis of such a couple is indicated in the figure by the broad lines parallel to the arrow. All of the centrifugal forces can therefore be replaced by equivalent forces in the reference plane A and couples.

We can now proceed to find the resultant of all the couples referred to the plane A.

A "couple" polygon is drawn, the sides of which, being drawn in the proper order, are proportional to the magnitudes of the couples, and in the same direction as the axes of the couples. The closing side of this polygon gives the magnitude MRX and direction α_B of the resultant couple.

Since X is given, we know MR, and can choose either M or R for

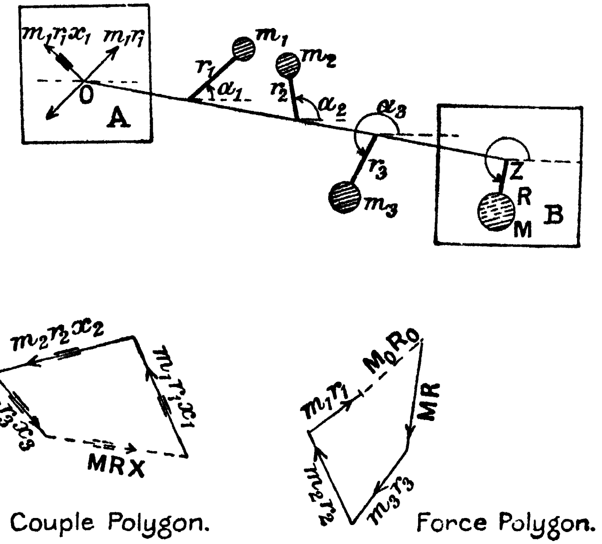


FIG. 119

convenience, so long as the product is constant, and we also know the direction of the arm of the balance weight M, namely, $(\alpha_B + 180°)$. This gives the position of balance weight, and its value, in plane B.

The resultant of all the centrifugal forces (including MR) referred to the plane A is similarly obtained by a diagram of forces, and the closing side gives the magnitude M_oR_o and direction α_A of the resultant force in plane A.

An equal force M_oR_o in an opposite direction $(\alpha_A + 180°)$ gives the magnitude and direction of the equilibrant, and the position of the balance weight in plane A is now known.

This graphical method is equivalent to solving the vector equations—

$$(a)\quad m_1r_1x_{1(\alpha 1+90)} + m_2r_2x_{2(\alpha 2+90)} + \text{etc.} + \text{M} . \text{R} . \text{X}_{(\alpha_B+90)} = 0$$

and $$(b)\quad m_1r_{1\alpha} + m_2r_{2\alpha} + \text{etc.} + \text{M}_o\text{R}_{o\alpha\text{A}} + \text{MR}_{\alpha\text{B}} = 0$$

(2) *The Analytical Method.* In order to obtain an analytical solution, the couples and forces are referred to the plane A, as before.

A pair of convenient axes OX, OY at right angles is taken, and the couples and forces resolved along them.

Since the resolute of any number of forces in a given direction is equal to the resolute of their resultant in the same direction, we get the following relations—

$$\text{MRX} \sin (\alpha_B + 90) = \Sigma m_1 r_1 x_1 \sin (\alpha_1 + 90) + m_2 r_2 x_2 \sin (\alpha_2 + 90) + \text{etc.}$$

$$\text{MRX} \cos (\alpha_B + 90) = \Sigma m_1 r_1 x_1 \cos (\alpha_1 + 90) + m_2 r_2 x_2 \cos (\alpha_2 + 90) + \text{etc.}$$

from which MR and α_B are obtainable.

Also for the resultant of the forces in plane A we have

$$M_o R_o \sin \alpha_A = \text{MR} \sin \alpha_B + \Sigma m_1 r_1 \sin \alpha_1 + m_2 r_2 \sin \alpha_2 + \text{etc.}$$

$$M_o R_o \cos \alpha_A = \text{MR} \cos \alpha_B + \Sigma m_1 r_1 \cos \alpha_1 + m_2 r_2 \cos \alpha_2 + \text{etc.}$$

from which $M_0 R_0$ and α_A are obtainable, so that a complete solution is thus effected, analytically.

Principles of Centrifugal Balancing

It is now customary to balance all high-speed rotating parts in order to obtain smooth running and freedom from harmful vibration effects. Typical instances of such components include the crankshafts of petrol and compression-ignition engines, the flywheels for these engines, high-speed gear-wheels, supercharger, and exhaust turbine impellers or rotors, electric motor and dynamo armatures, centrifugal pump impellers, high-speed shafting and couplings, etc.

Although, as mentioned previously, the presence of unbalanced centrifugal masses may not affect the stability of a complete engine or machine, owing to the damping effects of the relatively heavy mass of these units, the direct effects on the bearings and, possibly, on the components themselves, may become serious in regard to wear and fatigue results. It is therefore considered advisable to balance all high-speed rotating members in engineering members.

There are two different methods of balancing a rotary component, depending upon whether it is at rest or in motion, and known, respectively, as *static balance* and *dynamic balance.*

Static Balance

A body capable of rotating about a fixed axis is said to be in static balance when its centre of gravity lies on the axis of rotation.

Referring to Fig. 120, which shows a cylindrical body having a shaft mounted in bearings XY, the body has an unbalanced mass A at a distance m from the axis. Assuming frictionless bearings, the body will rotate until A is at the lowest position.

Static balance of a rotating member having parallel shaft bearings is usually determined by mounting it with the end shafts on parallel knife-edges or flat-topped runways located both level and horizontal (Fig. 121). The body is displaced on these ways, when it rolls along until the heavier portion is lowest. If, on the other hand, the body,

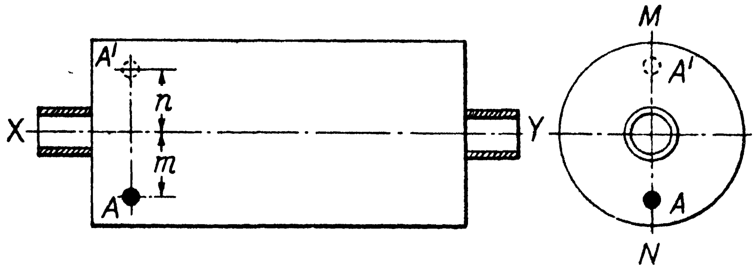


FIG. 120. STATIC UNBALANCE

after displacements, comes to rest in different positions each time it can be assumed to be very nearly in static balance. If the end shafts can be mounted on rollers, which are themselves located on ball or roller bearings (Fig. 122), the friction of the bearings is reduced; more

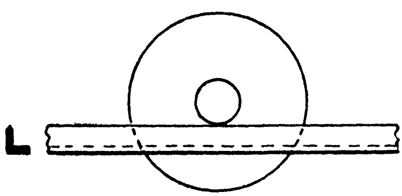


FIG. 121. PRINCIPLE OF STATIC BALANCE TESTING

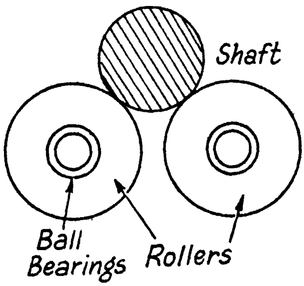


FIG. 122. METHOD OF REDUCING FRICTION IN STATIC BALANCING TEST

especially if both rollers and bearings are lubricated and greater accuracy is obtained.

In modern balancing machines, static unbalance is usually detected by rotating the body slowly and utilizing the centrifugal effect due to the unbalance to provide an indication of the unbalance. Referring again to Fig. 120, the body may be balanced statically by adding a weight A′ at a distance n from the axis such that the moments of A and A′ about the axis are equal. Thus,

$$Am = An'$$

Alternatively, material of weight A′ may be removed from the side of A, such that the weight of A′ multiplied by the distance of its C.G. from the axis equals the product Am.

It is not always possible to add or remove weight from the perpendicular plane to the axis through the unbalanced weight A, as, for example, in the case of a solid mass or an electrical armature, so

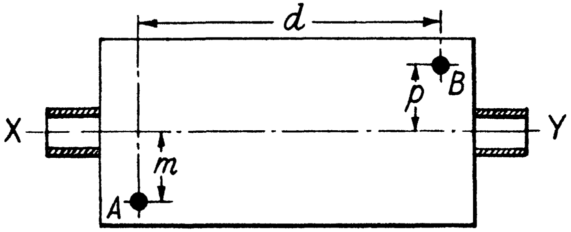


Fig. 123

that the balancing weight must be applied in some other plane, as indicated in Fig. 123 at B, where

$$Am = Bp$$

It is here assumed that both A and B are in the same diametral plane.

Dynamic Balance

A rotating body is said to be in dynamic balance when the couples set up by the centrifugal forces are in balance, i.e. the algebraic sum of the moments is zero.

Referring to Fig. 123, it will be seen that, although the system is in static balance, when it is rotating the centrifugal forces due to A and B give rise to an unbalanced couple of moment equal to the centrifugal force on A multiplied by the distance d. Thus

$$\text{Moment of unbalanced couple} = \frac{A\omega^2 m}{g} \times d \quad . \quad . \quad (1)$$

where A = weight of A; ω = angular velocity $= \dfrac{2\pi N}{60}$ where N = r.p.m. and $g = 32{\cdot}19$.

Since Am = Bp the moment (1) can also be written as

$$\frac{B\omega^2 p}{g} \times d \quad . \quad . \quad . \quad . \quad (2)$$

The values given in (1) and (2) represent the absolute ones, but in many balancing considerations it is unnecessary to take the speed ω into consideration, since it is constant for all the masses; neither

need the masses themselves be considered, so that the unbalanced couples can be represented by the product of weight × distance of C.G. from axis of rotation × distance from given plane about which moments are taken. Thus, if A and B be the *weights* in Fig. 123, the moment of

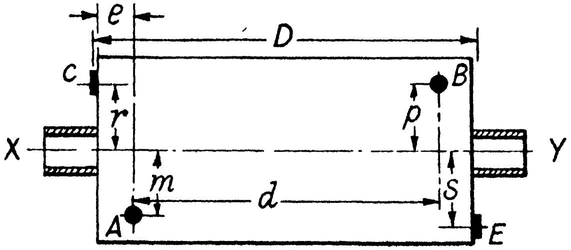


FIG. 124

the unbalanced couple can be represented by A*md* or B*pd*, in balancing problems.

It should here be mentioned that *when a rotating body is dynamically balanced it is also statically balanced*, but, as has previously been demonstrated, a statically balanced body is not necessarily in dynamic balance. In the example shown in Fig. 120, the body is actually in dynamic and static balance, since the centrifugal forces due to A and A′ are equal and opposite.

In the arrangement shown in Fig. 123, however, whilst the body is in static balance, it is not in dynamic balance.

A body which is not balanced dynamically—and numerous examples occur in practice of such bodies, e.g. flywheels, crankshafts, armatures, etc.—*can be properly balanced by the addition of a pair of balancing weights in two different planes*; alternately, material may be removed from the body at two different places to restore balance.

Fig. 124 shows a similar example to that of Fig. 123, but with the addition of balancing weights C and E at the ends of the body—for it is here assumed that weight cannot be added to or taken from the interior.

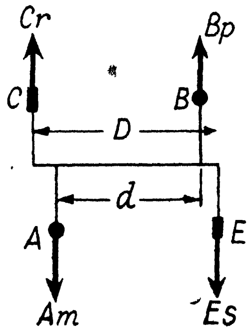


FIG. 125

For perfect dynamic balance the moment of the couple due to C and E must be equal and opposite to that due to A and B. It is also necessary for equality of the centrifugal forces due to C and E that the product C*r* = E*s*.

Thus, for dynamic balance C*r*D = A*md*, as indicated in Fig. 125. When thus balanced there will be *no additional loads on the bearings*

due to centrifugal forces, although if weights C and E are added for balancing purposes there will be a greater static load on the bearings. It is for this reason that whenever possible it is a better plan *to remove material from the same* side as that of the unbalanced mass or masses; the static load on the bearings is then reduced. Thus in the example shown in Fig. 124 it would be more advantageous to remove material

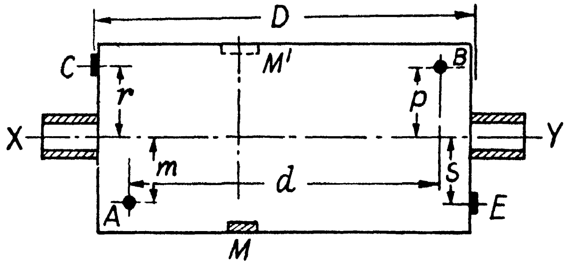


FIG. 126

of weights C and E from the same sides of the body as A and B, respectively. The total weight of the body would therefore be reduced.

Combined Static and Dynamic Unbalance

Cases occasionally occur in which a body as fabricated or machined has both static and dynamic unbalance. These conditions result when the amounts of unbalance at the ends of the body are different and in cases where the angles of unbalance at each end of a rotating body differ, although the amounts at each end are the same. If, however, the angles differ by 180° and the amounts are the same, the unbalance is dynamic only.

An example of static and dynamic unbalance is shown in Fig. 126, in which there are two masses at A and B representing uneven distribution of material giving rise to an unbalanced dynamic couple equal to A*md* or B*pd*. In addition, there is an unbalanced mass at M causing static unbalance, for A and B are actually in static balance. There are two alternative methods that are employed in balancing practice for effecting proper balance in such cases, namely—

(1) To add another balancing mass M′ on the opposite side to M to restore static balance and then to balance A and B dynamically—as before—by masses C and E.

(2) To add or remove weight at two places, C and E, in order to provide a balancing couple for M and also A and B, i.e. to correct both static and dynamic unbalance by balancing weights in two planes only.

It is not always possible or convenient to correct static unbalance at M (Fig. 126) by an equal and opposite balance weight M′, but most modern balancing machines, e.g. the Olsen-Lungren Type S or Vibro-Electric Type E–O, enable combined static and dynamic unbalance to be corrected by the addition of weight or its removal from two given planes, with no more trouble than for ordinary dynamic balancing.

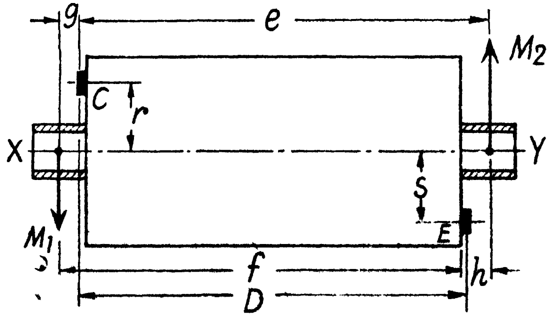


FIG. 127

As is described later, *in balancing machine practice* the body is arranged alternately *to oscillate about a pair of pivots* which can be adjusted along the axis of rotation. If these pivots can be arranged in the required correction planes, the process is much simplified. In certain types of balancing machines and in other instances where the shape and size of the body render it difficult to arrange the pivots under the balancing planes, the effects of unbalance may be measured at the centres of the bearings, as shown in Fig. 127, namely, at X and Y. Using the notation of this diagram, and taking moments about one of the correction planes through E, then

$$M_1 f + M_2 . h = CrD$$

from which
$$C = \frac{M_1 f + M_2 h}{Dr}$$

where M_1 and M_2 are the measured moments, due to unbalance at the centres of the bearings.

Again, by taking moments about the correction plane through C,

$$M_1 g + M_2 e = EsD$$

from which
$$E = \frac{M_1 g + M_2 e}{Ds}$$

From these two formulae, the values of the balancing weights C and E can be obtained.

Unbalanced Centrifugal Forces in Different Angular Planes

In the cases of all the preceding examples of static and dynamic unbalance it has been assumed that the unbalanced forces were in the same diametral plane, such as MN in Fig. 120. It is now proposed to consider the principles of balancing, dynamically, masses in different angular planes. Here it may be mentioned that provision is made in certain types of balancing machines to measure the magnitudes of

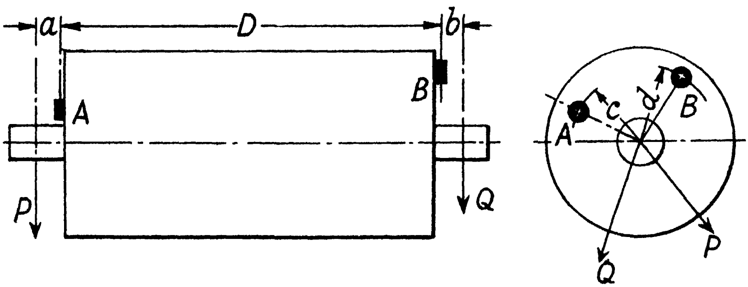


FIG. 128 BALANCING OF MASSES A AND B IN TWO DIFFERENT DIAMETRAL PLANES

the unbalanced forces and their angular positions, usually by reference to two given datum planes.

Referring to Fig. 119 on page 161, the method of balancing masses in different angular positions but in the same plane, i.e. perpendicular to the axis of rotation, was described and it was there shown that any two centrifugal forces could be balanced by a single force, using the vector method for obtaining the magnitude of the force, or equivalent mass at a given radius, and its direction.

An example that occurs frequently in balancing considerations is that of an equivalent rotating cylinder (Fig. 128), which has a couple due to two centrifugal forces which are not in the same diametral or transverse plane. The balancing machine, when the pivots are at the centres of the bearings, gives readings of the amounts and angles of the unbalance forces P and Q, as shown in Fig. 128.

It is usual to correct for these forces in given planes at the ends of the cylinder, such as by masses A and B in the planes indicated in Fig. 128 through A and B.

In order to determine the amount and angular locations of the corrections that must be employed in the planes A and B for dynamic balance, it is first necessary to transfer the amount and angle of the forces at P and Q to one resultant force in the right-hand plane B, assuming the part to be held by a fixed pivot at the plane.*

* *Static and Dynamic Balancing Machines*, T. Olsen.

Next, the amount and angle of forces at P and Q are transferred to one resultant force in the left-hand plane A, assuming the part to be held by a fixed pivot at the plane B.

If, then, these two resultant forces are neutralized by the addition or removal of material at two points only, the part will be in perfect static and dynamic balance.

It follows that if the pivots of the balancing machine cradle were

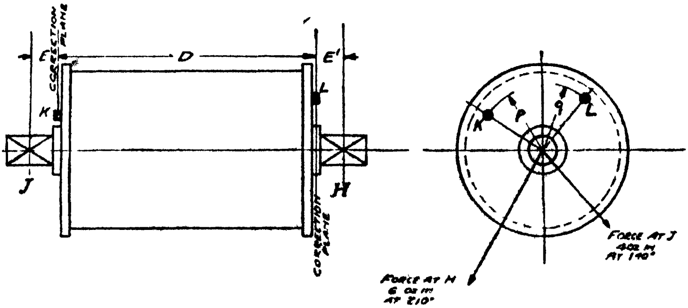


FIG. 129 EXAMPLE OF BALANCING COMPUTATION

actually located in the planes A and B, one pivot being locked and readings taken in the other plane, the reading thus obtained would be the resultant of all the forces due to unbalance outside the plane containing the locked pivot. All unbalanced forces in the actual plane of the locked pivot would have no moment about the pivot and need not, therefore, be taken into account.

It may here be mentioned that it is possible to lock the pivots in the planes selected for correction in balancing machines of the Olsen-Lundgren (Type S) and Olsen Vibro-Electric (Type E–O).

Example of Balancing Computations

As an example of the general method used in machines in which the pivots cannot be located in the planes K and L, but in the central bearing planes H and J (Fig. 129), it is assumed that after running the body in the balancing machine, the following results* are obtained—

(1) At the plane J, holding H as a pivot, a centrifugal force of – 4 oz.-in. at 140°.

(2) At the plane H, holding J as a pivot, a centrifugal force of – 6 oz.-in. at 210°.

* The component of the centrifugal force is here represented by the product of weight by radius, for the reason explained on page 165.

The following dimensions refer to the notation given in Fig. 129—

$E = 4$ in.; distance between centre of bearing J and correction plane K;

$D = 12$ in.; distance between correction planes K and L;

$E' = 3$ in.; distance between centre of bearing H and correction plane L;

$q = 3$ in.; radius at which correction is to be made in plane L;

$p = 4$ in.; radius at which correction is to be made in plane K.

It is first necessary to find the force which, if acting in the plane L and with the body pivoted on its axis at the intersection of the other

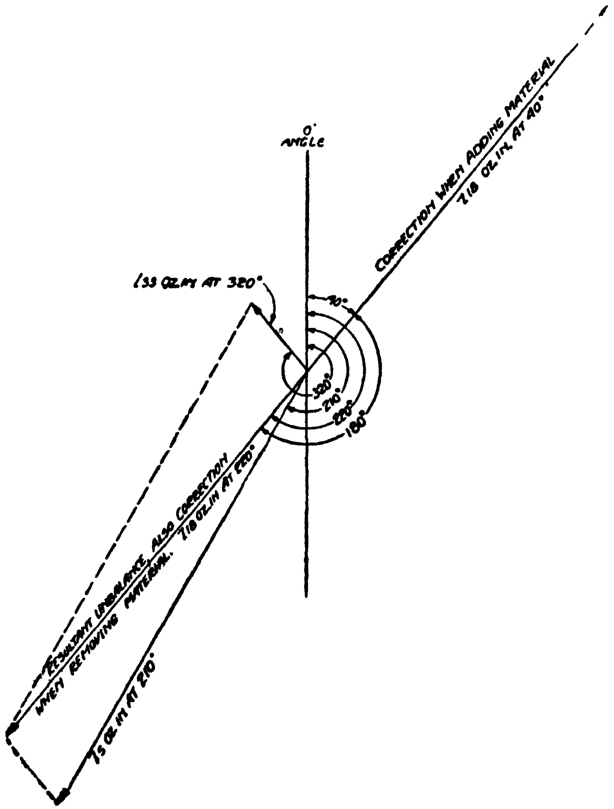


FIG. 130. RESULTANT CORRECTION FORCE FOR PLANE L

correction plane K, will have the same disturbing effect as the readings of H and J taken from the balancing machine.

Taking moments about the other correction plane K, obtain the amounts of the forces at H and J as they would be if working in the plane L where one of the corrections is to be made. Thus—

The equivalent force at plane L to H $= 6 \text{ oz.-in.} \times \dfrac{D + E'}{D} = 6\left(\dfrac{12 + 3}{12}\right)$

$= 7{\cdot}5$ oz.-in. at L and at the same angle of 210° as H.

Similarly, equivalent force at plane L to J $= 4 \text{ oz.-in.} \times \dfrac{E}{D} = 4\left(\dfrac{4}{12}\right)$

$= 1{\cdot}33$ oz.-in. at the opposite angle to J

$= 140° + 180° = 320°$

Note. This angle is taken as opposite to J, since L and J are on opposite sides of the neutral plane K about which moments are taken.

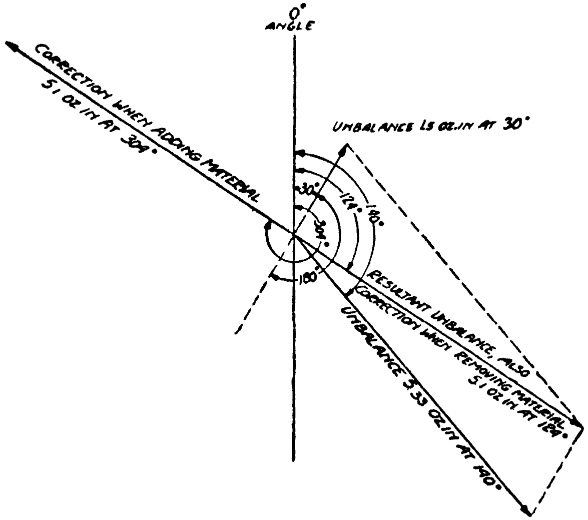


FIG. 131. RESULTANT CORRECTION FORCE FOR PLANE K

The resultant of 7·5 oz.-in. at 210° and 1·33 oz.-in. at 320°, by the parallelogram of forces (Fig. 130), is found to be 7·18 oz.-in. at 220°.

This result indicates that if the body to be balanced were pivoted at the centre of the shaft where it is intersected by the plane K, a single force of 7·18 oz.-in. at an angle of 220° will have the same effect as the combined forces 7·5 oz.-in. at 210° in plane J and 1·33 oz.-in. in plane H.

To apply the correction a force of 7·18 oz.-in. applied by adding material at the opposite angle $(220^\circ - 180^\circ) = 40^\circ$ will balance. If, however, the correction is to be made by drilling or removing metal, 7·18 oz.-in. should be removed at the same angle 220°.

If the place at which the correction is to be made is at a radius $q = 3$ in., then the weight to be removed will be $\frac{7 \cdot 18}{3} = 2 \cdot 39$ oz. If the balance is to be made by adding weight at 3 in. radius on the opposite side, namely, 40°, the weight will be 2·39 oz.

The amount and angle of correction at the correction plane K is determined by a similar method, the results being shown graphically in Fig. 131. The resultant force is one of 5·1 oz.-in. at 124°, and if the correction is made at a radius of 4 in. the amount to be added on the opposite side $(124^\circ + 180^\circ) = 304^\circ$ will be $\frac{5 \cdot 1}{4} = 1 \cdot 275$ oz. If weight is to be removed by drilling, the amount will be the same, namely, 1·275 oz. at the same angle 124°, and radius 4 in.

CHAPTER VIII

BALANCING MACHINES

Static Balancing Machines

In the case of engineering components, such as circular discs, flywheels, clutches, impeller rotors, and similar parts of relatively small thicknesses in relation to their diameters, it is usually necessary only to balance these statically.

The reason for this is that it is generally possible to add or remove weight for balancing purposes very nearly, if not entirely, in the same plane as that containing the C.G. of the unbalanced mass. Referring to Fig. 132 (1), which shows a rotating disc with equivalent unbalanced

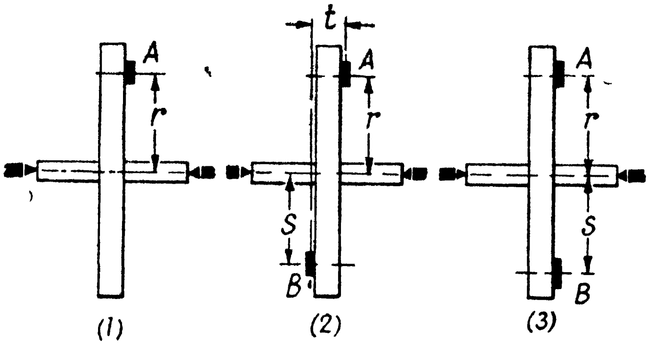


FIG. 132. STATIC BALANCING OF NARROW CYLINDRICAL DISCS

weight A at a distance r from the axis, this can be balanced, very nearly, by means of a weight B at a distance s from the axis (Fig. 132) (2)), such that $\mathrm{A}r = \mathrm{B}s$.

The unbalanced centrifugal couple will be of moment $\dfrac{\mathrm{A}\omega^2 rt}{g}$ where ω = angular velocity, t = distance shown in Fig. 132 (2), and $g = 32{\cdot}2$.

Since t is very small in regard to the other quantities the magnitude of the unbalanced couple will be very small and in most practical examples can be disregarded. When, however, it is possible to apply the balance weight B in the same plane as the unbalanced mass A, the centrifugal couple becomes zero and if the disc is in correct static balance it will also be in dynamic balance.

Balancing Flywheels

Flywheels of petrol and high-speed Diesel engines can usually be balanced in this manner by drilling holes right through their rims on

the same side as the unbalanced mass; but even if drilled only part of the way through, the balance is generally satisfactory. If the flywheel is held by a key, the latter should be inserted in the flywheel boss and balancing shaft, so that its effect may be included in the complete balance of the flywheel. If the shaft to which the flywheel

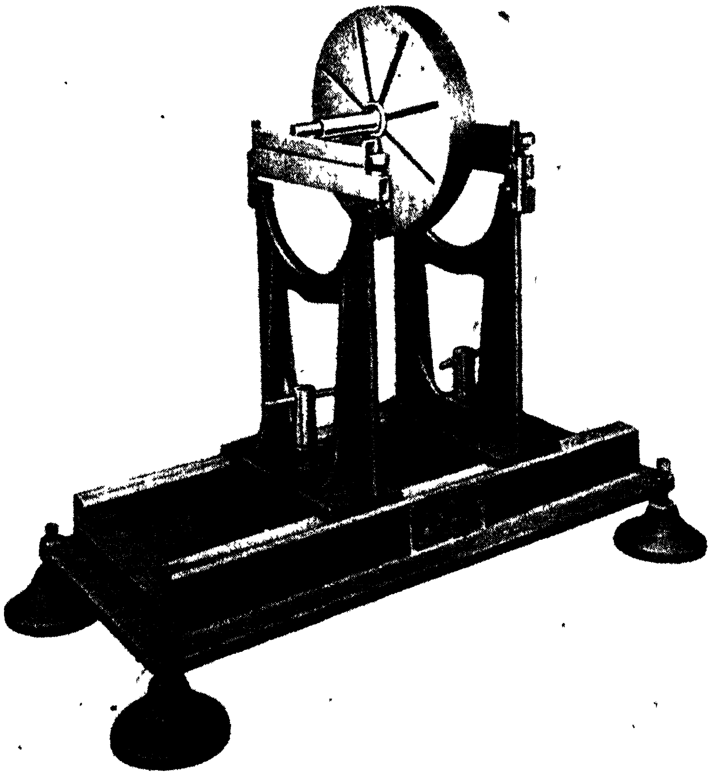


FIG. 133. A FLYWHEEL-TYPE STATIC BALANCING MACHINE

is attached is a long one, with bearings on one side of the flywheel and at the other end of the shaft, the effect of the small dynamic unbalance shown in Fig. 132 (2) will be still further reduced.

Narrow flywheels, lathe face-plates and similar parts, can be balanced statically by means of level ways of the Olsen-Lundgren type shown in Fig. 133. These are levelled up with the aid of a spirit level applied along and across the ways in turn, by means of the screw adjustments provided for the purpose.

In using this method the operator usually judges the degree of static unbalance by the speed with which the heavier portion of the

disc or rim tends to turn to the bottom after displacement. A method sometimes used for correction purposes is to place a piece of plastic modelling clay or putty on the opposite side to the unbalanced part and to adjust its amount and radius until the part is properly balanced, i.e. will stay in any position on the ways. The product of the weight of the clay or putty and the distance of its C.G. from the axis gives the unbalanced moment. From this it is a simple matter to work out the

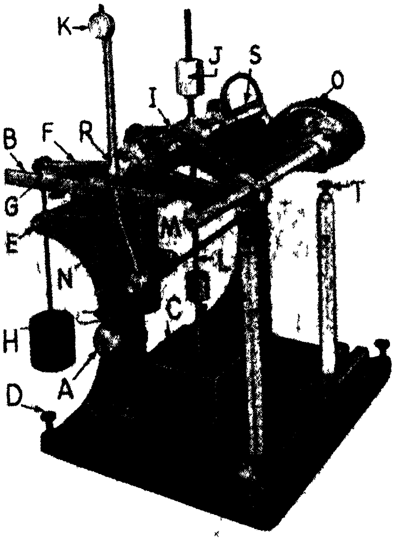


FIG 134. THE OLSEN-LUNDGREN STATIC BALANCING MACHINE

A—motor switch, B—scale beam, C—oil dashpot; D—levelling screws, E—anvil for knife-edge pivot, F—balancing frame, G—poise weights for calibrating machine, H—counterweight to compensate for weight of part, I—angular unbalance pointer, J—calibrating weight to adjust C G of balancing frame, K—indicating dial, L—dashpot paddle rod, M—extended mandrel for mounting part to be balanced, N—locking handle, O—motor drive unit, R—knife edge pivots, S—rocking beam connecting bar, T—stops to control maximum swing of balancing frame.

weight of metal to be drilled, at a given radius, from the heavier side, in order to obtain static balance.

Machines of the type illustrated in Fig. 133 are available in the following ranges: 0·5–15 lb.; 10–300 lb., and 250–7000 lb.

In the case of comparatively short parts, with respect to their diameters, a type of automatic weighing machine designed for static balancing is shown in Fig. 134. Its chief advantage is that it provides quick and accurate unbalance indications with much greater sensitivity than is possible in the level ways method. The principle of operation is illustrated in Figs. 135 and 136. This type of static balancing machine runs at a very low speed, namely, 3 or 4 r.p.m., so that the centrifugal

forces do not have any disturbing effects. The mandrel M on which the part G, to be balanced, is mounted is driven by a small motor. All of the parts are mounted on the balancing frame F, which is supported at the front and back on hardened and ground knife edges R which rest on anvils E. The counterweight H is adjustable to suit the weight of the part being balanced and keeps the balancing frame assembly horizontal. The dial indicator K is secured to the main

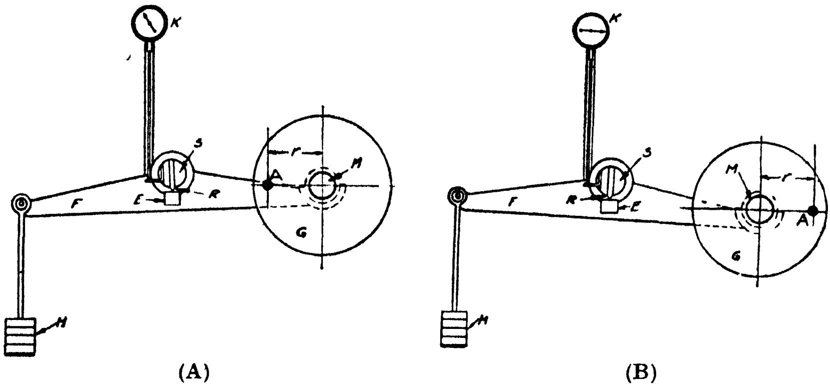


FIG. 135. PRINCIPLE OF STATIC BALANCING MACHINE

frame of the machine and its stem is actuated by the balancing frame, so that very small oscillations can be registered.

The action is as follows: Assuming the part G has a static unbalance A, the C.G. of the part is not at its geometrical centre but is somewhere on the radial line through A. Thus, as the part rotates its C.G. describes a circle about the geometrical centre of rotation. When A is in the position shown in Fig. 135 (A), where it lies directly between the mandrel M and the pivot R, the part will apparently weigh least; this is shown by the indicator scale minimum reading. When A is in the extreme outward position shown in Fig. 135 (A). the maximum apparent weight reading will be obtained. The total movement of the dial indicator pointer from maximum to minimum is therefore a measure of the amount of the unbalance A. The scale of the indicator K can be calibrated in equivalent ounce-inches, so that the unbalance due to A can be read off. The position of this unbalance is readily determined by the position shown in Fig. 135 (A) or (B), of least and greatest indicator readings.

It is therefore a comparatively simple matter to remove metal from the side A or add it to the diametrically-opposite side in order to effect the correct static balance. Thus, if the machine shows an unbalance reading of x ounce-inches and it is desired to remove metal,

by drilling on the radial line through A, at a distance, or radius, r in., the amount of metal to be removed is given by $\frac{x}{r}$ oz.

Use of Dynamic Balancing Machines

It will be apparent that the static balancing machines described do not show the location of the unbalance in the axial direction, but only in a radial sense. Since these considerations have been confined to the balancing of parts of small thickness-to-diameter ratio, the axial unbalance position is not usually important. In cases, however, where it is necessary to locate the static unbalance, in order to correct for it in its own plane, as in the instance of parts that have to rotate at very high speeds, it is usual to employ a dynamic balancing machine to ascertain the amount of the unbalance, its angular and also axial location.

Some dynamic balancing machines cannot be used to show the axial position of the unbalance for narrow parts, owing to their design, so that in selecting a machine that is intended for both narrow and long cylindrical components this point should not be overlooked.

Propeller Balancing Machines

Propellers belong to the class of component having a small width-to-length ratio, so that any unbalance of the usual order due to variations in density of the material or small manufacturing irregularities can be allowed for by the static balancing method; usually, the amount of unbalance is such that the corresponding dynamic unbalance of the subsequently static balanced propeller is negligible. It is necessary for static balance tests of propellers to provide a suitable hub, with hub flanges, bolts, nuts, washers, and other parts identical to those of the engine upon which the propeller is to be used.

The static balancing of propellers is effected on special machines designed for the purpose, of which the Avery and the Olsen types are well-known examples.

A static balancing machine for small propellers and spinners is made by Messrs. W. & T. Avery, Ltd., and another model (Fig. 136*) for the blades of variable pitch propellers, whereby the centre of gravity or predetermined turning moment of the propeller blade can be ascertained in comparison with a standard blade. A frame *a* has a pair of bearing mountings *c* for a rotatable carrier block *d*, which is secured on one side to a counterbalancing arm *e* provided with slidable poise-weights e^1. The other side of the carrier block has provision for the reception of a hub adapted to receive the butt of a propeller blade *f*, the arrangement being such that the dead weight of the

* *Engineering*, 21st July, 1944.

propeller blade can be counterbalanced on the arm *e* by setting the poise-weights e_1 about a fulcrum knife edge, the face line of which is co-axial with the axis of the bearings in the brackets *c*.

The knife edge fulcrum forms part of a horizontal spindle on which the carrier block *d* and the parts associated with it are mounted. The carrier block can be clamped to the spindle by the friction between a fibre shoe and the periphery of the spindle so that when required the carrier block can be rocked with the spindle or the block can be

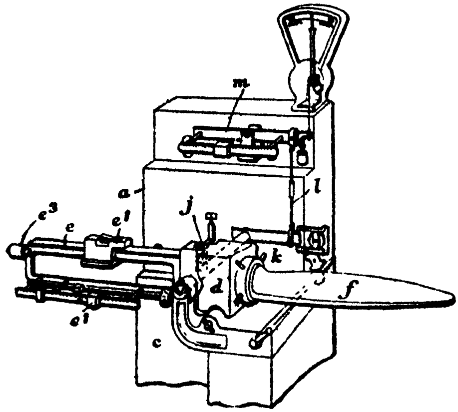


FIG. 136. MACHINE FOR STATIC BALANCING OF SMALL PROPELLERS AND SPINNERS

rotated about the spindle. The amount of the friction between the carrier block and the spindle is determined by the adjustment of a loading screw *j* in relation to a coil spring which determines the frictional pressure exerted by the fibre shoe so as to permit the independent rotation of the carrier block *d* about the spindle to effect a resetting of the propeller blade *f* from the horizontal to the vertical position. The knife edge spindle is extended at right angles at one end by means of the lever *k* to form a torsion lever which is connected by a connecting rod *l* to the lever system *m* of a weighing apparatus. The lever system also incorporates pendulum mechanism embodying an index pointer. For counterbalancing the adaptor hub, an adjustable balance weight e^1 is provided.

After the apparatus has been accurately calibrated and a standard propeller blade tested by the requisite setting and locking of the poise-weights e^1 on the counterbalancing arm *e*, the standard propeller is replaced by the blade *f* which is to be tested, the butt of this blade being mounted in the adapter hub in the carrier block *d*. The weighing lever system is freed and an observation is made to ascertain whether there is any departure from the balance position of the indicator in

either direction which will indicate a plus or minus error. If an error is indicated, corrections are effected on the blade *f* and after the necessary correction has been made in the horizontal plane, the carrier block *d*, with the counterbalancing arm *e* and the blade mounted in

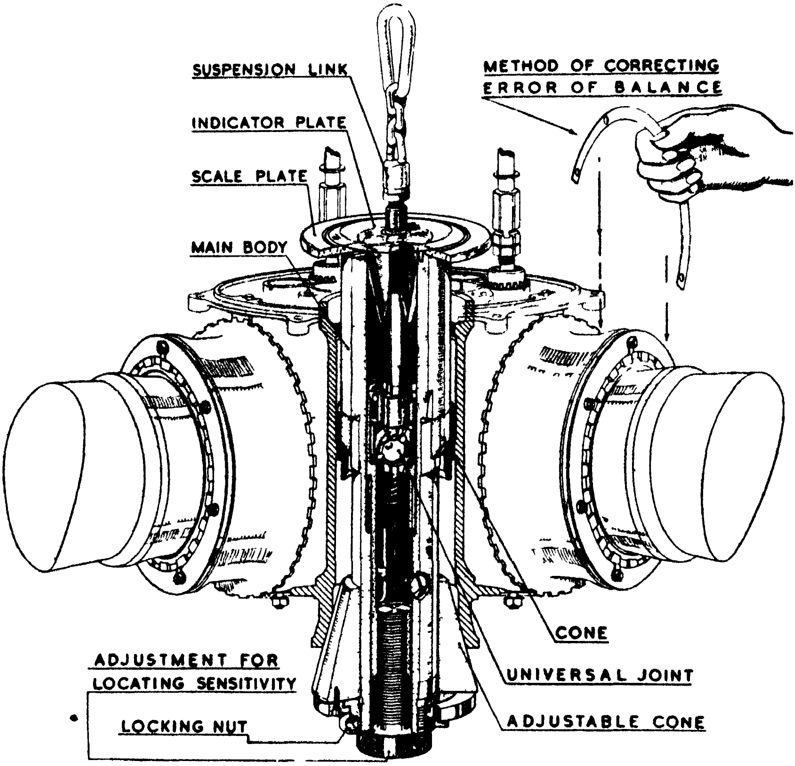


FIG. 137. THE ROTOL PROPELLER BALANCING METHOD

the block, are turned through 90° about the knife edge spindle and a further test is effected.

The Rotol propeller balancing machine, the principle of which is shown in Fig. 137, can also be used for other engineering parts of small thickness-to-diameter, e.g. flywheels and rotors. It consists of a hollow cylinder which has on its outside cone, supports that locate upon the front and rear cone seats of the propeller hub, the rear cone being made removable for fitting the propeller. Inside the cylinder a spherical joint is fixed in relation to the cone seats but adjustable vertically to bring the point of support as near as possible to the C.G. of the assembly. An indicator plate located on the suspending rod slides over a scale plate inscribed with a concentric ring just a

little larger in diameter than the former and gives a relative reading showing by eccentricity the heavy side of the propeller. The propeller blades are first set at approximately the same angle.

In operation, an unbalanced propeller will come to rest after the oscillation with the scale plate, the point of greatest eccentricity showing the heavy side. Balance shims, one of which is shown in the process of insertion, in Fig. 137, are placed as near as possible to the shim housing of the blade or blades which are "light," so as to bring the scale plate concentric with the indicator plate. For practical purposes the concentric ring has been calibrated in such a way that, provided the indicator plate is visually within the confines of the inscribed line on the scale plate, the propeller will be within the specified limits of balance.

With this method the propeller can be properly balanced without the necessity of more complicated apparatus and a draught-proof room. For checking the balance of propellers at aerodromes a portable form of this balancing machine, mounted on a road trailer designed especially for this purpose, is available.

Propellers can also be statically balanced on a special machine of the Olsen weighing static balancing type, based upon the same principle as that used in the machine shown in Fig. 134. The chief modification when this machine is used for propellers is in the type of adapter for mounting the propeller and in the installation of the machine. A splined-type adapter is usually supplied and is provided with hardened and ground tapered rings which fit the tapered seats in either end of the propeller hub. One machine of this pattern will deal with propeller assemblies up to 600 lb. weight; the maximum diameter is limited only by the height of the machine above the floor level, or the depth of pit provided.

Blade Traction Balancing Machine

Apart from the static balance requirement, which ensures that there is no resultant centrifugal force that would cause vibrations, it is necessary, also, to check the propeller for uniformity of thrust or traction, i.e. equality of thrust in the axial direction of each blade, since any inequality will result in a rotating centrifugal couple that will tend to promote vibrational effects on the engine unit.

A special machine, known as the Olsen blade-traction balancing type for propellers (Fig. 138) is based upon the principle of locating and measuring the dynamic couple of the rotating propeller in a somewhat similar manner to that of the Olsen-Carwen static-dynamic balancing machine, described later in this chapter. The machine is, however, modified by shortening the vibrating bed in order to locate the pivots as closely as possible to the propeller. The static pivots on the bed and

the static compensator on the spindle have been dispensed with. A splined adapter with tapered seating rings is used on this machine.

The machine runs at a comparatively low speed and operates in the "critical" or natural period of vibration of the bed.

In operating the machine the propeller is placed on the adapter in the same manner as it would be placed on the engine shaft, so that the tendency is for the propeller to pull away from the machine when running. The machine is then started and the speed adjusted until it is in synchronism with the natural period of vibration of the bed. When this "critical" speed is reached the bed will vibrate with a uniform rocking motion, giving maximum amplitude of vibration as seen on the dial indicator. Assuming that the propeller is in static balance, as it should be

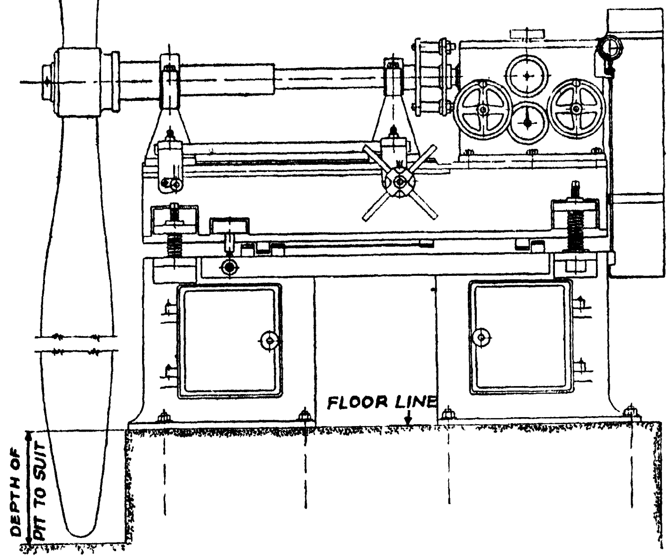


FIG. 138. PROPELLER TRACTION BALANCING MACHINE

when checking for variation in blade traction, the vibration is caused by the unequal tractive effort occurring alternately above and below the axis of the propeller, which acts upon the bed, causing it to oscillate about the pivots located near the left end.

When the compensating blocks have been adjusted to the proper angle and amount to counteract exactly the variation in blade traction, the bed will cease vibrating. The machine is then stopped and turned

by hand until the scribed line marked "Light Traction Front," on the angle reference disc at the top of the headstock assembly, aligns with the index. The light traction plane is then in the horizontal plane through the hub of the propeller and toward the front or operating side of the machine.

In a two-blade propeller this plane will generally come on or near the centre line of one of the blades, but in a three- or four-blade propeller, it is likely to come between the blades, requiring correction in the two adjacent blades.

Correction for uneven traction depends upon the particular type of propeller being checked. If it is the adjustable blade type, it is usually possible to turn the individual blades independently, in which case the blade or blades on the light traction side can be turned as necessary to increase the traction. In the case of the adjustable pitch type of propeller, with automatically or mechanically operated blades adjustable during flight, the blades may be set at a fixed pitch when readings are taken, but it is advisable to take readings for different blade settings, to see that the blade turning device functions properly.

In propellers with fixed blades, where the traction cannot be altered by turning the individual blades on their axes, correction for variation in traction becomes more difficult. It is usually necessary to remove metal at the most efficient place to reduce the traction of one or more blades, but it may be necessary to recheck and correct again for static balance, correcting for static where it will not affect blade traction.

Accuracy of Static Balance Tests

Of the two general methods of making static balance tests, namely, (1) the stationary one, in which the part is mounted on level ways or in bearings and allowed to roll or rotate until the heavier portion reaches its lower position, and (2) the rotation method, the former is open to the objection that friction between the body and the ways or bearings tends to reduce the accuracy of the results. In order to minimize the effect of friction the shaft of the part to be balanced may be mounted on friction reducing bearings or on "wheel" bearings. In the case of the Losenhausenwerk static balancing machine,* frictional effects are minimized by mounting the balancing spindle on two sets of ball bearings situated at some distance apart within a sleeve which is itself mounted in ball bearings in the main casting of the machine. The sleeve is given a small oscillation by means of a crank and connecting rod mechanism, from a small electric motor, in order to overcome the static friction or "sticktion" of the spindle in the bearing. It is possible, in this way, to obtain a more accurate reading of the angular position of the out-of-balance portion.

* *Machinery*, 23rd December, 1937.

In the rotary types of static balancing machines, the degree of measuring accuracy is usually higher, since the frictional effects mentioned previously do not affect the results. The greatest accuracy is obtained in machines which enable the leverage or the centrifugal effects of the out-of-balance to be magnified appreciably. It is necessary, also, that the balancing machine should be suitable for the size and weight of the part to be balanced, statically. Thus, if the latter is much smaller than the average size for which the machine is designed

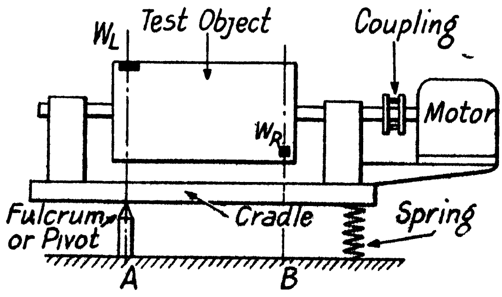


FIG. 139. SHOWING PRINCIPLE OF DYNAMIC BALANCING MACHINES

W_L and W_R denote out-of-balance, A and B positions of reference planes

both the sensitivity and accuracy of measurement will be reduced; if too large, or heavy, the accuracy may also be lessened.

Dynamic Balancing Machines

From earlier considerations in this chapter the general principles of dynamic balancing will have become apparent to the reader. In this connexion it has been shown that dynamic unbalance causes centrifugal couples, giving rise to vibration effects, and that these may be balanced by means of suitable counterweights in two given planes. The amounts, radial distances and angular positions of these counterweights are determined in special balancing machines, so that the parts under test can be suitably balanced. Alternatively, the method of removing weight in the two balancing planes, by drilling, can be employed.

The method generally adopted in commercial balancing machines is to mount the part to be balanced in bearings on the machine and to rotate the part at a given speed. The out-of-balance then causes vibrations of the frame or mounting—which is usually provided with flexible supports to amplify the effect. (Fig. 139.) In order to locate and measure the out-of-balance the machine is provided with a parallel, or axial, extension shaft driven at the same speed as that of the rotating part. This shaft has an out-of-balance weight the radius and

angular position of which can be varied during the test. Since it runs at the same speed as the part to be balanced it tends to set up vibrations of the frame. If the angular position of the weight on the shaft is varied a position will be found at which the centrifugal force on the weight is opposite in phase to that of the out-of-balance part under test. If, in addition, the radius of the C.G. of the weight is correctly adjusted the centrifugal forces will be equal, and the moments due to the unbalance of the rotating part and the compensating weight will be equal upon the machine frame, so that the vibrations will cease. Means are provided in balancing machines *to vary the positions of the pivots of the mounting or frame*, so that they lie in the planes of the part under test in which balancing is to be effected by weight addition or removal, i.e. in the selected *correction planes.*

Balancing machines are provided with dials, the moving pointers of which show the amount of out-of-balance (usually in ounce-inches) and the angular position for each correction plane. The vibrations set up by the unbalanced part under test are amplified and indicated on special instruments, or vibration meters, which are located at the two shaft bearings of the part. During the test the operator manipulates the balancing machine controls so as to reduce the amplitudes of the vibrations to zero.

In tests of this type, the pivots are first adjusted to the correction planes and one of them is then locked and readings are taken in the plane of the other pivot. In this way any unbalance in the plane of the locked pivot has no effect on the free end and the readings of the amount and angular position of the correction apply direct to the plane of the free pivot. The fixed pivot is then released, the free one locked in position and the readings taken in the former plane in order to ascertain the amount and angular position of the correction in the second correction plane.

Instead of using a vibration meter to amplify and indicate the vibrations of the machine during balancing tests it is possible to employ the electrical method, based upon *the use of electrical vibration pick-ups* mounted in two selected axial planes. These generate voltages which are proportional to the amplitudes of the mechanical vibrations. The voltages are amplified and indicated on a large voltmeter. The process of balancing consists in noting the "amounts" on the voltmeter and the angular positions, by means of a stroboscopic device. From this information the unbalance can be corrected. The Gisholt balancing machine of the Westinghouse Company, U.S.A., operates on this principle. Another balancing machine that employs an electrical method is the Olsen "E–O" machine. In this example, small permanent magnets and coils are used at each end of the balancing cradle. Vibrations, due to out-of-balance, cause alternating currents to be

generated, the amount and phase of which indicate the amount and angle of unbalance.

Balancing Machine Selection Factors

Before proceeding to describe some typical modern balancing machines it is proposed to consider certain factors that may affect the choice of the balancing machine and results obtained therefrom.

One of the most important items in this connexion is the speed at which the balancing test is carried out. Whilst, theoretically, a rotating part which has been correctly balanced at one speed will also be in perfect balance at any other speed, the selection of the speed of test may influence the results in several ways.

Thus, if *too low a speed* is chosen, the magnitude of the unbalanced centrifugal force may be too small to be indicated accurately on the vibration meter. On the other hand, if *too high a speed* is selected, the centrifugal force may cause excessive vibration and wear effects on the machine, for which it is not designed.

Again, if certain parts such as crankshafts and plain cylindrical members of relatively high length-to-diameter ratio are operated at high speeds in balancing machines, *deformation, due to lateral bending* caused by the unbalanced centrifugal forces, may—and sometimes does—occur. If, therefore, the part is balanced correctly at one high speed, the results may not apply at other speeds, owing to changes in the distortion of the part. It is necessary, therefore, to select a balancing machine that is designed to effect balancing of such "slender" rotating parts, at speeds well below those causing lateral bending distortion. This is important, since certain balancing machines are designed to operate at high resonance speeds and others at much higher speeds than these.

Since many balancing machines make use of the presence of mechanical vibrations as the indication of unbalance, and effect proper balance by adjustments of the components until these vibrations cease, one class of machine that is in current use arranges for the operating speed to synchronize with the "critical" or *resonance speed* of the machine itself, so that the natural period of the vibration of the vibratory support corresponds with the rotational speed of the balancing machine. As the maximum amplitude of vibration occurs at this speed, the out-of-balance indication is well defined, so that an accurate means exists for measuring and correcting out-of-balance.

Fig. 140 shows a typical curve of vibration amplitudes on a frequency or speed base, with a well-pronounced resonance increase in the amplitude at a speed of about 1038 r.p.m. At this critical speed the amplitude of vibration is 15 times as great as that at 940 r.p.m. The angle of phase or lag of the vibrations is denoted by the dotted curve,

and it indicates that between about 970 and 1020 r.p.m. there is a change of phase of 160°, with the most rapid rate of phase lag occurring at the critical speed of the machine. Owing to this wide variation in angle lag, it is difficult to obtain accurate angle measurements around the critical speed, but in certain balancing machines, e.g. the Olsen-Lundgren Type "S," the lag has no effect on angle indication, since

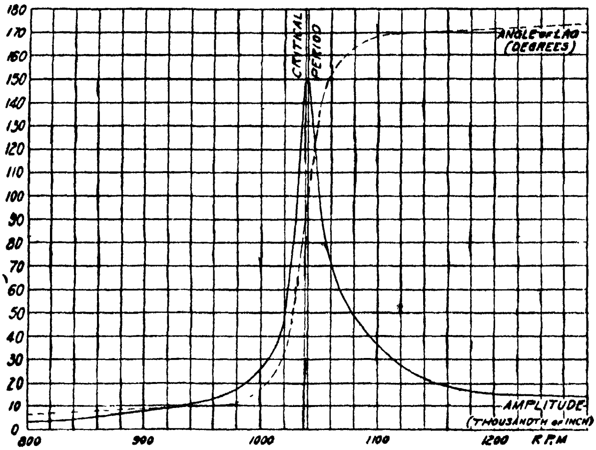


FIG. 140. DYNAMIC BALANCING MACHINE AMPLITUDES AND FREQUENCIES, SHOWING RESONANCE AND PHASE LAG CONDITIONS

the unbalance angle is determined from the position of the compensating weight which is subject to the same lag conditions as the unbalance that is to be determined.

The Carwen Balancing Machine

The Carwen static-dynamic machine, as originally manufactured by the Carlson-Wenstrom Co., America, is shown, schematically, in Fig. 141. The object to be balanced, A, is supported in bearings on a bed B, which is fixed at one end E but is spring-supported at its other end F. Underneath the bed and with its axis parallel to that of the axis of rotation of A is another shaft C, which is suspended from the same bed. A motor G drives the shaft H of A and the shaft C at the same speed and in the same direction.

Compensating weights D are adjustably mounted on the shaft C, such that their axial distance can be varied, as well as their angular relation. The axial distance N can be varied by means of the right- and left-handed turnbuckle shown in Fig. 141. The method of using this machine is to vary the angular and axial distances of the weights D, whilst the machine is operating until the out-of-balance vibrations

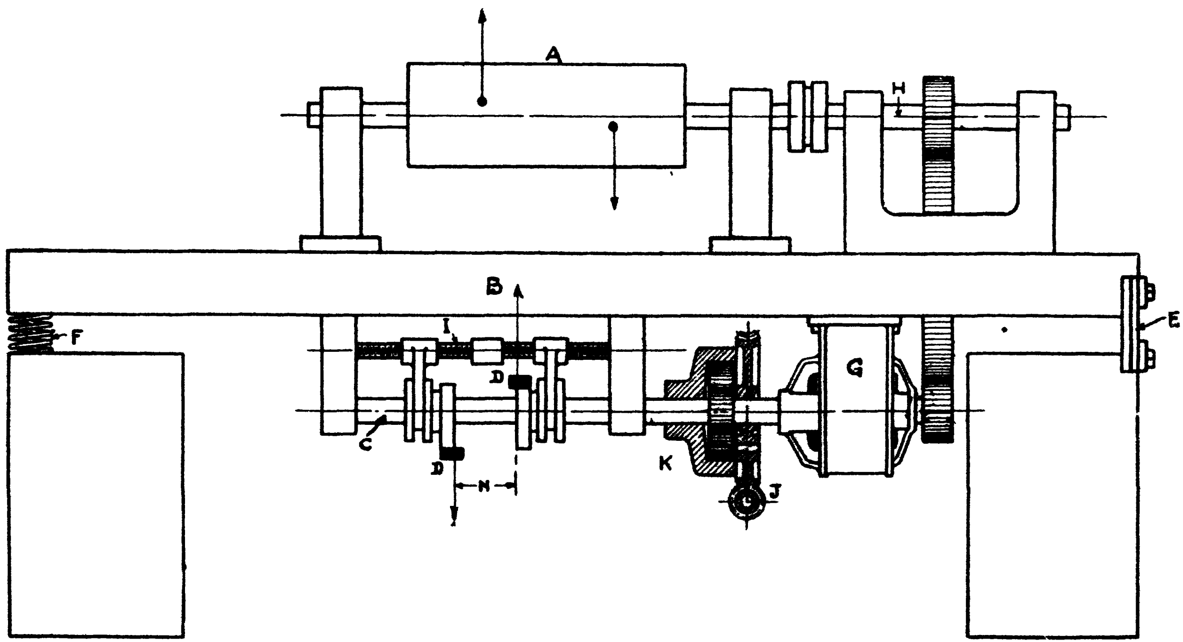


FIG 141 ILLUSTRATING THE PRINCIPLE OF THE CARWEN BALANCING MACHINE

of the bed B cease entirely. The angular position of the weights D can be varied by means of the differential gear K by means of a hand control worm and wheel device J.

The value of the compensating moment of the centrifugal forces due to the weights D is given by the following relation—

$$\text{Compensating moment} = \frac{D\omega^2 R \,.\, N}{g}$$

where D = compensating weight,

ω = angular velocity,

R = radius of C.G. of D,

g = 32·19,

and N = distance between weights D.

In the Carwen machine ω is the same for both shafts and R is constant (since the weights D are fixed at a given radius), so that the compensating moment is proportional to the product DN. As D is constant also, the moment is therefore proportional to the distance N.

Balance may therefore be effected by adding or removing weight equal to m at two places at equal radii r and distance apart d, such that the moment $m \,.\, r \,.\, d = D \,.\, R \,.\, N$.

It is necessary that the angular positions of m should correspond with the positions indicated by the balance weights D. In the original Carwen machine scales were provided to read the amount of the out-of-balance in ounce-inches, at a given speed, and charts were available to show the exact amount of metal to be added or removed at a given radius to effect correct balance of the member under test.

The Carwen machine is particularly suitable for instruction and demonstration purposes in engineering institutions, as various principles of balancing can be investigated and both static and dynamic methods dealt with separately or in combination.

The more recent version of this machine, known as the Olsen-Carwen (Type "C") has several improvements on the original model. It is shown schematically in Figs. 142 and 143, and in external view in Fig. 144. In the latter illustration the various components are described in the caption. It should here be mentioned that the compensating weight system in this design forms an extension of the shaft driving the out-of-balance member under test; both are therefore driven at the same speed, without gears. They are mounted on the top of a common bed which is provided with flexible supports at its two ends.

In reference to Fig. 144, the rigid bed J carries adjustable bearing brackets N in which the part to be balanced is supported. At the

right end the headstock assembly R is secured, with the driving motor mounted to the rear of this assembly and connected to a pulley on the main spindle by means of an endless woven belt. One of the flexible supports for the bed J is shown at K, there is a similar support

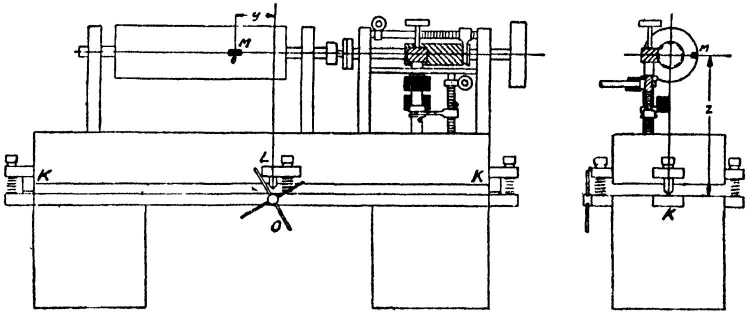


FIG. 142. ILLUSTRATING PRINCIPLE OF OLSEN-CARWEN TYPE C BALANCING MACHINE

at the other end of the bed. Alternatively, the bed can be supported on "dynamic" pivots L at the front and back of the bed (see also Figs. 142 and 143), this changeover is effected by means of the shifting mechanism O.

The rigid bed with its spring supports, in use, has a certain natural vibration frequency, and in tests on this machine the speed is chosen

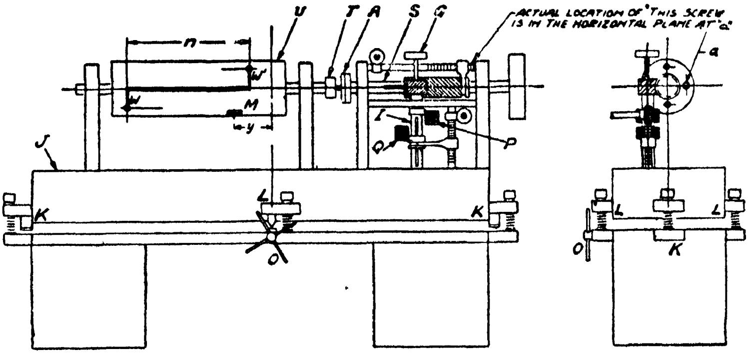


FIG. 143. THE OLSEN-CARWEN TYPE C BALANCING MACHINE

so that the critical conditions are obtained; *all unbalance readings are taken under these critical speed conditions.* A variable speed control is provided on the electric motor driving the rotatable members to produce the critical speed during each test.

The amount of unbalance of the compensating weights is controlled by the handwheel C, which adjusts the separation of the compensating blocks on the vertical spindle in the headstock. The angular relation of these blocks with respect to the work is varied by means of the handwheel D. Adjustment of both amount and angle is made whilst the machine is running at the critical speed. The dial F indicates the amount of created unbalance and the dial E the angular position of the compensating weights.

With regard to Figs. 142 and 143, the part U to be balanced contains a static unbalance weight M and a dynamic unbalance couple WW′. By turning the capstan O in an anti-clockwise direction the pivot-shifting mechanism operates to release the anvils for the pivots L and the bed or vibrating unit will rest on the static pivots K. The springs adjacent to the pivots L support the bed flexibly and allow it to vibrate crosswise on the static pivots K. These springs have a screw adjustment for proper tension.

The compensating blocks P and Q, shown in Fig. 143, are located diametrically opposite on the vertical spindle I in the headstock of the machine. Both blocks are of the same weight and their centres of gravity are the same distance from the centre of spindle I. Block P is fixed vertically, but block Q may be adjusted vertically along the spindle, by manipulation of handwheel C, shown in Fig. 144. When the blocks are together, their centres of gravity are in the same plane normal to the axis of the vertical spindle, hence as they rotate no dynamic couple is present.

When block Q is moved away from block P a dynamic unbalance couple is introduced which acts as a static unbalance on pivots K, or as a counteracting dynamic couple on dynamic pivots L, depending upon which pivots are engaged.

For the compensating blocks exactly to counteract either static or dynamic unbalance in the part, they must rotate in the proper angle, or phase relation, to the part being balanced. Handwheel D, Fig. 144, permits variation of this angular relation, by moving, longitudinally, a splined sleeve on the main spindle in the headstock. This sleeve is actually a wide-faced helical gear meshing with a gear on the vertical block spindle, so that any longitudinal movement of the sleeve will change the angular relation between the two shafts. Now, as the part U is rotated the centrifugal force due to static unbalance M will cause the bed J to vibrate about pivots K. The dynamic couple WW′, however, will not cause any vibration on these pivots, since the moments of each element of the couple about the pivot axis KK are equal and opposite. Therefore, if the compensating blocks in the headstock are adjusted, as shown in Fig. 142, until they create a couple equal in moment to the moment of the static unbalance M

and opposite in direction, the bed will cease to vibrate. As previously mentioned, dials record the amount and angle of the compensating unbalance, thus determining the unbalance in the work.

The static compensator A, Fig. 144, on the main spindle may be adjusted to create an artificial static unbalance, equal and opposite

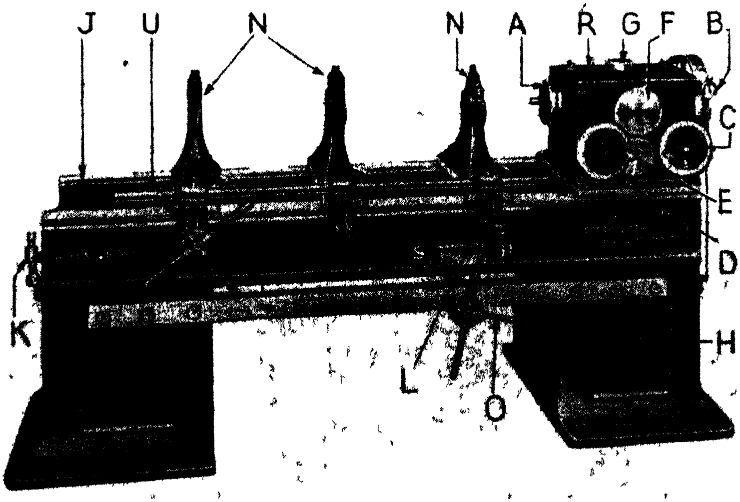


FIG. 144. THE OLSEN-CARWEN TYPE C BALANCING MACHINE

A—static unbalance compensator, B—dial indicating any vibration due to unbalance, C—handwheel for regulating amount of compensating weight applied to balance D—handwheel to adjust angular position of compensating weights E—dial indicating angular position of unbalance, F—dial indicating amount of unbalance, G—angle reference disc, H—tool compartment J—rigid bed with inverted "V" way to align bearing supports, K—static pivot, L—dynamic pivot N—any number of bearings may be used to support the body being balanced Two furnished as standard equipment, O—pivot shifting mechanism R—headstock, U—spider for moving, longitudinally along the bed, the part to be balanced and all its bearing supports

to the static unbalance in the work, when taking readings for dynamic unbalance. It may be set to any angle and amount of static unbalance within the limits of the machine.

To determine the dynamic unbalance in the work, the shifting spider O is turned clockwise, releasing the static pivots K and allowing the bed to rest on dynamic pivots L, as shown by Fig. 143. The bed will then vibrate about pivots L due to the moment of the dynamic couple WW′ about the pivot axis. If the compensating blocks P and Q are now separated sufficiently to set up a dynamic couple equal in amount to the couple in the work, and are adjusted angularly so that the couple acts opposite in direction to that in the work, the bed will cease vibrating. The amount and angle of the couple are recorded on dials on the headstock, thus enabling the determination of the amount and location of unbalance in the work.

The Avery Rapid Dynamic Balancing Machine

This machine, which is made in different models for light and medium size members, has been designed to determine the magnitude and position of the out-of-balance in the member under test in order to show, automatically, by means of a chart the amount of the out-of-balance in ounce-inches, and on an angular disc the angular position

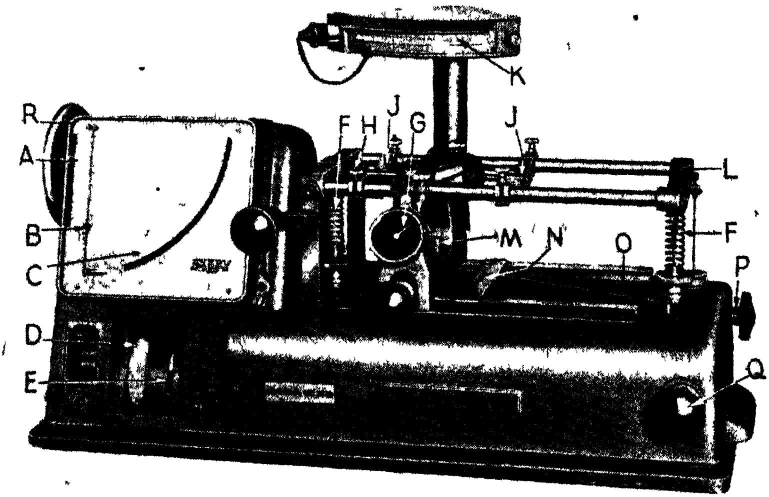


FIG 145. THE AVERY RAPID DYNAMIC BALANCING MACHINE FOR LIGHT ROTORS, ARMATURES, AND SIMILAR PARTS

A—balance indicator knob, B—self adjusting indicator, C—measuring chart for balance results, D—knurled regulation wheel for axial movement of compensating disc, E—angle position locator F—supporting springs, G—mechanical vibration indicator, H—drive coupling for test body, J—bearing traverses for test body, K—optical vibration indicator, L—oscillating cradle, M—fulcrum support, N—graduated ruler for use with measuring cord O and chart C, P—speed control for attaining resonance, Q—angular position of compensating disc control, R—angular position locating control, used in conjunction with red-and-white disc E

of the out-of-balance force. The machine has been constructed so that the procedure of balancing can be carried out with facility and speed by non-technical operators, as in mass-production balancing of similar parts, such as armatures, rotors, crankshafts, etc.

The smaller machine, Type 3001/A00, with a capacity of $3\frac{1}{2}$ oz. to 10 lb., is illustrated externally in Fig. 145 with the various lettered parts explained in the caption below.

The principle upon which the machine is based is shown schematically in Fig. 146, from which it will be observed that the member to be balanced, termed the "test body," is mounted on bearing traverses on a frame or "oscillating cradle" having supporting springs at either end. The lower ends of the springs are attached to "ground" members

and do not, therefore, have any movement. A parallel shaft having a compensating disc with an out-of-balance weight is driven from the main shaft that drives the out-of-balance member. Upon proper adjustment of the balance weight the vibrations of the cradle, which are indicated by an optical and a mechanical indicator, will cancel out. The speed of the drive from the electric motor is varied during a test until the critical speed, giving maximum amplitude of vibration, is attained; a variable resistance is provided for the purpose.

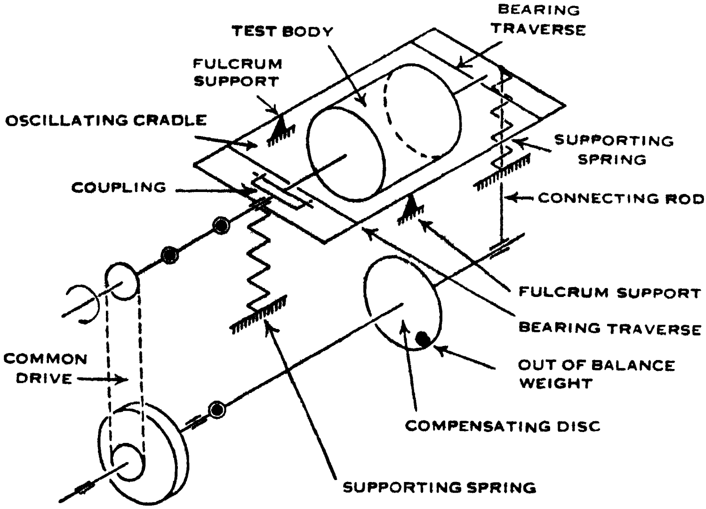


FIG. 146. THE AVERY LIGHT BALANCING MACHINE PRINCIPLE

The machine is fitted with fulcrum supports, the axial position of which can be adjusted in order to bring them into the compensating planes of the test body, thus eliminating any calculation in regard to the uncompensated residual balancing moments.

The angular position of the out-of-balance weight on the compensating disc can be altered by means of a differential gear actuated by means of a control wheel.

In order to counterbalance the effects of unbalance in the test body, the compensating disc is moved axially until the vibrations cease, so that its centrifugal force moment is equal and opposite to that caused by the centrifugal force on the unbalanced test body. For the purposes of a test the fulcrum supports are first moved beneath one of the vertical compensating planes of the test body and the compensating disc adjusted until all vibrations die out. The measuring chart C (Fig. 145) then shows the balancing results in ounces or ounce-inches by means of a knob A and a self-adjusting indicator B, which,

before the balancing operation, can be set to the pre-selected position of the vertical compensating plane and to its diameter. A measuring cord O is used for locating the compensating plane by an adjustable graduated ruler N, which is set along the base The various parts of

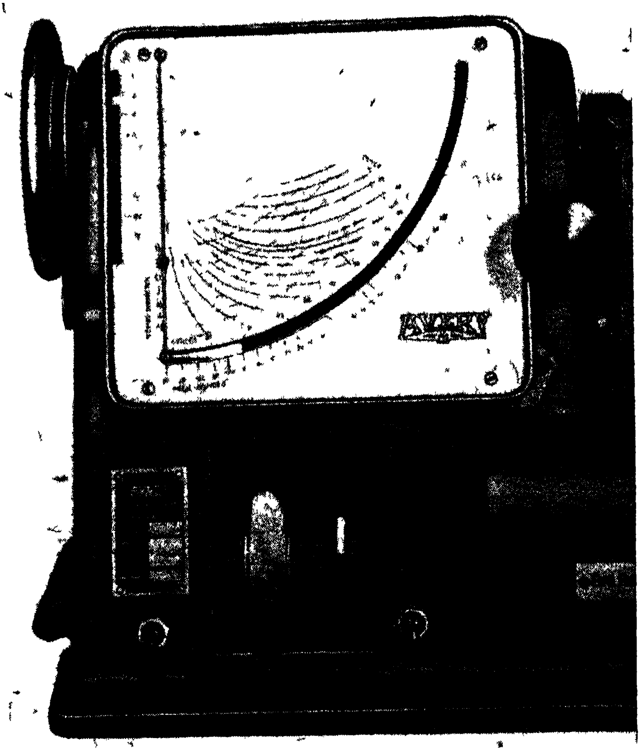


FIG 147. ENLARGED VIEW OF MEASURING CHART AND SELF ADJUSTING INDICATOR

the balancing machine are lettered in Fig 145, and a key to these letters is given in the caption.

After the test body has been measured for out-of-balance with the fulcrum support M beneath one vertical compensating plane it is moved along so as to lie in the second vertical compensating plane and the procedure repeated. In this manner the dynamic unbalance is measured, and the results show the required counterbalance weights or moments and their respective angles in the compensating planes of the test member.

Fig. 149 illustrates a larger model of the Avery rapid dynamic balancing machine, intended for crankshafts, medium size rotors, and

armatures. It utilizes the same principle of the compensating disc with its out-of-balance weight, provided with axial and angular adjustments, as that employed in the smaller model shown in Fig. 145. It is only necessary to operate two handwheels, L, L for actuating the compensating disc, in order to measure the out-of-balance amount and angular position. As in the smaller model, balancing is effected after adjusting the speed to the critical value.

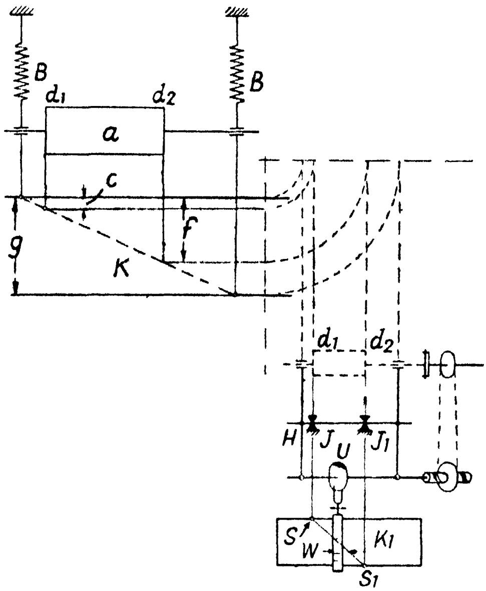


FIG 148. ILLUSTRATING THE PRINCIPLE OF AVERY DYNAMIC BALANCING MACHINE

A refinement in the larger model is the provision of a rocking bar at the front of the machine, which is held in position by two adjustable fulcrum points. These are set to positions relative to the distance between the two compensation planes of the test body (Fig. 150)—a crankshaft in the present example—previously selected for removal or addition of material for balancing purposes. Referring to the diagram given in Fig. 148, it will be seen that the actual test body *a*, with its compensation planes at d_1 and d_2, and flexible end supports B, as shown in the upper left-hand diagram, is reproduced to a reduced scale on the rocking bar between the adjustable fulcrum points, coincident with its compensation planes, as shown by the lower right-hand diagram. In this case the fulcrum points are indicated at J and J_1, the compensating disc, with its out-of-balance weight at U, and the actual

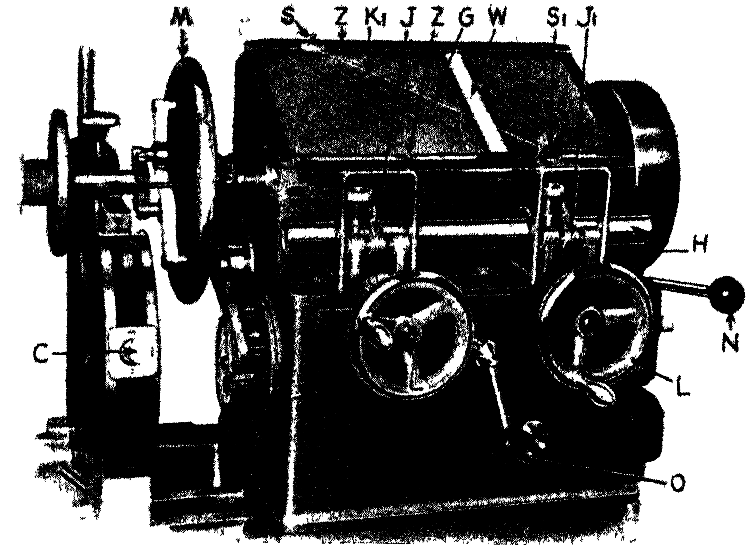


FIG. 149. THE AVERY LARGER MODEL DYNAMIC BALANCING MACHINE FOR CRANKSHAFTS, ARMATURES, ETC

C—bearing roller tension spring adjuster, J, J_1—fulcra K₁—measuring cords, L—control handwheels, M—telescopic disc coupling N—hand brake O—clutch lever, G—measuring table, S, S_1—slides for cord K_1, W—measuring chart Z—ruler

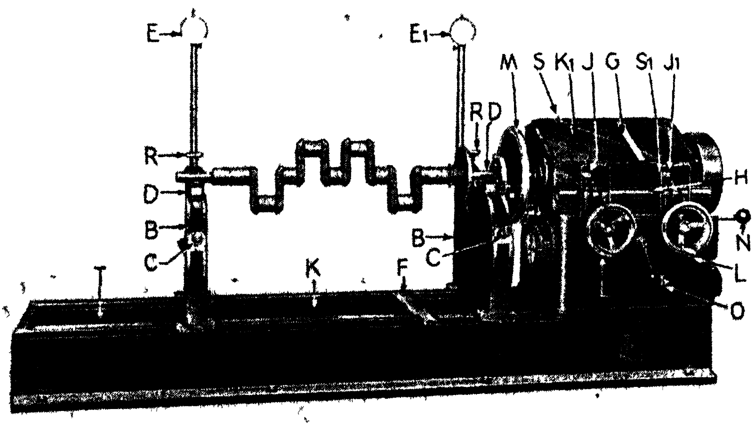


FIG. 150 THE AVERY DYNAMIC BALANCING MACHINE, SHOWING CRANKSHAFT UNDER TEST

B—bearing supports, C—bearing supports tensioning adjuster, D—open bearing rollers, E, E_1—dial gauges, F—scale ruler, G—measuring table, K, K_1—measuring cords, L—control wheel for compensating "out-of-balance" disc, M—telescopic disc coupling, N—hand brake, R—aligning screws for test body and coupling M alignment, S, S_1—slides for cord K_1, O—clutch lever, J, J_1—fulcrum points, T—bed scale

out-of-balance measuring device with its inclined cord, held by the sliders S and S_1 and scale W. This arrangement K_1 is a smaller scale replica of that shown at K in the upper left-hand illustration.

For the balancing of a large number of similar test members, as in mass-production testing, an exact scale drawing of the test member

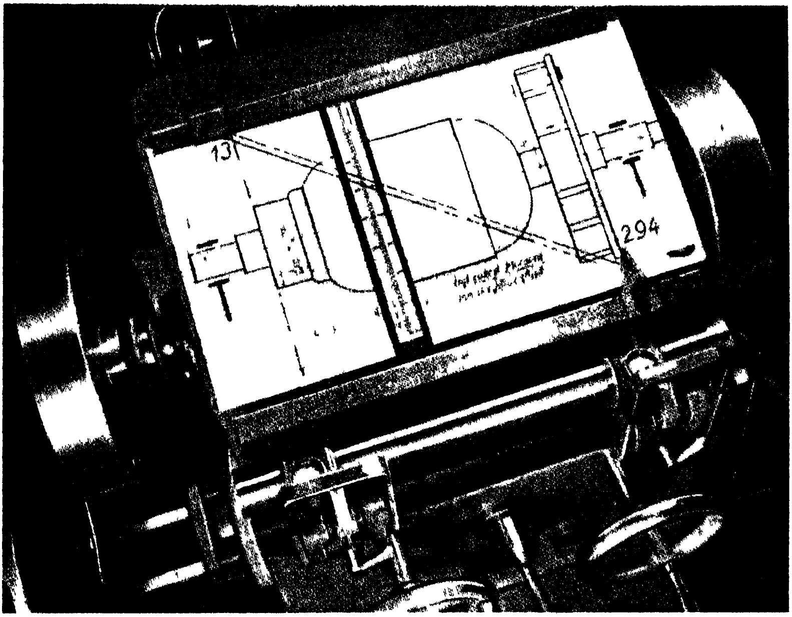


FIG. 151. CHART SETTING DEVICE, SHOWING SCALE DRAWING OF BODY TO BE BALANCED FIXED TO TOP OF MACHINE HOUSING

can be attached to the measuring table G (Fig. 151) of the machine and used for setting the machine to the correct compensation plane without reference to the test body itself, namely, by the position of the indicator S_1 (Fig. 149).

With regard to Fig. 150, the two bearing uprights B each contain a spring C, which is adjustable to support the open bearing rollers for accommodating the test body (crankshaft), and a device R for adjusting the height of the bearing rollers to suit the shaft diameter under test. Two dial gauges, E and E_1, are provided for vibration indication purposes, when the test object is being set, prior to balancing. The measuring device comprises an adjustable ruler F engraved with a double scale, which is set to the selected vertical compensation plane of the test body; a measuring table G; a rocking bar H positioned

by two adjustable fulcrum points J and J_1 which can be locked or released, individually; two measuring cords K and K_1 between the machine base and across the measuring table G, respectively, and two slides S and S_1 for adjusting the inclination of the cord K_1 to correspond with the values indicated by the adjustable ruler F and its intercepting cord K.

The compensating device (Fig. 152) consists of an oscillating shaft with differential gear, synchronously driven with the test body by electric

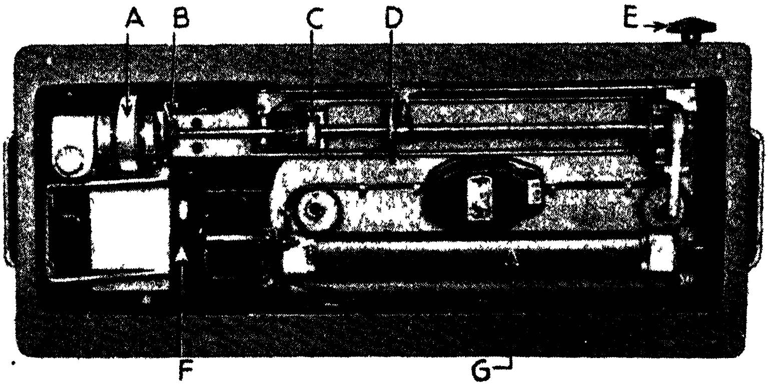


FIG. 152. THE COMPENSATING SYSTEM ON AVERY BALANCING MACHINE

A—regulation wheel, B—angular disc, C—differential gear, D—compensating disc, E—knurled knob, F—driving motor, G—variable resistance

motor and coupled with the rocking bar H in front of the table G to obtain synchronous oscillations of the test body, compensating device, and rocking bar. The oscillations of the bearings D are transmitted to the oscillating shaft and rocking bar by two hexagonal shafts T running inside the machine base. The out-of-balance compensating disc is set in angular relation to the test body by the hand wheel L. There is a telescopic disc coupling M with coupling belt and centring pin to connect with the driver of the test body, a hand-brake N to stop the rotation of the test body; and a clutch O for disconnecting the test body and compensating device for checking the balance of the test member after correction.

After certain initial adjustments of the test body and fulcrum points, slides, etc., the fulcrum point in one of the two selected compensating planes is locked, the other fulcrum point remaining "open." This causes the test body and rocking bar to oscillate about the locked fulcrum point and the appropriate dial (E or E_1, Fig. 150) will indicate

the vibrations. The handwheels L (Figs. 149 and 150) are then adjusted to bring the amplitudes of the vibrations to zero, and the amount of unbalance and its angular position for the selected compensation plane are read off. The procedure is then repeated with the other fulcrum "open" and the former "open" one locked, in order to effect balance in the other compensation plane. The machine is arranged to indicate the positions for adding weight to correct the unbalance or for removing weight on the opposite side, for the same purpose.

After the test body has been corrected in the two compensating planes it should be given a test run, preferably over its normal operating speed range to check the accuracy of the balancing, on the machine itself.

The Lundgren Balancing Machine

This type of machine is widely used for the commercial balancing of similar production components in the automobile and electrical industries and for many other engineering parts. It has been developed to a high degree of measuring accuracy and at the same time its manipulation has been simplified by the Tinius Olsen Co., in their more recent Model "S" balancing machines.

The Olsen-Lundgren machine operates at a speed considerably lower than the critical speed, as in this type the amplitude of vibration is not relied upon for the determination of unbalance. Further, the phase angle or lag effect of the induced vibrations has no influence whatever on the unbalance angle determination, as the latter is obtained from the position of the compensating weight, which is subject to the same lag conditions as the unbalance to be determined.

The Lundgren machine utilizes the principle of a created unbalance in the compensating portion of the machine to counteract the unbalance of the test body. Fig. 153 is a schematic drawing, to show the principle of the machine. Essentially, the test body is supported on rollers on a vibratory table or cradle, and is driven by the same drive and at the same speed as the compensator, shown at the right-hand side in Fig. 153. The two correction planes are selected in the test body and the cradle locking pivots or fulcrum points adjusted along the bed in each of these planes. The resultant dynamic unbalance in the test body can be represented by the weight W_R and W_L in the planes selected for correction, as indicated in Fig. 153. When the compensator W_C is set at zero unbalance, it can have no effect upon the vibration of the cradle; further, the unbalance weight which lies in the plane of the locked pivot cannot cause vibration of the cradle. Thus, the only vibration is that due to the centrifugal force of the resultant unbalance weight in the plane of the open pivot. This is,

of course, the same principle as that described previously in this chapter.

Since the unbalance-weight W_L, when the right-hand pivot is locked, is that causing vibration of the cradle, its moment about the right pivot is given by $W_L \cdot R_L \cdot B$ where R_L is the radius of C.G. of the unbalance weight and B is the distance between the pivots. The

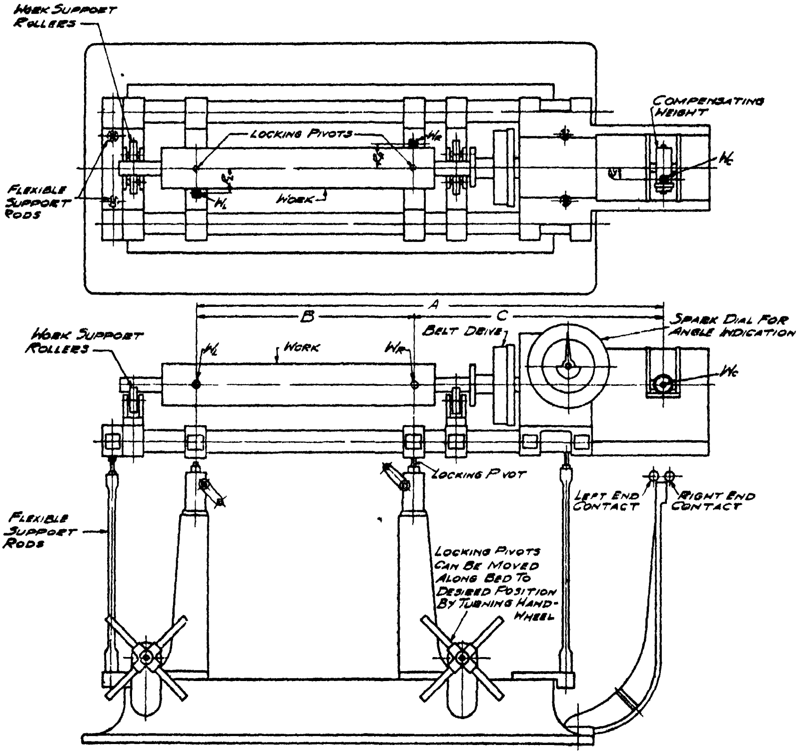


FIG. 153. PRINCIPLE OF LUNDGREN BALANCING MACHINE

compensator weight W_C at radius R_C is then adjusted so that its opposing couple is equal and opposite to the unbalance W_L. Thus,

$$W_C R_C \cdot C = W_L R_L \cdot B$$

where C is the distance between the plane of the compensating weight W_C and the fixed right-hand fulcrum. The value $W_C R_C$ is known for the machine, as also are the distances B and C, so that the unbalance, expressed in ounce-inches, in the left-hand plane, is therefore directly determinable. If the left-hand fulcrum is then locked and the

right-hand one freed, the amount of unbalance W_RR_R in the right-hand correction plane can be determined in a similar manner.

Thus $$W_RR_R = \frac{W_CR_C \times A}{B}$$

where $A = B + C$.

Fig. 154 illustrates the Olsen-Lundgren Type "S" machine, the various parts indicated by the letters being described in the caption.

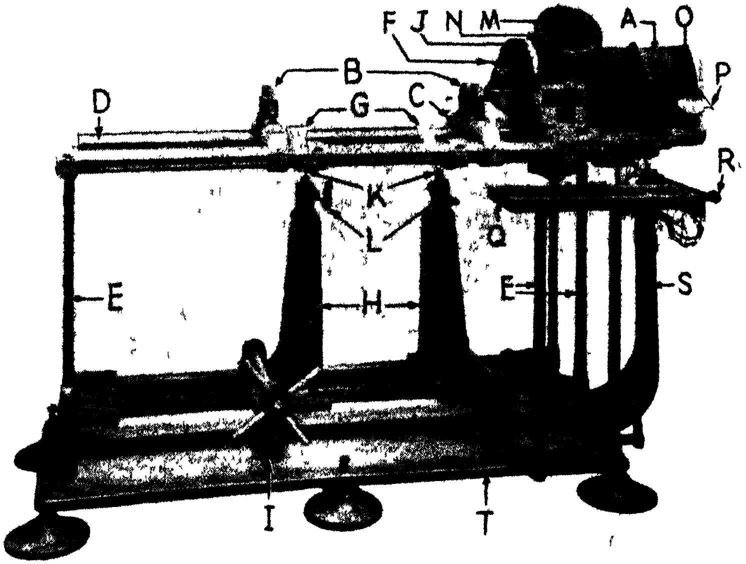


FIG. 154. THE OLSEN-LUNDGREN TYPE "S" BALANCING MACHINE

A—compensator; B—work support rollers; C—support roller brackets, D—vibratory cradle, E—flexible support rods for D; F—floating type driving pins, G—cradle pivot brackets, H—pivot locking stands, I—mechanism for adjusting pivot positions, J—driving pulley connected to motor by an endless belt, K—locking pivots, L—pivot locking handles, M—spark dial pointer, N—high tension spark dial; O—headstock cover; P—dial indicator for unbalance amount, Q—switchboard, R—spark contact adjusting screws for cradle contacts, S—contact screw supporting bracket, T—machine base

The two pivot locking stands H are adjustable along the length of the base by a rack and pinion actuated by the capstan arm unit I. The four flexible cradle support rods E are located at the ends of the base, two at each end; these support the cradle and restrain it to vibrate in the horizontal plane. The two cradle pivot brackets G are adjustable along the cradle to the selected correction planes. Two support roller brackets for the journals of the test body are shown at C; these are adjustable along the cradle to suit different lengths of the test body. The support rollers are shown at B; they consist of hardened and

ground steel rollers mounted on ball bearings and are adjustable to suit different diameters of the test body journals. The test body shaft or journal is driven by an adapter by driving pins F in the driving pulley J, the latter is belt-driven from the electric motor which is mounted on the rear of the headstock bracket. The spindle on J drives a pointer M on a high-tension spark dial N, in synchronism with the test body. This spindle also connects through a magnetic

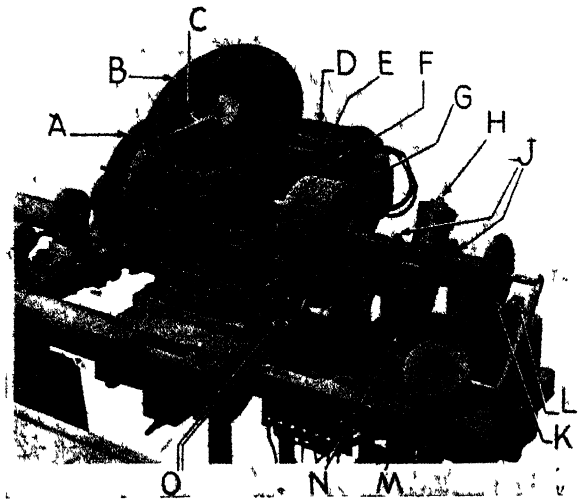


FIG. 155. CLOSE-UP VIEW OF HEADSTOCK UNIT

A—pulley, B—spark dial, C—angle indicator pointer, D—electric motor, E—compensator angle shifting clutch F—driving member, G—driven member, H—compensator, J—friction discs, K—single spark cam, L—spark contactor, M—dial indicator, N—magnets for shifting compensator weight, O—angle shifting clutch rollers

release friction clutch with the compensator, so as to drive the latter in synchronism with the test body.

A close-up view of the headstock unit is given in Fig. 155 to show the various components more clearly. In particular the clutch for altering the angle of the compensator weight arm with relation to the test body is illustrated. It is controlled by means of two buttons which are marked "*Angle*" on the switchboard; one advances and the other retards the weight arm. The actual angle of the weight arm is shown by the spark dial. The single spark cam on the end of the compensator shaft makes contact once every revolution and causes a single spark to show on the dial as it rotates; the corresponding angle at which the spark occurs is then read off. The compensator weight is controlled from the operator's switchboard by two buttons marked "*Unbalance.*" Pressing one of these buttons causes the created unbalance to be

increased, whilst the other button decreases it. The compensator, as shown in Fig. 155, is cylindrical in shape and has a screw running through it. One projecting end of the screw carries a friction disc and the other end a graduated cap for reading the finer limits of unbalance. As the screw is turned the weight moves along inside the barrel, which has a graduated slot showing the unbalance in ounce-inches; the cap acts as a kind of micrometer for the finer reading. The compensator screw is caused to move by two bevelled friction discs concentric with the compensator spindle. These discs are free to slide in an endwise direction on the compensator spindle. They are rigidly attached to a yoke which carries a small armature at its lower extremity. On either side of the armature are electromagnets which are controlled by the *unbalance* buttons.

A spring centres the yoke so that normally neither friction disc on the yoke contacts the friction disc on the compensator screw. However, when either of the unbalance control buttons is pressed, the magnets move the yoke in an endwise direction, causing one or the other of the friction discs on the yoke to contact the compensator screw friction disc. The yoke discs are stationary; hence as the compensator assembly rotates, the planetary action of the friction disc on the compensator screw, rolling around the stationary yoke disc, causes the screw to turn, which moves the compensator weight radially, thus changing the compensator unbalance. The weight will continue to move only so long as the button is held down by the operator.

The compensator weight has a wedge-shaped piece on one side. This acts against a plunger extending through the centre of the compensator shaft which carries the single-spark cam. Acting through a lever, this plunger actuates a dial indicator which moves simultaneously with the compensator weight, and enables reading the compensator unbalance without stopping the machine to be effected if so desired.

The vibrations of the cradle are amplified and indicated by an electrical method, utilizing platinum-iridium alloy contacts which are adjusted so that the slightest motion of the cradle, due to unbalance vibrations, causes the cradle contacts to "make" an electrical circuit at one extreme of the forced vibration and to cause a spark on the same spark dial N (Fig. 154) as is used for angle indications.

During the length of time this contact is closed, the pointer had made a partial rotation around the spark dial, and the spark over the gap between the end of the pointer and the annular ring on the dial gives the appearance of a blue arc of sparks, denoting the position of the unbalance.

Fig. 156 shows how this may appear on the dial. The heavy arc extending from 265° to 15° on the dial represents the arc of sparks set up by the unbalance. The single spark denotes the angular position

of the compensator. It is shown in the sketch at 140°, or diametrically opposite the centre of the arc, indicating that the compensator has been adjusted to oppose the unbalance. Owing to persistence of vision, both the arc and the single spark appear to occur at the same time, making it a simple matter to adjust the position of the single spark accurately by eye, without it being necessary actually to note the angular values.

The Balancing Procedure. The angular position of the compensator is shown by the position of the single spark on the dial as previously explained. This angular position is adjusted, by the two *angle* buttons on the switchboard, until the single spark appears diametrically opposite the centre of the unbalance spark arc. This means that the compensator has been placed in the same angular plane as the unbalance. The unbalance to be set up in the compensator will now be opposite in moment to the unbalance in the part. Moments, of course, are taken about the locked pivot.

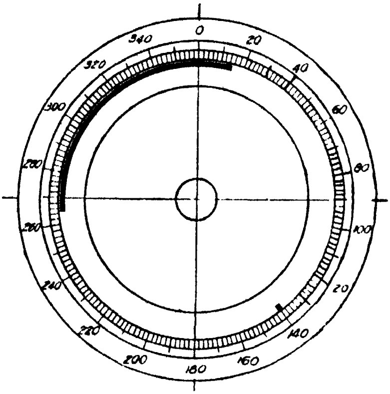


FIG 156. THE SPARK DIAL (B, FIG. 155) SHOWING "ARC" IN BLACK

The weight in the compensator is then adjusted by the two *unbalance* buttons on the switchboard, until the unbalance spark arc on the dial either entirely disappears or shows up as a scattered spark around the dial. This indicates that the cradle has come to rest and that the moment of the created unbalance in the compensator is exactly equal and opposite to the moment of the unbalance in the part. The machine is then stopped and the amount of the compensator unbalance can be read directly in ounce-inches, or other units, on the compensator barrel. The angular location of the unbalance is obtained from the location of the compensator, since both must be in the same plane.

The part may then be marked for the location and amount of unbalance in the one correction plane, the pivots interchanged, and the process repeated for the other correction plane.

The simplicity of the method of operating this machine is such that no special skill is needed and balancing can be done on parts such as automobile crankshafts at a rate as high as 16 to 20 per hour; this time includes the making of corrections by drilling or otherwise removing metal. The machines are made in fourteen different sizes, from the smallest for parts of 15 to 100 lb. weight up to the largest

for 500 to 10,000 lb.; the latter type would be used for steam turbine rotors, printing rolls, large electric motor armatures, etc.

The Benrath Balancing Machines

An entirely different principle from those previously described for the mechanical type balancing machines is employed in the Benrath type "NUA" and "RJN" machines. In these the test object is mounted on a spring-supported work cradle and the latter is subjected to forced oscillations at the same frequency as those caused by the unbalance of the test object. These forced oscillations are imparted to the cradle by means of an eccentric and connecting rod; the eccentric operates at the same speed as the test object is driven. The amount and the phase of the oscillations can be varied by means of controls on the machine so as to oppose the vibrations of the test object and thus to cause the vibrations to cease.

The usual method adopted in these machines is to arrange for the fulcrum points to coincide with the correction planes, so that only two individual balancing operations are necessary, namely, one in each plane.

Fig. 157* illustrates the principle of the "RJN" type machine for production combined static and dynamic balancing of test objects such as crankshafts, impellers, armatures, gyroscope wheels, etc. It is made in several models, ranging from one for maximum and minimum test object weights of 3 lb. and ¼ lb., respectively, to that for weights of 100 lb. and 5 lb., respectively.

The electric motor drives the test object driving spindle through a pair of gears A and B. There is a differential gear unit C which drives the bevel gear D connected to the flexible coupling shown, which in its turn drives the test object. The flexible coupling and the test object can be adjusted by means of the hand wheel, shown on the upper right-hand side, so as to alter its angular relation to the drive shaft of B, by any amount up to 360°. The shaft to which B is attached drives an eccentric and sheath E, connected by a hinged rod J to a lever F having a fulcrum point G. The rod is thus oscillated at the test object drive frequency. A roller H can be moved along the lever F by means of the hand wheel shown below, through a nut and screw arrangement. This roller H imparts the oscillations to the lever R with its fulcrum at K in varying amounts from zero to maximum, according to its position in relation to the lever F. The two levers R and F are maintained in contact by means of the spring shown on the right. The lever M, with a fixed weight at its right end, is coupled to the cradle N, in which the test object is placed, by means of a

* *Machinery*, 16th December, 1937.

connecting rod. The cradle N is supported by the balancing spring on one end and its fulcrum at the other.

As long as the lever R is stationary the cradle N readily oscillates the lever M with its weight. As soon, however, as the lever R oscillates with consequent movement of the fulcrum point L of the lever M, the weight, on account of its inertia, has a tendency to act as a fulcrum point for M, thereby causing the oscillations of the cradle N, due to

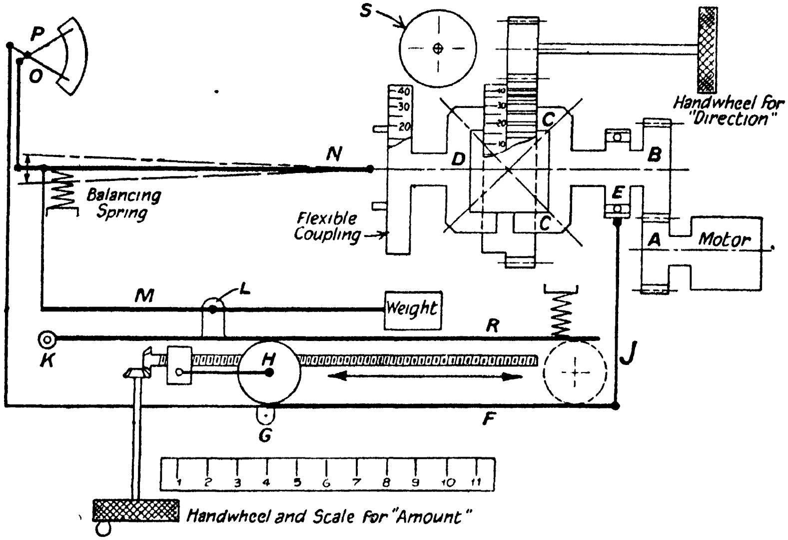


FIG. 157. PRINCIPLE OF BENRATH TYPE "RJN" BALANCING MACHINE

the unbalance of the test object, to cease. The lever F has an extension beyond G and is thence connected with an indicator needle O. The machine cradle N operates a similar indicator needle P on the same axis. The movements of the two vibrating needles are observed by means of a stroboscopic device, having a "creep" arrangement which enables the motions of the needles to appear to be slowed right down, thus making it possible accurately to synchronize the movements of the two needles.

The procedure for balancing the test object is to run the machine at the given test speed and observe the vibrations of the two needles by means of the stroboscope. The "direction" handwheel (Fig. 157) is then operated so as to bring the vibrations due to the test object and the eccentric mechanism into synchronism—and at the same time to measure the phase difference or angle. Next, with the two needles vibrating synchronously, the handwheel for "amount" (Fig. 157) is adjusted in order to bring the amplitude of the vibrations of O and

P down to zero. Thus, both the angular position and amount of unbalance in one correction plane can be read off the scales provided for the purpose. The procedure is then repeated for the fulcrum adjusted so as to coincide with the second correction plane.

Another model Benrath balancing machine, known as the "HAK" type and used for *high production dynamic balancing*, measures the unbalance by means of a pair of counterweights, revolving synchronously with the test object and controlled for amount and angular direction by stationary-type hand wheels, so as to stop the resonance vibrations of the machine cradle, as shown by a precision dial type indicator.

The Olsen Electric-type Balancing Machine

In principle the Olsen type "E–O" machine operates in a somewhat similar manner to the Lundgren machine, but, instead of observing the amount of unbalance and its angular position by mechanical vibration indicators, it utilizes an electrical method. The machine has the usual cradle and drive for the test object and two axially adjustable pivots which can be set in the correction planes of the test object. Each pivot is locked in turn and the amount and angle of the unbalance measured electrically. The method employed is to utilize the vibrations of the free pivot to actuate magnetic pick-ups which produce an alternating current proportional to and in phase with the vibration. Means for rectifying the alternating current output of the pick-ups and obtaining its phase relation to the rotation of the test object are provided.

For this purpose the magnetic pick-up is attached by means of a push-rod to each end of the cradle; this converts the vibrations of the latter into alternating current, proportional to the amplitude of vibration. The pick-up consists of a coil actuated by the push-rod, which moves in the magnetic field set up by a heavy permanent magnet, secured to the base of the machine. The alternating current is proportional to the vibration amplitude, and this, in turn, varies with the out-of-balance amount, so that the alternating current value is proportional to the latter. Further, the phase relation of the alternating current output of the pick-up to the rotation of the part being balanced is an indication of the angle of unbalance.

A mechanical type rectifier is employed. It consists of a special cam type of commutator located in the headstock of the machine. The cam for actuating the contacts in this commutator rotates in synchronism with the test object. The cam has two dwells, each 180° in length and at slightly different radii. The cam followers and contacts are so arranged that one set of contacts remains closed for 180° or one-half turn of the cam; then this set opens and the other

set opens for the next 180°, and so on. As there are two contact assemblies located on this commutator, spaced 180° apart, full wave rectification is obtained. The commutator block which carries all of the contacts and cam followers is arranged so that it can be turned about the axis of the cam by turning a knob at the front end of the machine. This knob has a graduated dial to indicate its relative position. The commutator contacts are so connected with each other

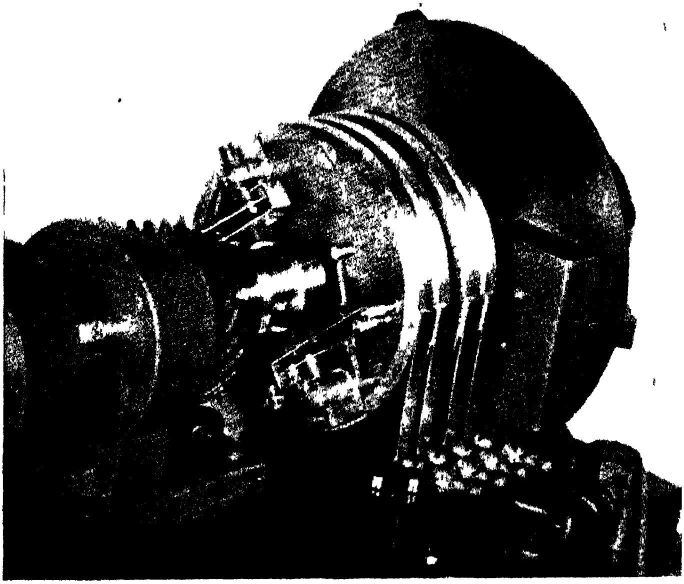


FIG. 158. CONTACT TYPE COMMUTATOR USED ON OLSEN TYPE "E-O" ELECTRICAL BALANCING MACHINE

and with the current meter that for a given setting of the commutator the direction of the current from the pick-up to the meter A (Fig. 160) is reversed every half-turn of the cam and, therefore, of the part being balanced.

The principle employed is illustrated by the alternating current output graphs of the pick-up coils shown in Fig. 159, *A*, *B*, and *C*. In Graph *A*, the solid line shows a simple harmonic variation current curve on an angle base. Assume that the commutator is set so that contact change takes place at *a*, *b*, *c*, etc., i.e. at the points where the curve crosses the axis.

For such a setting the direct contacts are closed from *a* to *b* and the reverse contacts are open for the same period. From *b* to *c* the direct contacts are open and the reverse contacts closed. This has the effect of throwing the negative part of the curve between *b* and *c* up into

the position shown by the dotted line. Hence the meter, which is a D.C. microammeter, receives two direct-current impulses for every revolution or cycle from a to c, and will show maximum reading for this particular setting of the commutator. If, now, it is assumed that the setting of the commutator is moved by a slight amount as shown in Graph B, the solid line will still represent the output from the pick-up coils and the dotted line the reversed output part. From this setting of the commutator the meter will receive two direct current impulses per revolution equivalent to the shaded positive area minus the shaded negative area. Hence the meter will show a lower reading than the maximum of the Graph A, the actual reading depending upon how far the commutator has been shifted from the position of the maximum reading (Graph A).

If the commutator is moved 90° from the position for maximum movement as shown by Graph C, the positive and negative areas will be equal and the D.C. meter A (Fig 160) will read zero. Any further movement, in the same direction would result in a negative D.C. reading but for the fact that there is a zero stop pin slightly below the zero graduation, so that the meter will not register.

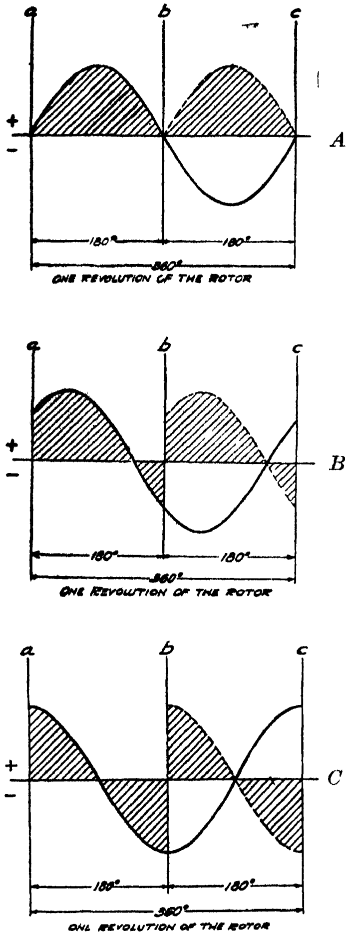


FIG. 159. ILLUSTRATING PRINCIPLE OF ELECTRICAL METHOD USED FOR TYPE "E-O" BALANCING MACHINE

The angle of unbalance can be read from the commutator dial opposite the angle marked "*Angle*" when the commutator setting is such as to give a zero reading with the meter pointer moving in the same direction as the commutator is turned. The angle between the "*Angle*" and "*Amount*" indices is 90°, and the pointer over the commutator dial serves to indicate the *angle* and also the *amount* of unbalance. In obtaining the latter the knob is turned to the right (Fig. 161), carrying the angle indicating pointer with it until the assembly is turned 90°, which will be indicated when the pointer

is opposite the "*Amount*" index located at the top of the dial. The electrical system is provided with four different sensitivity positions, numbered 1, 2, 10, and 20.

After the angle of unbalance on the commutator dial and the amount on the meter have been read, the machine is stopped and the actual location of unbalance of the test object is found by turning it around

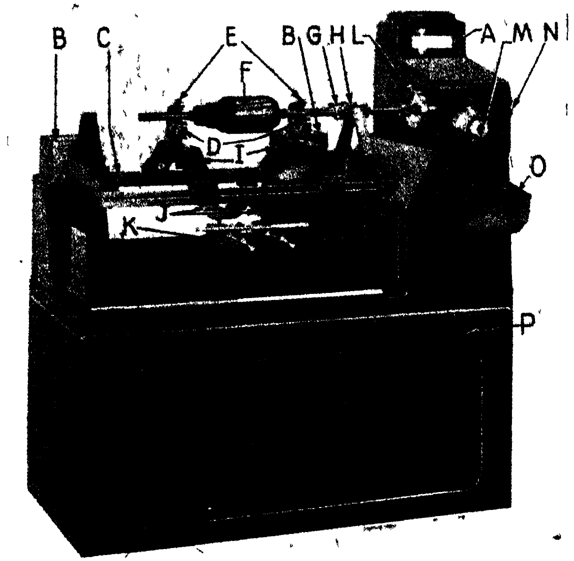


FIG. 160. THE OLSEN ELECTRICAL TYPE "E-O" HORIZONTAL DYNAMIC BALANCING MACHINE

A—microammeter to show unbalance; B—pick-up unit; C—vibratory cradle; D—work support rollers; E—retaining rollers; F—part to be balanced; G—drive adapter; H—driver; I—work support brackets; J—pivot brackets; K—pivot control knob; L—angle reference disc; M—commutator control; N—spindle hand wheel; O—control panel; P—machine base.

by means of the hand wheel N (Fig. 160), until the index on the head-stock cover indicates the same angle on the graduated disc on the spindle as that on the commutator dial. The unbalance angle on the test object then lies in the horizontal plane through the axis.

In addition to the horizontal type machine shown in Fig. 160, there is a special vertical model designed for the convenient and quick static balancing of parts of relatively small width in relation to their diameter, such as petrol engine flywheels, clutch discs and other rotating parts, rotors, and supercharger impellers. In this machine the vibratory frame carries a vertical spindle and is so arranged that it is restrained to vibrate in a horizontal plane about a vertical axis. This design enables a convenient grouping of the controls and meter

on a sloping panel The same electrical principle is employed as in the horizontal dynamic balancing machine Type "E–O"

The Dynetic Balancing Machine

The Dynetic machine, of which the Gisholt type developed by the

FIG 161 THE COMMUTATOR DIAL FROM WHICH READINGS OF OUT OF BALANCE AMOUNT AND PHASE ANGLE ARE OBTAINED

Westinghouse Co of America* is a good example, is based upon a somewhat similar electrical principle to the Olsen "E–O" type, namely, in its use of magnetic pick-ups although it differs essentially in the balancing procedure and method of ascertaining the angle of unbalance

The mechanical analogue of the electrical method is indicated at (1), (2), and (3) in Fig 163, which shows an unbalanced test object

* "Balancing Rotating Machinery," F C Rushing, Westinghouse Research Laboratories, *The Machinist*, 1943.

In Diagram (1), an unbalance of W_L is shown causing amplitudes of vibration A and cA at the two places on the axis indicated, the place of zero vibration being to the right of cA. The factor c represents an assumed constant relationship between vibrations at these places, produced by any unbalance weight in the transverse plane of W_L.

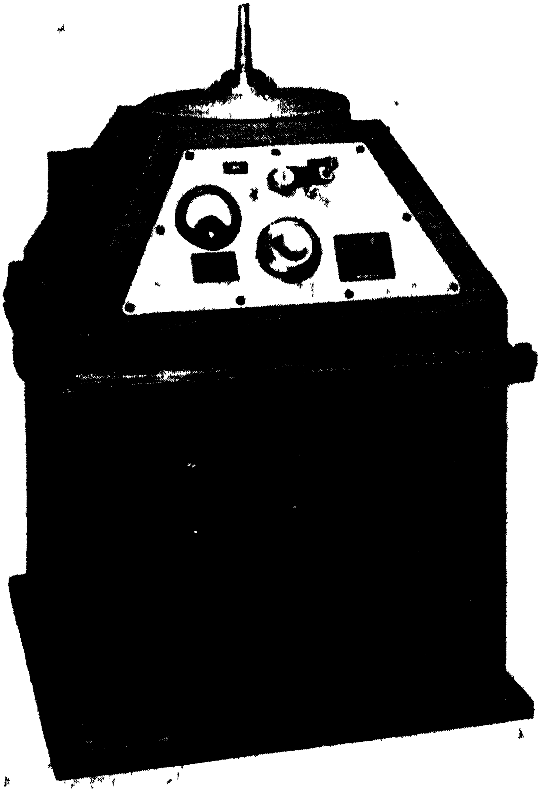


FIG. 162. VERTICAL MODEL STATIC BALANCING MACHINE FOR FLYWHEELS, ETC.
This machine operates on the electrical principle

Similarly, as shown in Diagram (2), an unbalance W_R on the right will cause vibrations kB and B at the positions indicated.

If these vibrations are added together, as shown in Diagram (3), the result will be vibrations $(A + kB)$ and $(cA + B)$ at the positions shown.

If these vibrations are combined in certain proportions, so as to eliminate B, then $(A + kB) - k(cA + B) = (1 - ck)A$, which is proportional to W_L.

Similarly, $-c(A + kB) + (cA + B) = (1 - ck)B$, which is proportional to W_R.

Referring next to Fig. 164, which illustrates the electrical equivalent of the mechanical arrangement shown in Fig. 163, Diagram (3), the pick-ups P_L and P_R are indicated at the two pedestals, and the electrical circuit shown enables the generated voltages proportional to A (or W_L)

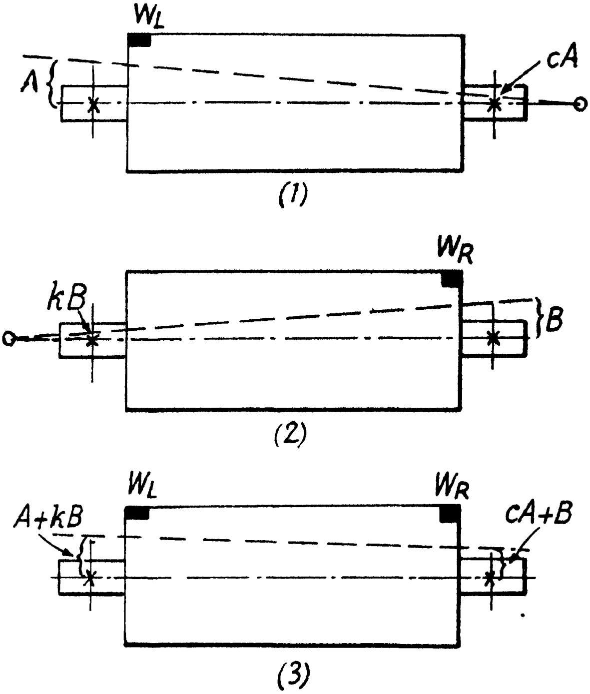


FIG 163. MECHANICAL ANALOGUE OF GISHOLT ELECTRICAL BALANCING PRINCIPLE

and B (or W_R) to be obtained, thus giving measurements of the unbalance W_L and W_R in the two chosen correction planes.

The electrical circuit setting can be determined experimentally by a rotor with a zero value of W_L but with a definite value for W_R; the adjustment needed to give zero voltage across the network output is the required one. A similar treatment of another electrical network switched across the same pick-ups gives an adjustment to indicate W_L but to cancel out W_R, so that voltages proportional to the unbalances in each of the two correction planes can be obtained by electrically measuring vibration at two arbitrary points along the axis of rotation if C and k are constant, and, further, if the vibrations

produced by each unbalance have either a 0° or 180° phase relationship. The Dynetic balancing machine is provided with the usual means for supporting and driving the test object and the electrical apparatus required to effect the measurements of voltage of the pick-up circuits, indicated previously.

The Gisholt Type "S" Dynetic balancing machine for combined static and dynamic balancing is shown, schematically, in Fig. 165.

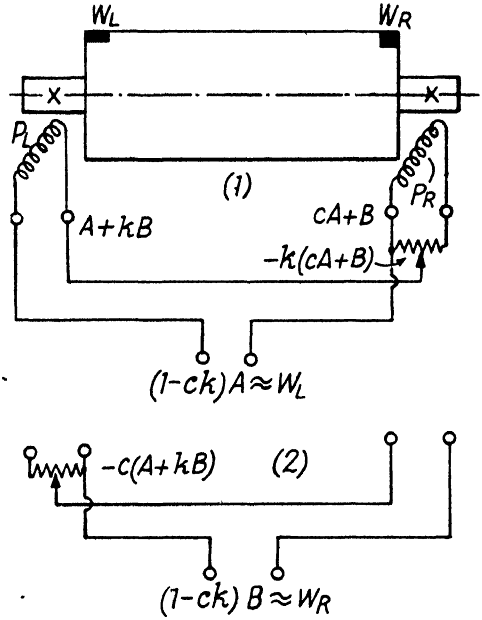


FIG. 164. ELECTRICAL EQUIVALENT TO ARRANGEMENT SHOWN IN FIG. 163

The test object is represented as a cylinder carried on flexible supports at A and B. Attached to supports A and B are coils which are in the field of powerful permanent magnets. The vibration of the supports A and B due to unbalance causes voltages a_2 and b_2 to be generated in the coils. The voltages thus generated are proportional to the vibrations of supports A and B.

Assume that the only unbalance in the workpiece is the unbalance W_2. When the work is rotated, the unbalance W_2 will cause a large motion of the support B and will cause to be generated a correspondingly large voltage b_2 by the coil in the field of the permanent magnet connected to support B. The lesser motion of the support A will cause a smaller voltage a_2 to be generated at the left-hand coil. By means of the voltage divider K, a portion of b_2 may be chosen which

will be equal to a_2, but opposite in value so that the resultant voltage V will be zero due to unbalances of any magnitude in plane 2.

An unbalance W_1 in plane 1 will cause a large motion of support A and a correspondingly large voltage a_2, and will cause a small motion of support B and a small voltage b_2. Now, if the voltage a_2 and a small part of the small voltage b_2 be added, there will be a definite

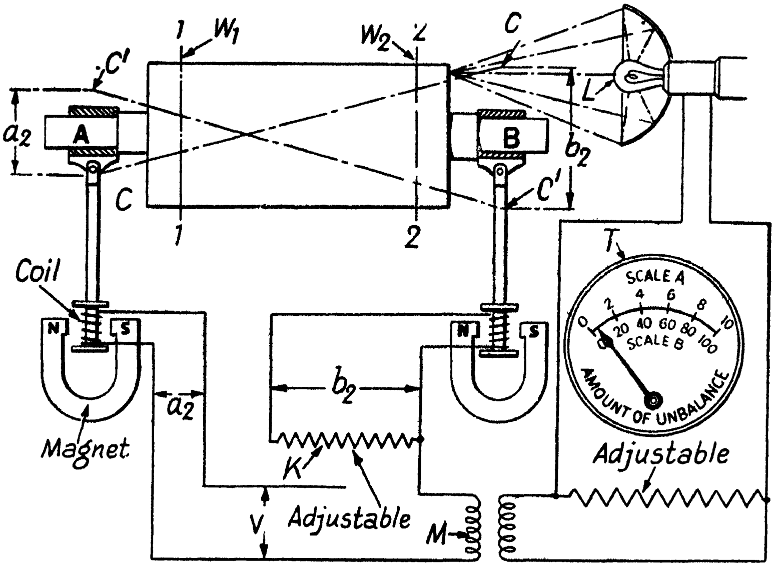


FIG. 165. SCHEMATIC LAYOUT OF GISHOLT TYPE "S" ELECTRICAL BALANCING MACHINE

voltage V due to unbalance W_1 with no effect from unbalance W_2. This voltage V may be amplified as much as 1,600,000 times by means of transformers, radio valves, or other devices at M. This voltage, or any desired portion, is supplied to the amount meter which will indicate the amount of unbalance in the work. The portion of voltage selected will be determined by the practical correction method selected so that the meter reading will indicate the depth in $\frac{1}{64}$ in. for a given size of drill or the number of $\frac{1}{32}$ in. of length of $\frac{1}{8}$ in. wire solder, etc.

The angular location of the unbalance W_1 is determined by means of a Stroboglow lamp L. This lamp is caused to flash for ten millionths of a second each time the voltage generated in the pick-up coils changes from negative to positive. The flashing of the lamp at each revolution of the work-piece will cause one point on the periphery apparently to stand still. If numerals are placed on the work-piece, the numeral which apparently stands still in front of the lamp will indicate the angular location of the unbalance correction.

In another model of the Gisholt Dynetic balancing machine, namely Model "U," which is used for heavier test objects such as turbine rotors, heavy armatures, aircraft and Diesel engines, crankshafts, blower and fan rotors, etc., a different method is employed for indicating the amount and angle of the unbalance. Referring to the schematic drawing given in Fig. 166, the general arrangement of the

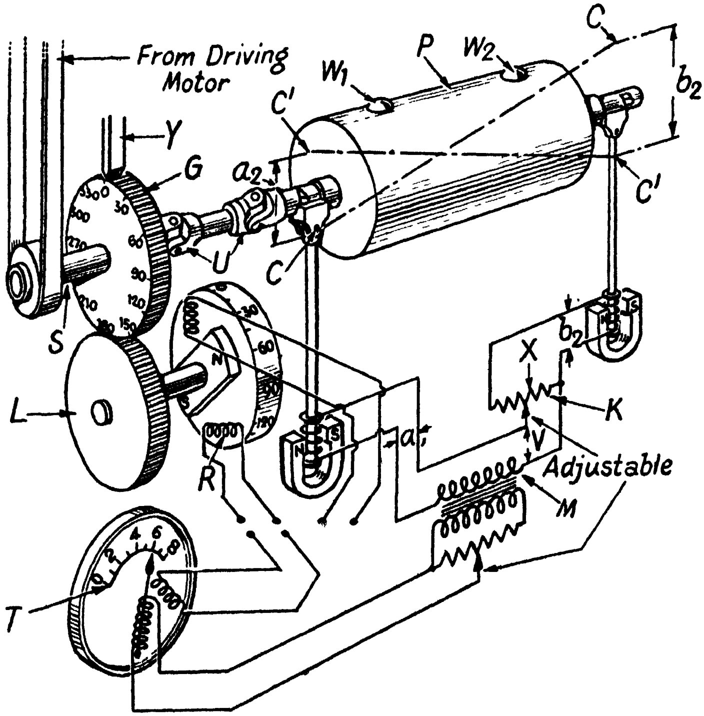


FIG. 166. THE GISHOLT TYPE "U" DYNETIC BALANCING MACHINE, (SCHEMATIC DIAGRAM)

supports and pick-ups is similar to that of Fig. 165. The voltage V due to the unbalance W, which can be amplified as desired, is used to supply the voltage coil of a wattmeter T.

The current coil of the wattmeter is supplied by means of voltages generated in the coils of the sine wave generator R. The rotor of this generator consists of a permanent magnet driven at the same speed as shaft S. The angularly adjustable stator of the generator carries two coils spaced 90° apart. When the stator is angularly adjusted until the voltage in one of the coils is 90° out of phase with the amplified voltage V, the wattmeter T will read zero. Then the angular location of the stator will indicate angular position of the unbalance.

The other coil in the generator R is used to supply current to the wattmeter, and the wattmeter T will indicate on its scale the amount of unbalance in the work-piece.

In regard to the performance of the Westinghouse-Gisholt balancing machine, it is claimed that the unbalance in test objects, such as

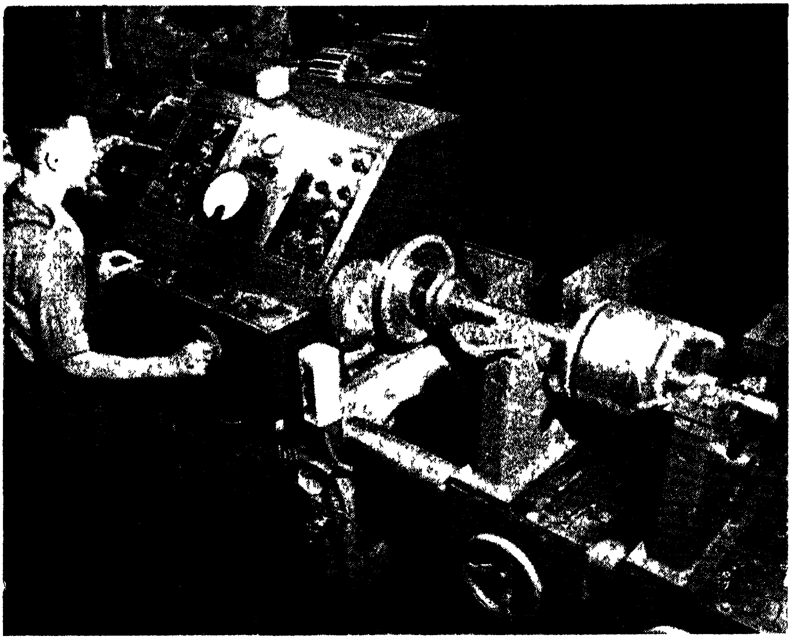


FIG 167. THE GISHOLT TYPE "U" BALANCING MACHINE, SHOWING CONTROL PANEL, TEST OBJECT (*on right*) AND PORTABLE ELECTRIC DRILL (*below*)

armatures, can be located in about one-eighth of the time required for the mechanical type balancing machine It is also stated to be possible to precision balance armatures for 50 h.p. to 200 h.p motors to $\frac{1}{10000}$ in. linear measurement.

Notes on Crankshaft Balancing

Usually, for production purposes, a special design of balancing machine is provided for crankshafts of a given design. The crankshafts are generally supported on two end bearings of the roller-assembly type and a simple drive coupling is employed. Typical machines for this purpose are the Avery rapid dynamic and the Olsen-Lundgren Type "S" machines. As mentioned previously in this chapter, special care is necessary when balancing relatively long crankshafts, namely,

those for certain six- and eight-cylinder in-line engines, to prevent *any "whip" or deflection occurring between the supports,* due to weight or unbalanced centrifugal force when rotating at moderate speeds. It is therefore necessary to eliminate any sag or "whip" by the provision of an additional intermediate roller bearing support, namely, at the centre journal bearing. The rollers require careful adjustment in regard to vertical height in order to ensure that the weight of the crankshaft is borne equally well by all three bearings; otherwise, when rotated it will tend to lift at the ends or at the centre.

Another precaution necessary when mounting a crankshaft on its support rollers is that the rollers should be adjusted so that *they clear all oil holes in the bearing surfaces* of the crankshaft journals. Even running too close to an oil hole may cause an error in unbalance readings, since the bearing surface immediately adjacent to an oil hole may not be a truly cylindrical surface due to the polishing operation over the hole. It is usual *to correct the measured unbalance* in automobile and similar types of crankshaft by drilling either in the ends of the counterweights or in the ends of the crank pins; and sometimes at both ends of the crankshaft. *In designing crankshafts for production engines* it is therefore important, in order to facilitate and expedite their balancing, to arrange for the necessary corrections to be made as nearly as possible in two particular planes normal to the axis of the shaft. With such correction planes it is possible to provide special charts showing the size and depth of drill to remove unbalance at a given radius.

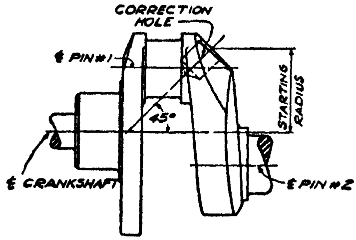


FIG. 168

In many instances, the counterweights on the crankshaft are forged integrally with the cranks, each throw being individually counterbalanced in this manner. This tends to minimize local unbalance and reduces the time required for balancing the crankshaft. With mass-produced crankshafts it is now customary to drop forge these to a fair degree of balance so that for final balancing purposes only a small amount of metal has to be removed at each of the two correction planes. In this connexion it is found that with a suitable drilling correction chart only a single drilling operation is necessary to effect balance within the prescribed limits for the job.

In regard to *crankshafts having inclined surfaces on their webs,* as shown in Fig. 168, it is not convenient—and often impossible—to drill either normal or longitudinally so that the correction hole must be made at an angle, as shown in Fig. 168. This design of crankshaft,

therefore, requires a special drilling chart for correcting measured unbalance.

When a crankshaft and its flywheel have been balanced separately, it will sometimes be found that on assembly the combination is not quite in correct dynamic balance. The most probable causes of this are: (1) Eccentricity of fit between the shaft and flywheel boss. (2) Additive balancing errors, bringing the total outside the prescribed limits for the assembly. (3) Effect of the key and keyway that may not have been taken into account on the balancing machines.

In this connexion, whilst it may be quicker to balance the complete assembly, instead of the crankshaft and flywheel, separately, if there is any possibility of having to replace one or the other component

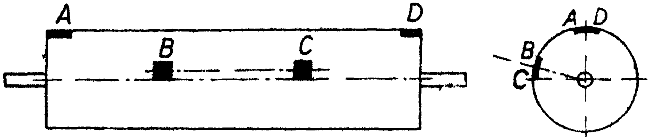


FIG. 169. BALANCING FOUR-THROW CRANKSHAFTS

alone after a period of service, it is better to have each part balanced, individually, so as to obviate re-balancing the assembly.

When balancing four-throw crankshafts of the type represented by weights A, B, C, and D in the positions indicated in Fig. 169, using the electrical method of *the Gisholm dynetic balancing machine* (Fig. 165), it is possible to arrange the controls so that the unbalances, due to B, C, and D, when A is to be indicated, can be reduced to zero meter readings. For this purpose the effect of unbalance at D is eliminated by an electrical circuit network similar to that shown in Fig. 165. Further, the effects of B and C, which are in a common axial plane in a different angular position to that of A and D, are eliminated by setting a sine wave generator stator at such an angle that the readings for the weights at B and C are zero. Then the unbalance at A is the only one that causes a meter reading. Three similar arrangements must then be used for determining the unbalance at B, C, and D, respectively.

In a special crankshaft balancing machine operating on this principle, two sine wave generators are provided, one for B and C, and the other for A and D; four electrical networks are also required. For metering purposes, special potentiometers are employed so that the meter readings can be brought down to zero when necessary. In operating this type of crankshaft balancing mechanism after the crankshaft is mounted on its roller bearings the position A is first selected and its potentiometer adjusted so as to read zero. This procedure is then

repeated for B, C, and D, with the aid of their own potentiometers. The angular positions of these potentiometers are then an indication of the amounts of unbalance in each of the four determined positions.

It is then possible to employ special "synchrotic" electric motors to couple each of these metering potentiometer shafts to a stop on a special drilling machine, thus causing the latter to be set up auto-

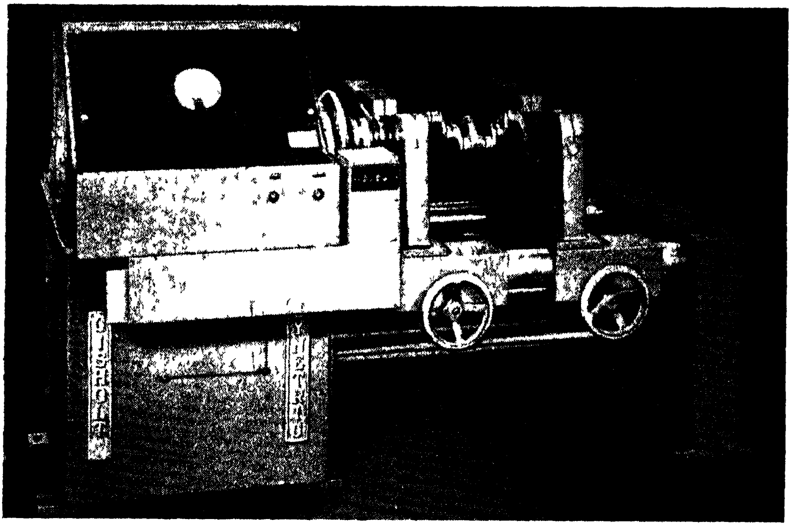


FIG. 170 THE GISHOLT TYPE "C" DYNETIC BALANCING MACHINE, WITH AUTOMOBILE CRANKSHAFT UNDER TEST

matically so that when the crankshaft is placed in it the correct amounts will be drilled from each of the four locations.

Electric Type Crankshaft Balancing Machine

Fig. 170 shows a special crankshaft balancing machine, known as the Gisholt Type "C" Dynetic machine, which enables corrections to be made at four places It is applicable to four-, six-, and eight-throw crankshafts in which it is usually not possible to find two transverse planes where corrections can be made through 360°. In this connexion it may be pointed out that in all crankshafts it is possible to select two radial planes and in each of these planes there will be provided on each end of the shaft a permissible correction point. For example, in a six-cylinder crankshaft (Fig. 171), one radial plane could contain No. 1 and No. 6 crank pins (one on either end of the shaft) and correction could be made by drilling in these two pins. The second radial plane would contain No. 2 and No. 5 crank pins, which would permit drilling corrections. Therefore, there are actually

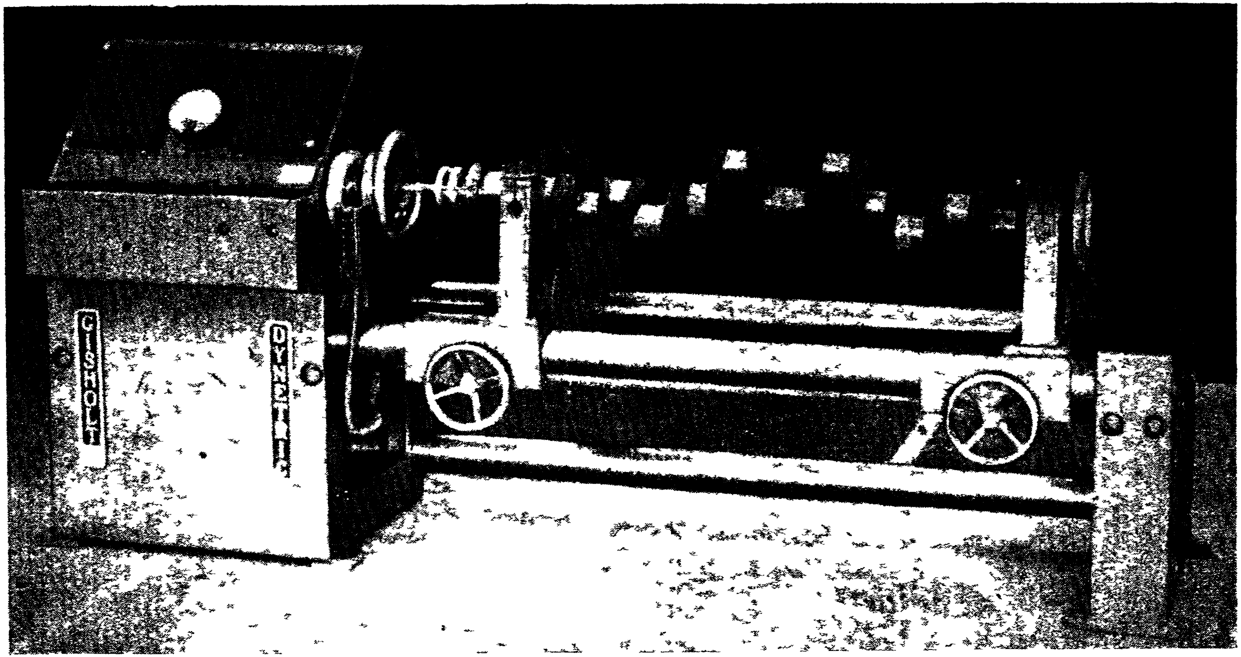


FIG 171 THE GISHOLT TYPE 'C' DYNETIC BALANCING MACHINE, SHOWING A SIX THROW CRANKSHAFT UNDER TEST
The time taken to completely balance such a crankshaft is about 15 min

four correction points in the six-cylinder crankshaft where corrections could be made, namely: in Nos. 1, 2, 5, and 6 pins. (No. 3 and No. 4 crank pins are normally made light, making it certain that unbalance will fall in the other four pins.) Likewise, the two radial planes may be so selected that corrections can be made at four points in the counterweights or cheeks at each end of the shaft, if desired.

The operating procedure, by which these four readings are obtained, is both simple and rapid. The crankshaft is placed in the balancing

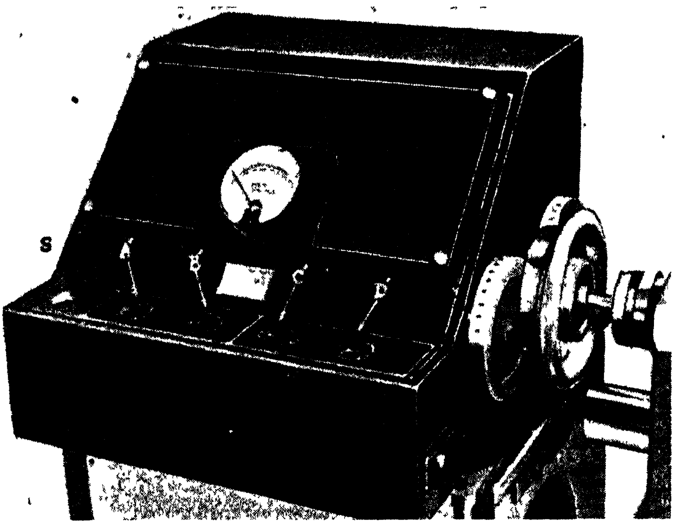


Fig. 172. Controls of Gisholt Crankshaft Balancing Machine

machine in plain half-bearings which correspond to the regular bearing liners in the motor block. The coupling to drive the work is then connected and the machine is started by a push-button, which starts the work rotating.

With the work rotating in the balancing machine, the four-position selector switch S at the left-hand side of the cabinet (Fig. 172) is moved to the first or A position, and the first knob, marked A′, is turned clockwise until the indicating needle of the meter reads zero. The four-position switch is then moved to the next, or B position, and knob on dial B′ similarly adjusted; and the same procedure is repeated for C and D and C′ and D′, after which the machine is then stopped by the push button. The four knobs A′, B′, C′, and D′ have pointers and there are flat circular dials over which these pointers move. When the machine is stopped, the pointer A′ will show a reading on this dial indicating exactly what depth of hole to drill in the first crank

pin A (No. 1 crank pin) and the other three pointers will likewise show the depth of correction hole required in each of the other crank pins (No. 2, No. 5, and No. 6 crank pins).

The average operator requires approximately 20 seconds to take these four readings and mark the work for correction.

If desired, the reading of dials A′, B′, C′, D′ may be transmitted to the individual spindles of a special four-spindle drilling machine so

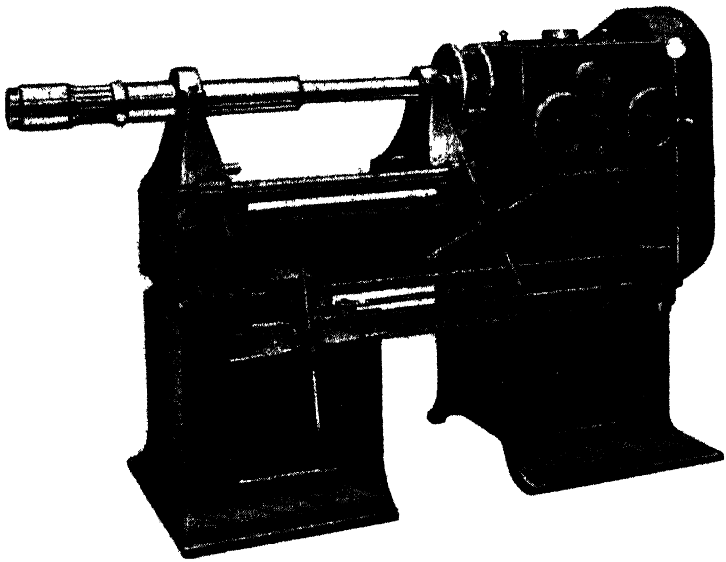


FIG. 173. THE OLSEN HORIZONTAL BALANCING MACHINE FOR SUPERCHARGER IMPELLERS, FANS, PROPELLERS, ETC.

that the depth of each spindle may be set directly and automatically from the dial positions. That is, dial A′ controls the depth of hole drilled by the spindle entering No. 1 pin, etc. This arrangement reduces the likelihood of errors due to the human element and permits one man to operate both the balancing and the drilling machines.

Balancing Supercharger Impellers

As the impellers or rotors of centrifugal compressors and also those of exhaust-turbines are of small width-to-diameter ratio and of more or less symmetrical design, their balancing is a comparatively simple procedure on a modern balancing machine, and, for most purposes, only one correction plane need be considered.

One difficulty that arises, however, is *the effect of any air streams,*

due to the high-speed of rotation of the impeller, upon the balancing machine's indicating mechanism. If run at fairly low speeds the effect is generally unimportant.

In balancing machines, such as the Olsen horizontal static-dynamic type (Fig. 173), giving their indications of unbalance by horizontal movement of the rotor, the effects of a horizontal air flow due to the rotor are the only ones to consider, and if the balancing machine is to be used chiefly for impellers it should be located so that there are no walls or other large obstructions near the ends of the machine. Further, the operator should stand well away from the impeller when the latter is being tested for dynamic balance.

Many centrifugal impellers and also exhaust turbine rotors—both of which types have to operate at speeds of 15,000 to 25,000 r.p.m. and above—require very careful balancing. If possible, they should be balanced on their own spindles or shafts, instead of on special mandrels, since slight errors due to eccentricity of fit on their spindles can be balanced out on the machine; further, the unbalance of keys and keyways can also be corrected for.

Impellers and other parts of relatively small width-to-diameter ratio may very conveniently be balanced on a machine of the Gisholt Type "E" static balancing type, shown in Fig. 174; this is made in several models for parts such as impellers, clutch plates, clutch assemblies, flywheels, automobile road wheels, aircraft propellers, fans, and pulleys. The operation of the machine is as follows.

The part to be balanced is mounted with its axis in a vertical position on an adapter carried on a spindle, which in turn is carried on a cradle hung from two vertical flat springs which act as pivots. The pivots allow the adapter to rock in one plane only.

The heavy side of the piece throws the cradle out of level as indicated on the sensitive level C. This out-of-level condition is corrected by turning the dial D until the bubble in the level is central. The reading on the dial is then noted. (Fig. 175.)

The vertical scale G on the "cutmeter" (Fig. 176) is then set so that this reading of the dial D is indicated on the horizontal scale H.

The table, with the part being balanced, is then turned 90° about its axis as indicated by the graduations E on the table. With the part in this position, the bubble in the level is again brought to a central position by turning the dial D.

The protractor scale K is then moved until the lower edge intersects on the vertical scale G, the number as shown on the second reading from the dial D.

The table E with the work is then turned to the angular position indicated on the protractor M.

With the table locked in this position the correction is made by

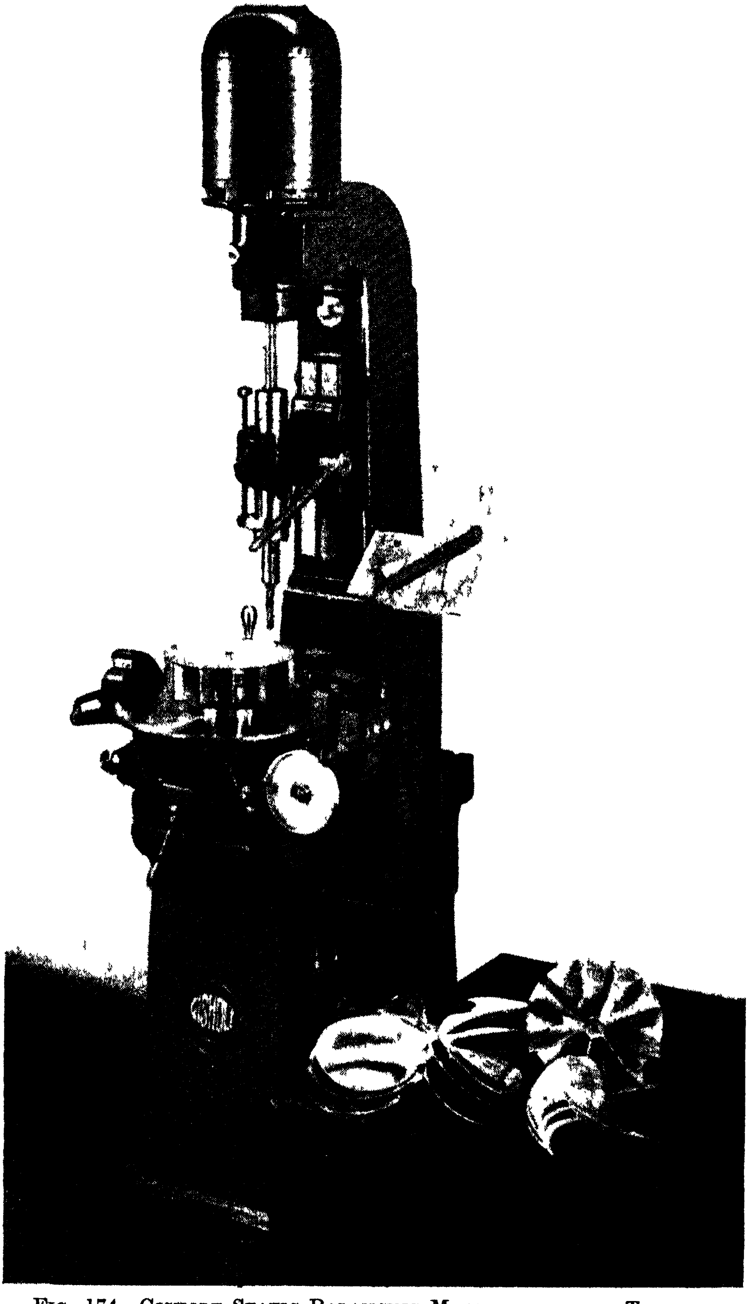


FIG. 174 GISHOLT STATIC BALANCING MACHINES, SHOWN TESTING BALANCE OF IMPELLER

drilling according to the depth and number of holes shown on the flat portion of protractor scale K

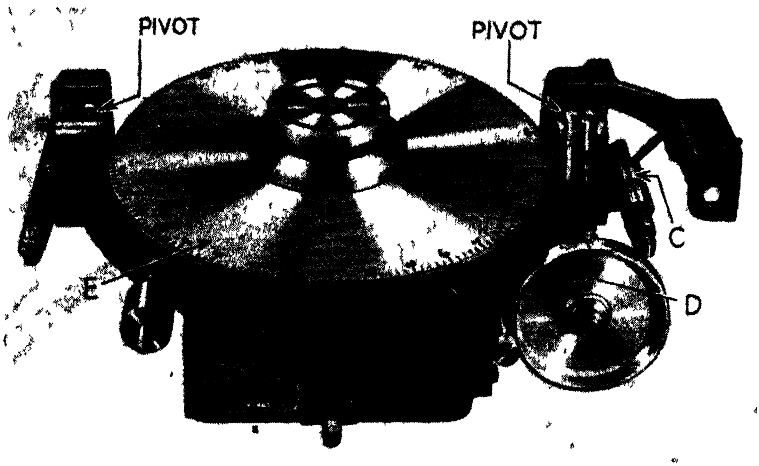


FIG 175 THE CRADLE, PIVOTS, LEVEL AND TABLE OF THE GISHOLT VERTICAL STATIC BALANCING MACHINE

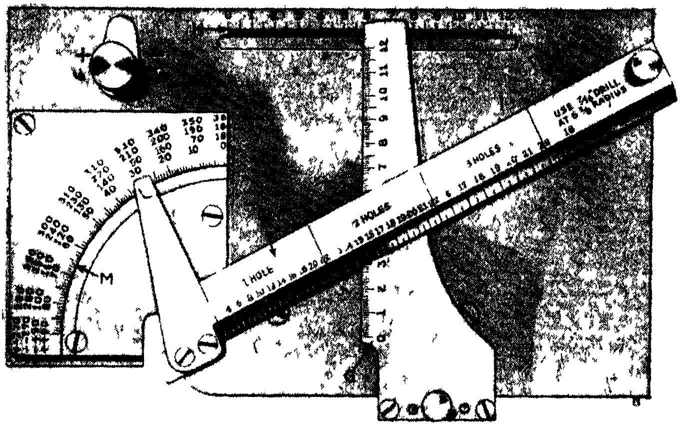


FIG 176 CUTMETER OF GISHOLT STATIC BALANCING MACHINE

After the correction has been drilled in the piece, the work may be checked before it is removed from the machine.

Dynamic Balance Checking Machines

The machines previously described have been concerned with the location and amount of dynamic unbalance, with the object of applying

corrections in two selected planes to enable the balance to be restored. In mass production balancing, it is not always convenient to re-mount the balanced test object in the balancing machine for checking purposes, after adding or removing weight, since the machine is usually employed exclusively for balancing purposes. When the errors of balance have been indicated the test object is removed, the corrections made, and the object is then checked on a simple dynamic balancing machine. This has a cradle supported on adjustable pivots which are set to the same correction planes as the test object. The latter is then rotated by a self-contained electric motor, first with one pivot locked and then the other, and the vibrations of the free pivot are observed with a vibration meter or other suitable means. If the amplitudes exceed the allowable limits for the particular test object, the latter is returned to the original balancing machine for further examination. Usually a satisfactorily balanced standard test object is first placed in the cradle of the checking machine and its vibrations observed. The mass-produced objects should then give vibration amplitudes within the prescribed tolerances.

CHAPTER IX

THE BALANCING OF ENGINES

ANY reciprocating type of engine has two possible main sources of unbalance, namely (1) the rotating masses, and (2) the reciprocating masses.

The principles of balancing the rotating masses of an engine, e.g. the crankshaft and flywheel, have already been discussed, and it now remains to consider the balancing of the reciprocating masses of typical automobile and aircraft engines. In this connexion it may be mentioned that whilst it is usually possible to effect almost perfect balance in the case of the rotating parts, it is more difficult and sometimes impossible, in practice, to balance the reciprocating masses of many types of engines.

The engine components having a reciprocating motion include the piston unit and to a lesser extent the valves (poppet or sleeve). The connecting rod has a combination of a reciprocating and rotating or rocking motion, which will now be considered.

The Connecting Rod

The general motion of a petrol engine connecting rod may conveniently be regarded as being due to a purely reciprocating movement at its small end and one of rotation, the same as that of the crank pin, at its big end.

Considering the movement of the centre of gravity G of the connecting rod in Fig. 177, this has a motion of translation together with one of angular oscillation about an axis through G and perpendicular to the connecting rod-crank plane.

Of the forces necessary to produce these motions, that which is required to give the translation of the connecting rod mass at G is identical with that needed for two masses m_p at P and m_c at C such that their total mass must equal that of the connecting rod, say, m_g, and their two moments about G must be identical.

Thus, $$m_p + m_c = m_g$$

and $$m_p \,.\, \mathrm{PG} = m_c \mathrm{CG}$$

or $$m_p = \frac{\mathrm{CG}}{\mathrm{PG}} \cdot m_c = \frac{\mathrm{CG}}{\mathrm{PC}} \cdot m_g$$

and $$m_c = \frac{\mathrm{PG}}{\mathrm{PC}} \cdot m_g$$

Hence, the translatory movement of the connecting rod may be regarded as being the same as a mass m_p of magnitude $\frac{CG}{l}$ times the connecting rod's weight (where l = length of connecting rod); having the same motion as the piston; and a rotating mass m_c of magnitude $\frac{PG}{l}$ times the connecting rod's weight at the crank pin, which can be balanced in the manner previously considered. The mass m_p is added

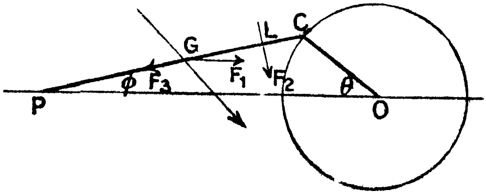


FIG. 177. INERTIA OF THE CONNECTING ROD

to the mass of the piston and the whole is treated as a reciprocating mass.

The angular oscillation of the connecting rod, and transverse couple, involves the acceleration of the rod in the angular sense, and it can be shown* that when the crank is at an angle θ the angular acceleration is given by the following expression—

$$\alpha_c = -\omega^2(C_1 \sin \theta + C_3 \sin 3\theta + C_5 \sin 5\theta + \ldots .)$$

where ω = angular velocity and C_1, C_3, C_5, etc., are constants.

If the ratio $\frac{r}{l}$ of the crank to connecting rod be denoted by q, then this expression reduces to the following form, which although not a complete solution, is sufficiently accurate for all practical purposes—

$$\alpha_c = -\omega^2(q \sin \theta - \tfrac{3}{8}q^3 \sin 3\theta + \tfrac{15}{128} q^5 \sin 5\theta -, \text{etc.} \ldots .)$$

The transverse couple causing this acceleration is obtained by substituting the value of α_c in the following well-known relation—

Torque or couple = moment of inertia $\times$ angular acceleration

$$= \frac{I}{g} \times \alpha_c$$

where I = moment of inertia of the connecting rod about an axis through its centre of gravity parallel to the crankshaft, and g = acceleration due to gravity.

* *Balancing of Engines*, A. Sharp (Longmans, Green, Ltd.).

Alternative Method. For an exact solution of the motion of the rod, it is necessary to consider the whole of the separate motions to which it is subjected.

The motion of the rod may be analysed as consisting of—

(*a*) A translation of its C.G. with the velocity and acceleration of the piston at any instant $= f$.

(*b*) An angular acceleration about P, the gudgeon pin (see Fig. 177) of $\frac{d^2\phi}{dt^2}$, where ϕ is the angle CPO.

And (*c*) an angular velocity $\left(\frac{d\phi}{dt}, \text{ at any instant}\right)$ about P.

It will be apparent that the rod will be under the influence of three separate forces acting as shown in Fig. 177, and consisting of—

(1) A force F_1 acting at the C.G. of rod, causing it to accelerate similar to the piston, the magnitude of this force being given by

$$F_1 = \frac{m_g f}{g}, \text{ where } m_g = \text{mass of rod}$$

(2) A force F_2 which acts through the centre of percussion, and which may be arrived at as follows: The angular acceleration of the rod about P may be considered as being due to a torque T such that

$$T = \frac{I}{g}\frac{d^2\phi}{dt^2}, \text{ where I = moment of inertia of rod about P}$$

$$= \frac{m_g}{g} k^2 \frac{d^2\phi}{dt^2}, \text{ where } k \text{ is the radius of gyration of rod about P}$$

The distance of the centre of percussion from P is $\frac{k^2}{PG}$ (the same as the length of the simple equivalent pendulum). Hence the force F_2 acting at L, the centre of percussion, is given by

$$F_2 \times PL = T = \frac{m_g}{g} k^2 . \frac{d^2\phi}{dt^2}$$

whence $$F_2 = \frac{m_g}{g} . PG . \frac{d^2\phi}{dt^2}$$

(3) The other remaining force acting upon the rod consists of the centrifugal force towards P, and which is given by

$$F_3 = \frac{m_g}{g} . PG . \left(\frac{d\phi}{dt}\right)^2$$

The methods for obtaining the values of angular velocities and accelerations of the connecting rod in terms of the crank angle are given on page 229.

Knowing now the magnitude, direction, and points of application of these forces acting upon the rod, the resultant force R may be obtained by the ordinary graphical (or analytical) methods.

The resolute of the resultant force along the line of stroke will give the connecting rod inertia force at any instant in this direction.

Also, by resolving R along the rod and perpendicular to it, the resultant thrust due to the inertia effect of the rod can be obtained, and *the thrust due to the piston pressure must be corrected for this inertia effect*, exactly as the piston pressure was corrected for piston inertia.

In obtaining the torque of the engine at any instant, the resultant of the piston thrust and inertia effect of the connecting rod resolved along itself is obtained, as indicated above, and multiplied by the perpendicular distance of the rod (or its prolongation) from the crank-shaft centre.

The torque curves should therefore be corrected both for piston and connecting rod inertia, in order to obtain accurate results, for these two latter effects become of great importance at high engine speeds.

The weight of the rod itself should be taken into account by treating it as an additional force to F_1, F_2, and F_3 in finding the resultant force R, if greater accuracy is required, but in relation to these three forces it is generally negligible.

Balancing the Reciprocating Parts

In the case of the reciprocating parts, of which the piston assembly is a good example, the inertia forces to which the motion of these parts gives rise will, if unbalanced, occasion rocking and vibration in the engine framing and its supports.

It has been shown that if in the case of the single-cylinder engine the obliquity of the connecting rod be neglected, then the periodical variation of velocity and direction of the moving parts give rise to inertia forces which are equal in magnitude, but opposite in direction, at the two ends of the piston stroke. If, however, the obliquity of the connecting rod be allowed for, then the inertia forces at the two ends of the stroke are unequal, and this factor is of considerable importance in connexion with the balancing of the reciprocating parts.

It will be shown later that the motion of the piston can be represented by the displacement (or component along the line of stroke) of a number of simple harmonic motions, of different amplitudes and of different periods.

The harmonics of the same period as the piston can be allowed for

and balanced, but the secondary, tertiary, and higher orders of harmonics either have to be arranged to balance themselves in different types of engine or left unbalanced. The secondary forces, due to the second harmonic of the piston's motion, may, in some cases, be of appreciable importance.

Often the secondary forces in an engine are, by a suitable choice of crank positions, made to balance themselves, and the higher harmonics reduced to negligible proportions.

The balance of a mass having a reciprocating motion can only be effected perfectly by the introduction of an equal and opposite reciprocating mass, in the proper phase and same line of motion as the original.

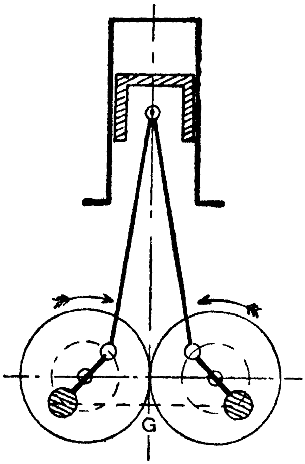


FIG. 178. METHOD OF BALANCING SINGLE-CYLINDER ENGINE

An interesting method of balancing, approximately, the reciprocating forces due to the mass of the piston, and part of the connecting rod, is illustrated in Fig. 178, and is due to Lanchester.* Taking the example illustrated of a single cylinder engine, two flywheels of equal moments of inertia were provided with balance weights placed opposite to the cranks on each flywheel, and were geared together so as to rotate in opposite directions, as indicated by the arrows.

It will be seen that the C.G. of the two balance weights has a simple harmonic motion along the centre line of stroke of the piston, but opposite in direction.

Thus the primary reciprocating forces can be balanced by this means.

A perfectly balanced engine is obtained by having the cylinders upon opposite sides of the crankshafts, as shown in Fig. 179, which represents the arrangement of the old Lanchester type of engine of 1896 to 1903.

The two opposed cylinders were arranged symmetrically upon opposite sides of the crank axes, and the connecting rods formed a symmetrical parallelogram system in all positions. The balance weights A and B were made to balance the rotating portions of the cranks, and also the reciprocating parts of the engine. It will be noticed that the whole system of pistons and connecting rods is symmetrical about the point C along the line of strokes, and that the motion of this system is strictly simple harmonic, and can be balanced by oppositely rotating weights on the crankshaft at A and B.

* *Vide* "Engine Balancing," F. W. Lanchester, *Proc. I.A.E.*, 1914.

The higher harmonics balance themselves, provided that they all lie in the same plane, and therefore do not introduce rocking couples. In this particular type of engine this was actually the case, for, as Lanchester remarks, "the engine was not troubled with any rocking moment owing to the fact that the whole of its reciprocating parts had 'looking-glass symmetry' about the transverse vertical plane."

Approximate Method for Balancing Problems

The following method, which may be employed for most practical balancing problems, takes account of the primary and secondary

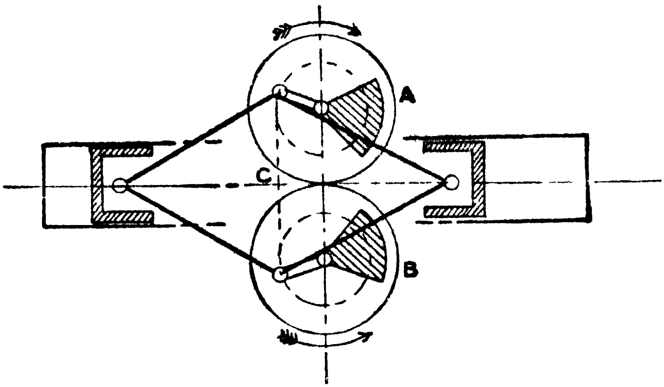


FIG. 179 LANCHESTER'S EARLY TWO-CYLINDER ENGINE

harmonics only, and is based upon a close approximation being used in place of the expression for the piston's acceleration corresponding to that given upon page 17, namely—

$$A_p = \frac{V_c^2}{r}\left\{\cos\theta + \frac{rl^2\cos 2\theta + r^3\sin^4\theta}{(l^2 - r^2\sin^2\theta)^{\frac{3}{2}}}\right\}$$

The position of the piston in its stroke is given by—

$$x = r(1 - \cos\theta) + l\left\{1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}\right\}$$

or putting $n = \dfrac{l}{r}$,

$$x = r(1 - \cos\theta) + nr\left\{1 - \sqrt{1 - \frac{\sin^2\theta}{n^2}}\right\}$$

If now, as an approximation, to the quantity under the surd sign, there be added the term $\dfrac{\sin^4\theta}{4n^4}$, it becomes a perfect square.

Thus $$\sqrt{1 - \frac{\sin^2 \theta}{n^2} + \frac{\sin^4 \theta}{4n^4}} = 1 - \frac{\sin^2 \theta}{2n^2}$$

The term added is of the second order, and is therefore very small in value, so that no appreciable error is introduced.

Then the expression for the piston's position becomes—

$$x = r(1 - \cos \theta) + nr\left\{1 - 1 + \frac{\sin^2 \theta}{2n^2}\right\} \quad . \quad . \quad (a)$$

$$= r\left\{1 - \cos \theta + \frac{\sin^2 \theta}{2n}\right\}$$

The piston velocity $$V_P = \frac{dx}{dt} = r\left[\sin \theta \frac{d\theta}{dt} + 2\frac{\sin \theta \cos \theta}{2n}\frac{d\theta}{dt}\right] \quad . \quad (b)$$

For $\frac{d\theta}{dt}$ may be written the angular velocity, namely $\frac{2\pi N}{60}$ radians per sec., where N is the crankshaft revs. per min., which are assumed to be constant.

Hence $$V_P = \frac{\pi N r}{30}\left(\sin \theta + \frac{\sin 2\theta}{2n}\right) \quad . \quad . \quad . \quad (c)$$

The piston acceleration $$A_P = \frac{dV_P}{dt} = \frac{\pi N r}{30}\left[\cos \theta + \frac{\cos 2\theta}{n}\right] \cdot \frac{d\theta}{dt}$$

$$= \left(\frac{\pi N}{30}\right)^2 . r\left[\cos \theta + \frac{\cos 2\theta}{n}\right] \quad . \quad . \quad . \quad (d)$$

The inertia force at any angle θ is then given by—

$$F_\theta = \frac{M}{g} . \left(\frac{\pi N}{30}\right)^2 . r\left[\cos \theta + \frac{\cos 2\theta}{n}\right] \quad . \quad . \quad (e)$$

These approximate expressions for the position, velocity, acceleration, and inertia of the piston or reciprocating parts may be used in place of the exact expressions given later.

The Single-cylinder Engine

The expression (e) given above can be directly applied to the case of a single cylinder engine, and may be regarded as being made up of two parts, namely—

$$\frac{M}{g}\left(\frac{\pi N}{30}\right)^2 . r \cos \theta, \text{ and } \frac{M}{g}\left(\frac{2\pi N}{30}\right)^2 . \frac{r}{n} . \cos 2\theta$$

The former reciprocating force is the primary force, which follows a complete cosine* curve of variation per engine revolution.

* That is, a sine curve displaced $\frac{\pi}{2}$ in phase.

The latter reciprocating force is a secondary or octave force, which has a cosine* curve of variation of *twice* the frequency of the primary; the maximum values and the variations of the secondary forces are smaller than in the case of the primary forces, however.

Fig. 180 shows the relative phases and magnitudes of these forces for one complete revolution of the crankshaft.

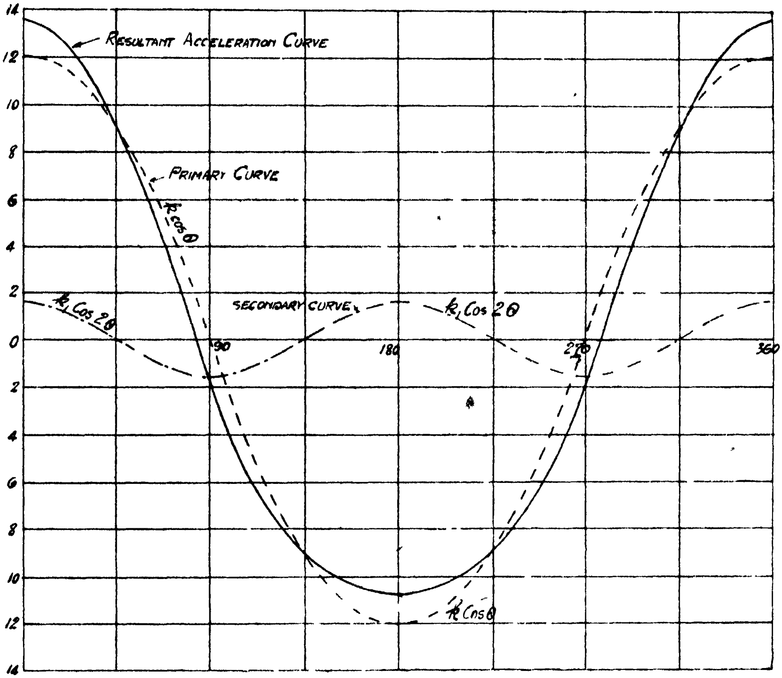


FIG. 180. HARMONIC CURVES FOR SINGLE-CYLINDER ENGINE

Consider the case of an engine of 4-in. bore by 5-in. stroke with a connecting rod-to-crank ratio of 4. The piston and reciprocating parts are assumed to weigh 4·5 lb.

For an engine speed of 2000 r.p.m., the maximum value of the primary force will be given when $\theta = 0$, so that $\cos\theta = 1$.

Its value will then be: $\dfrac{4{\cdot}5}{32{\cdot}19}\left(\dfrac{\pi\,.\,2000}{30}\right)^2 \cdot \dfrac{5}{2 \times 12}$ lb.

that is, 1285 lb.

* That is, a sine curve displaced $\frac{\pi}{2}$ in phase.

The maximum value of the secondary force is given by—

$$\frac{1}{n} \times \text{primary force} = \frac{1285}{4} = 321{\cdot}2 \text{ lb.}$$

Expressed in terms of piston area, the above values are as follows—

Maximum primary force = 102·2 lb. per sq. in.
,, secondary force = 25·6 ,, ,,

These forces, in the ordinary way, are unbalanced ones and give rise to vibrations. As will be shown later, the primary force may be partially balanced, but it is not usually practicable to balance the secondary force; in any case, the latter is comparatively small in value.

The Four-cylinder Vertical Engine

The approximate method previously outlined affords a convenient one for examining the balance of the ordinary four-cylinder vertical engine, similar to that shown in Fig. 205.

Here the two outside pistons work in the same plane, the cranks being in the same positions, whilst the two inner pistons move oppositely to the outer ones; the centre lines of the two sets of pistons coincide.

Considering the inertia forces due to the two outer piston assemblies, then, using the same notation as before—

$$\left.\begin{array}{l}\text{Total reciprocating force due}\\ \quad\text{to outer pistons, etc.}\end{array}\right\} = 2\left[\frac{M}{g}\left(\frac{\pi N}{30}\right)^2 r \,.\left\{\cos\theta + \frac{\cos 2\theta}{n}\right\}\right]$$

For the two inner pistons, the crank angle becomes $\theta + 180°$, so that—

$$\begin{array}{l}\text{Total reciprocating}\\ \quad\text{force due to inner}\\ \quad\text{pistons, etc.}\end{array} = \frac{2M}{g}\left(\frac{\pi N}{30}\right)^2 . r\left[\cos(180° + \theta) + \frac{\cos 2(180° + \theta)}{n}\right]$$

$$= \frac{2M}{g}\left(\frac{\pi N}{30}\right)^2 r\left[-\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Since the plane of the forces is the same, they may be added, algebraically; the resultant force acting vertically can be written as—

$$F_R = 4 \,.\, \frac{M}{g}\left(\frac{2\pi N}{30}\right)^2 \frac{r\cos 2\theta}{4n}$$

This result shows that the primary forces are balanced, but that

there is an unbalanced secondary force of the above magnitude of frequency equal to *twice* the engine speed

The same result might have been obtained from the inertia diagrams for the four cylinders by algebraically adding the ordinates at corresponding crank angles. The resultant diagram obtained would be a curve similar to that shown in Fig. 180 by dotted $k_1 \cos 2\theta$ line; it will be seen that it has twice the frequency of the primary forces (i.e. the engine speed).

The maximum values of the unbalanced secondary forces occur when $\cos 2\theta = +1$ or -1, that is, at 0°, 90°, 180°, 270°, 360°, etc., and the values are given by—

$$F_{max} = \frac{Mr}{ng}\left(\frac{2\pi N}{30}\right)^2$$

For a four-cylinder engine having cylinders of the same dimensions as the single-cylinder engine example considered on page 235, the maximum total unbalanced secondary force will be 1285 lb., or 102·2 lb per sq. in. of single piston area.

The Six-cylinder Vertical Type Engine

The cranks in this engine are arranged at 120° to each other, with the two centre cranks in the same plane, as shown in Fig. 211.

Considering the three pairs of cranks in the same plane, these reciprocating masses of cylinders (Nos. 1 and 6) can be regarded as being on a single crank, whilst those of Nos. 2 and 5, and of 3 and 4 respectively, are also on single cranks

Employing the same notation as before, the primary forces for the three pairs are given by—

$$F_1 = 2k \,.\, r \cos \theta$$

$$F_2 = 2kr \cos (\theta + 120^\circ)$$

$$F_3 = 2kr \cos (\theta + 240^\circ)$$

Now $\quad \cos (\theta + 120^\circ) = \cos \theta \,.\, \cos 120^\circ - \sin \theta \,.\, \sin 120^\circ$

$$= -\tfrac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$$

And $\quad \cos (\theta + 240^\circ) = \cos \theta \,.\, \cos 240^\circ - \sin \theta \,.\, \sin 240^\circ$

$$= -\tfrac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta$$

The primary forces may be added, since the cylinders are all in the same line.

$$\text{Hence } F_1 + F_2 + F_3 = 2kr\left[\cos\theta - \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] = 0$$

so that the primary forces are in perfect balance.

The secondary forces are given by—

$$f_1 = \frac{2kr}{n} \cdot \cos 2\theta$$

$$f_2 = \frac{2kr}{n} \cos 2(\theta + 120°)$$

$$f_3 = \frac{2kr}{n} \cos 2(\theta + 240°)$$

These may be added in the same way as before and, if this is done, the result will be found to be zero, so that the secondary forces are also in perfect balance.

It can be readily shown that the twelve-cylinder Vee-type engine is also in perfect balance, for the primary and secondary forces, since it consists of two systems of six-cylinder engines.

The Eight-cylinder V-type Engine

One common eight-cylinder arrangement consists of two sets of ordinary four-cylinder engines placed at right angles to each other, with a common crankshaft. As in the case of the four-cylinder engine, the primary reciprocating forces balance themselves in each four-cylinder set.

Using the same notation as before, and the arrangement shown in Fig. 213, there is then left an unbalanced secondary force of amount* equal to—

$$4k \cdot r \cdot \frac{\cos 2\theta}{n}$$

where $k = \frac{M}{g}\left(\frac{\pi N}{30}\right)^2$ acting in the plane of the left-hand cylinder block, and for the right-hand cylinder block there is an unbalanced secondary force of the following value, namely—

$$4k \cdot r \cos 2(\theta + 90°) = -4k \cdot r \cos 2\theta$$

* This force always acts in the plane of reciprocation, i.e. the plane of the cylinder block considered.

These two unbalanced forces do not act along the same line; the left-hand force acts downwards along the line of the cylinder axis, whilst the right-hand force acts upwards along the line of its cylinder axis. The two forces are, therefore, mutually at right angles, and by the parallelogram of forces are equivalent to a single resultant of amount equal to—

$$2\left[4kr \cdot \frac{\cos 2\theta}{n}\right] \cdot \cos 45^\circ$$

$$= \frac{4\sqrt{2}kr \cdot \cos 2\theta}{n} = \frac{5{\cdot}656kr \cdot \cos 2\theta}{n}$$

which acts in a horizontal plane (i.e. at 45° to the cylinder block axial planes), and has a frequency equal to twice engine speed. Its maximum

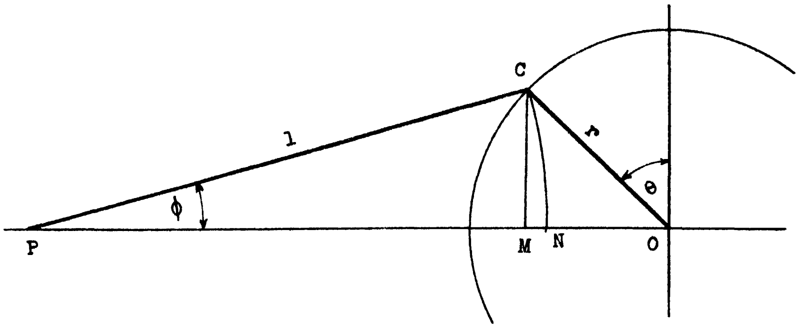


FIG. 181

value is 1·414 times the unbalanced secondary force of a four-cylinder engine of the same dimensions.

The General Case of Engine Balance

It is proposed to consider the general case for the motion of the piston in the case of the single-cylinder engine, and for this purpose both analytical and graphical methods will be employed. The results of a detailed study of the above primary example will then be used in considering the balance of types of engine.

It will be necessary to consider the expressions for the displacement, velocity, and acceleration of the piston somewhat more fully than in the earlier portion of this chapter. Let OC be the crank arm of length r, PC the connecting rod of length l (Fig. 181).

Assume $\frac{r}{l} = m = \frac{1}{n}$.

Let x = ON = piston displacement from equivalent stroke centre O.

Then
$$\text{ON} = \text{OP} - \text{PN}$$
$$= \text{OM} + \text{MP} - \text{PN}$$
where $\text{PN} = \text{PC} = l.$
$$= r \sin \theta + l \cos \phi - l$$

But
$$\frac{\sin \phi}{\cos \theta} = \frac{r}{l} = m$$

and
$$\cos \phi = \sqrt{1 - m^2 \cos^2 \theta}$$

Hence
$$x = r \sin \theta + l\sqrt{1 - m^2 \cos^2 \theta} - l$$

or
$$x = r \sin \theta + l \left\{ - \frac{1}{2} m^2 \cos^2 \theta - \frac{1}{8} m^4 \cos^4 \theta - \frac{1}{16} m^6 \cos^6 \theta - \text{etc.} \right\}$$
$$= r \left\{ \sin \theta - \frac{1}{2} m \cos^2 \theta - \frac{1}{8} m^3 \cos^4 \theta - \frac{1}{16} m^5 \cos^6 \theta - \frac{5}{128} m^7 \cos^8 \theta - \text{etc.} \right\}$$

Now, for any index n,
$$2^{n-1} \cos^n \theta^* = \cos n\theta + n \cos (n - 2)\theta + \frac{n(n-1)}{1 \cdot 2} \cos (n - 4)\theta + \text{etc.}$$

And substituting in the general expression definite values for n

$$\cos^2 \theta = \frac{1}{2} (\cos 2\theta + 1)$$

$$\cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3)$$

$$\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$$

$$\cos^8 \theta = \text{etc.}$$

Hence, we may express the displacement of the piston by a "Fourier Series," by substituting the multiple angle cosine terms for the powers of the cosine terms obtained.

The most convenient method is to write the displacement as

$$x = r\{\sin \theta - (k + \rho_2 \cos 2\theta + \rho_4 \cos 4\theta + \rho_6 \cos 6\theta + \text{etc.})\}$$

* When n is even, the last term is $\dfrac{n(n-1)(n-2) \ldots \ldots (\frac{1}{2}n+1)}{2 \,\underline{|\tfrac{1}{2}n}}$; note that this is one-half the apparent value given above.

Then the values of the constants are obtained by substituting for the cosine power terms, and are—

$$k = \frac{1}{4} m + \frac{3}{64} m^3 + \frac{5}{256} m^5 + \frac{175}{(121)^2} m^7$$

$$\rho_2 = \frac{1}{4} m + \frac{1}{16} m^3 + \frac{15}{512} m^5 + \frac{35}{2048} m^7$$

$$\rho_4 = \frac{1}{64} m^3 + \frac{3}{256} m^5 + \frac{35}{642} m^7$$

$$\rho_6 = \frac{1}{512} m^5 + \frac{25}{2048} m^7$$

$$\rho_8 = \frac{5}{(128)^2} m^7$$

$$\rho_{10} = \text{etc.}$$

In order to afford some idea as to the values of these constants for certain cases occurring in practice the following table has been prepared—

TABLE X

TABLE OF FOURIER CONSTANTS

$\frac{\text{Crank}}{\text{Connecting Rod}} = m$	$\frac{1}{4}$	$\frac{1}{4\frac{1}{2}}$	$\frac{1}{5}$
k	0·0632	0 0561	0 0504
ρ_2	0·0635	0 0563	0 0505
ρ_4	0·000261	0 000177	0·000140
ρ_6	0·00000205	0 00000112	0·0000065
ρ_8	0 00000000	—	—

It will be found that the analysis of the piston displacement gives very accurate results if taken to four terms only, as the higher harmonics die away very rapidly after the fourth term.

Hence we can express the piston's displacement as

$$x = r\left\{k + \sin\theta + \rho_2 \sin\left(2\theta - \frac{\pi}{2}\right) + \rho_4 \sin\left(4\theta - \frac{\pi}{2}\right) + \text{etc.}\right\}$$

It will be noticed that after the first term only the *even harmonics* are present.

From the above expression it will be seen that the displacement of

the piston may be regarded as being due to the component displacements upon the line of stroke of—

(*a*) a constant displacement rk
(*b*) a crank of radius r, moving at crank velocity

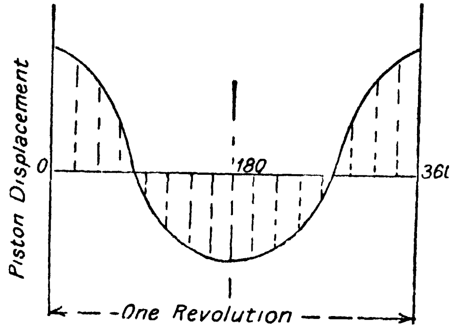


FIG. 182

(*c*) a crank of radius $r\rho_2$, moving at twice crank velocity and out of phase by 90°
(*d*) a crank of radius $r\rho_4$, moving at four times crank velocity and out of phase by 90°

and so on.

The total displacement x may be expressed by means of a number of sine curves of different amplitudes and periods, somewhat as illus-

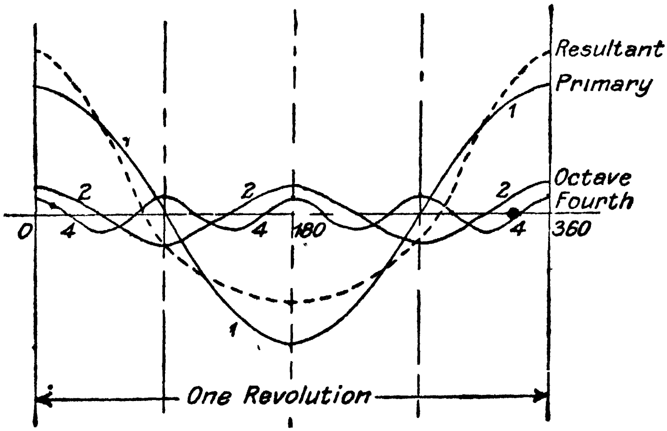


FIG. 183. ANALYSIS OF PISTON'S MOTION

trated in Fig. 183; here the dotted line shows the resultant piston displacement curve.

Piston Acceleration

By differentiating the expression for the piston displacement twice, with respect to the time, we obtain the acceleration ; thus we have—

$$\frac{d^2x}{dt^2} = \omega^2 r \left\{ \sin(\theta + \pi) + 4\rho_2 \sin\left(2\theta + \frac{\pi}{2}\right) + 16\rho_4 \sin\left(4\theta + \frac{\pi}{2}\right) + 36\rho_6 \sin\left(6\theta + \frac{\pi}{2}\right) + \ldots \right\}$$

The harmonics here become of greater importance as they are magnified, 4, 16, 36, etc., times respectively, and these harmonics become important in engine balancing problems, since the inertia forces are proportional to the above acceleration.

Corresponding to the table previously given, a similar one has been prepared for the values of the acceleration amplitudes.

TABLE XI

TABLE OF ACCELERATION AMPLITUDES

$\frac{\text{Connecting Rod}}{\text{Crank}}$ Ratio	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5
$4\rho_2$	0·2918	0·254	0·225	0·202
$16\rho_4$	0·0062	0·0041	0·0028	0·0021
$36\rho_6$	0·0001	0·000074	0·000040	0·000023

The results of the preceding investigation can now be applied to obtain the piston acceleration effect, in which the obliquity of the connecting rod is taken into account.

The acceleration may be considered to be due to the displacement of—

(*a*) a crank of radius $\omega^2 r$, rotating at crank velocity, and out of phase by $+\pi$

(*b*) a crank of radius $4\omega^2 r\rho_2$, rotating at twice crank velocity, and out of phase by $+\frac{\pi}{2}$

(*c*) a crank of radius $16\omega^2 r\rho_4$, rotating at four times crank velocity. and out of phase by $+\frac{\pi}{2}$

(*d*) a crank of radius $36\omega^2 r\rho_6$, rotating at six times crank velocity, and out of phase by $+\frac{\pi}{2}$

etc.

This is represented graphically (Fig. 184) by a vector O1, proportional in magnitude to $\omega^2 r$ and 180° in advance of the main crank,

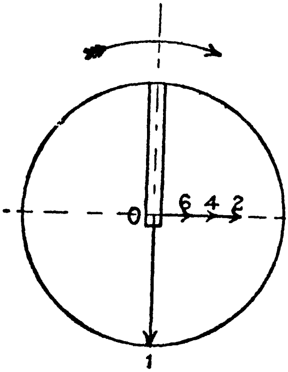


Fig. 184

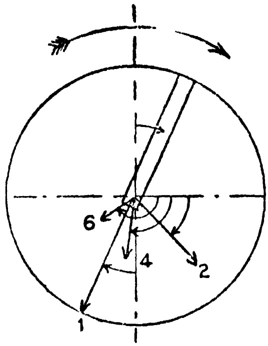


Fig. 185

and vectors O2, O4, O6, etc., proportional to $4\omega^2 r\rho_2$, $16\omega^2 r\rho_4$, $36\omega^2 r\rho_6$, etc., respectively, and rotating at twice, four, six, etc., times the crank velocity.

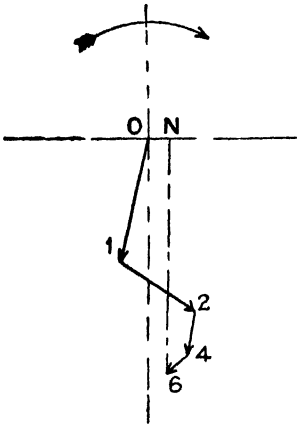


Fig. 186

For any position θ of the main crank, the first, second, third, etc., vectors go through angles θ, 2θ, 4θ, etc., respectively (Fig. 185), and the resultant acceleration may be determined graphically by finding the displacement ON due to separate displacements, that is, by algebraically summing the respective ordinates or projections upon the base line (Fig. 186).

Applications to Balancing Problems

The expression for the acceleration force in the case of a single-cylinder engine is—

$$F = M\omega^2 r \left\{ \sin(\theta + \pi) + 4\rho_2 \sin\left(2\theta + \frac{\pi}{2}\right) + 16\rho_4 \sin\left(4\theta + \frac{\pi}{2}\right) + 36\rho_6 \sin\left(6\theta + \frac{\pi}{2}\right) + \cdots \right\}$$

where M = mass of reciprocating parts.

If the $\dfrac{\text{connecting rod}}{\text{crank}}$ ratio be denoted by n, then by substituting

$$\rho_2 = \frac{1}{4n},\ \rho_4 = \frac{1}{64n^3},\ \rho_6 = \frac{1}{512n^5},\ \text{etc.}$$

we have

$$F = M\omega^2 r\left\{\sin(\theta + \pi) + 4\cdot\frac{1}{4n}\cdot\sin\left(2\theta + \frac{\pi}{2}\right) + 16\cdot\frac{1}{64n^3}\sin\left(4\theta + \frac{\pi}{2}\right)\right.$$
$$\left. + 36\cdot\frac{1}{512n^5}\sin\left(6\theta + \frac{\pi}{2}\right) + \text{etc.}\right\}$$

The first harmonic may be represented by a mass of $\dfrac{M}{4n}$ at the crank radius r rotating with velocity 2ω for the accelerating force $= \dfrac{M}{4n}(2\omega)^2 r = \dfrac{M\omega^2 r}{n}$. Similarly, for the second harmonic we have an equivalent mass $\dfrac{M}{64n^3}$ at crank radius, and so on.

Single-cylinder Engine

In the case of the single-cylinder, in so far as the crank and crank pin are concerned, these may be balanced by means of suitable counterbalance weights upon the opposite side of the shaft.

In the balancing of the connecting rod as before stated, it is usual to consider the big end of the rod as a rotating part, and to balance it by an additional counterbalance weight; the small end of the rod is treated as a reciprocating part, and a corresponding addition is made to the weight of the piston and its component parts.*

For a single cylinder, the balance of the reciprocating parts in respect to the initial simple harmonic component is usually effected by introducing a counterbalance weight equivalent to a portion of this mass, *not* to the whole of the reciprocating mass.

This is an important point, for if a rotating balance weight equal to that of the reciprocating parts be employed to effect a balance for the latter, then resultant unbalanced forces will be introduced having a reciprocation at right angles to the line of stroke, and equal in magnitude to the original unbalanced forces.

This will be evident from Fig. 187, from which it will be seen that whilst the resolute along the line of stroke balances the initial harmonic movement of the reciprocating parts, yet the perpendicular resolute is unbalanced.

In some cases, the construction of the engine is such that it is better

* See p. 228.

able to resist vibration forces in the direction perpendicular to the line of stroke; but, generally speaking, a compromise has to be adopted between wholly balancing the reciprocating parts by an equal rotating

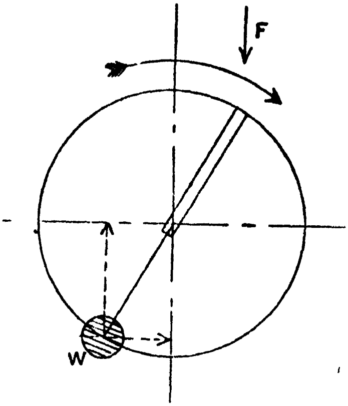


FIG 187

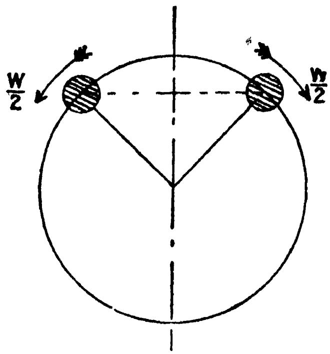


FIG. 188

mass on the opposite side of the crankshaft and not balancing them at all.

The usual compromise is to balance half of the reciprocating weight by a rotating counterbalance weight, in which case the unbalanced force is that due to a mass equal to the added balance weight rotating in the direction opposite to the engine's motion.

A proof of this statement may be obtained by assuming the primary harmonic motion as equivalent to two masses of half the reciprocating weight revolving at crankshaft velocity, one in the direction of, and the other in a reverse direction to, the main crank, as illustrated in Fig. 188.

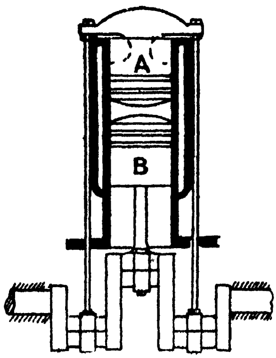


FIG. 189. THE GOBRON-BRILLE ENGINE

The mass moving in the direction of the main crank can be balanced by an equal and opposite counterbalance weight, and we are left with an unbalanced effect due to the mass rotating in a reverse direction.

The degree of compromise in attempting to obtain the balance of the reciprocating masses is entirely a question of the conditions of mounting, usage, etc., of the engine.

An interesting arrangement for balancing the reciprocating forces in a single-cylinder engine is that of the Gobron-Brille, once used in both car and aircraft engines (Fig. 189). There are two pistons

A and B working in the same cylinder, but in opposite directions, the compression and combustion space being between the two. The top piston is arranged to work by means of an overhead crosspiece and long side connecting rods, on to a pair of cranks symmetrically disposed in relation with and at 180° to the main crank.

This is an example of balancing a reciprocating mass by a similar one, moving in an opposite direction.

The engine, necessarily, occupies a greater vertical height than

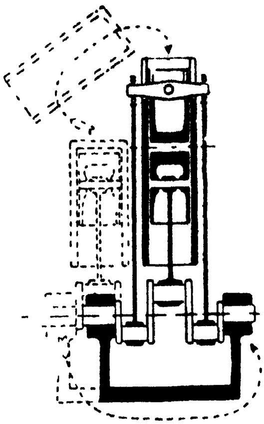


Fig. 190. Earlier Type of Junker's Engine

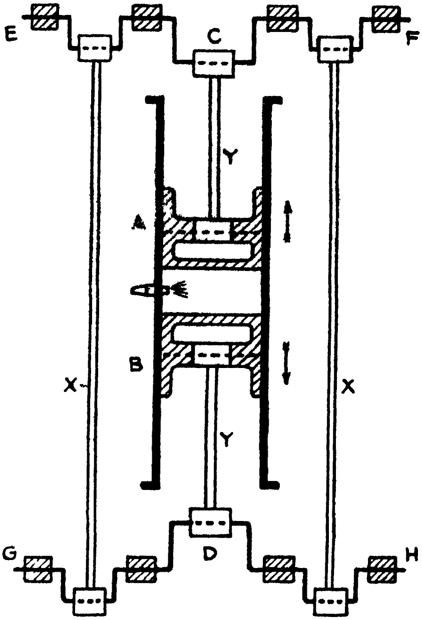


Fig. 191. Later Model Junker's Engine with Balanced Piston and Rod Motions

normal types, but otherwise it possesses advantages over other types, in the matter of balance.

In the case of the Junkers opposed-piston Diesel engines, the reciprocating forces on the pistons are in balance, but certain unbalanced forces due to the coupling rods between the crosshead used in the earlier design of engine (Fig. 190) and the crankshaft were introduced. In a later design there are two identical crankshafts, one above and the other below, as shown diagrammatically in Fig. 191. Thus, the two pistons A and B, when in their nearest positions, were separated by a compression space forming the combustion chamber. Each piston was connected to its crankshaft at C and D by its own connecting rod

Y. The upper crankshaft ECF was connected by long connecting rods X to the lower crankshaft GDH, so that both crankshafts rotated at the same speed and in the same direction.

In another alternative arrangement (Fig. 192), the upper and lower crankshafts are connected by helical gears, and the second gear from the top of the engine drives the propeller. It is thus possible to

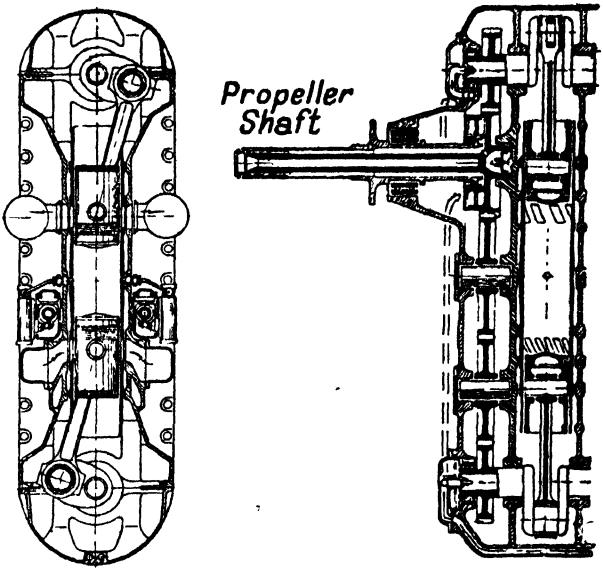


FIG. 192. GEARED DRIVE MODEL JUNKER'S AIRCRAFT ENGINE

obtain proper dynamic balance of the engine, with its single cylinder double piston units.

Other variations of the two-piston single-cylinder type engine have been made in the past, the main object in each case being to balance the inertia forces of the two pistons, by arranging for these to move symmetrically in opposite directions at all times.

Two-cylinder Engine, Cranks at 180°

The line diagram shown in Fig. 193 illustrates the arrangement of this type.

Dealing with the fundamental motion effect first, there will be seen to be two forces of magnitude $M\omega^2 r$ acting in the directions (1) shown. These constitute a disturbing couple of magnitude $\frac{M\omega^2 r \cdot b}{g}$, with its axis at C, the direction of the latter being normal to the crankshaft axis.

There will, therefore, be no resultant vertical force, but a resultant couple tending to act about an axis perpendicular to the line of stroke and centre of crank axis, the magnitude of the couple at any instant about this axis being the resolved part of the main rotating couple in this direction.

The maximum value of the resolved couple $\frac{M\omega^2 r \,.\, b}{g}$.

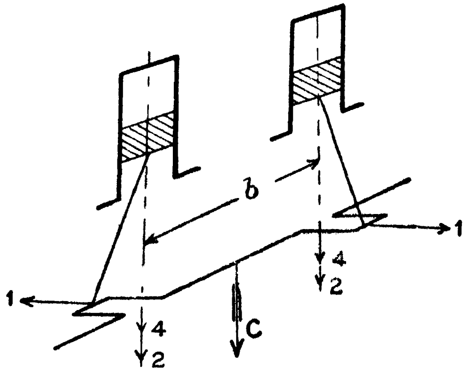


FIG. 193. THE TWO-CYLINDER ENGINE (CRANKS AT 180°)

The axis of this couple is always perpendicular to the plane of the cylinders.

Effect of the Harmonics

The octave harmonic effect is illustrated in the same diagram in Fig. 193.

This harmonic is such that the mass $\frac{M}{4n}$ at equivalent crank radius for the two cranks will have moved through twice the crank angle (being of double the fundamental period) and will thus be as shown at (2), (4) in the diagram, in their relative positions.

These harmonics lag by $\frac{\pi}{2}$ behind the fundamental, which for convenience is taken in phase with the main cranks.

The net effect of the octave components is equivalent to two forces $\frac{M\omega^2 r}{ng}$ acting at right angles to the respective cranks in the same direction, and revolving at twice crankshaft speed, giving rise to a hammering action of twice the fundamental frequency.

Similarly, the effect of the next, or 4θ component, can be studied and will be seen to be equivalent to two forces at right angles to the

respective cranks, in the same direction, and revolving at four times engine speed. These harmonics also give rise to a hammering action, but of four times the fundamental frequency.

Two-cylinder Engine, Cranks at 90°

In this type of engine, which is illustrated diagrammatically in Fig. 194, the fundamental component of the pistons' motion is represented by forces $\frac{M\omega^2 r}{g}$ acting in the directions shown.

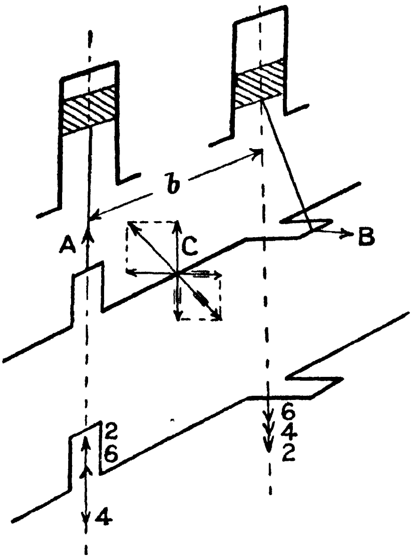


FIG. 194. THE TWO-CYLINDER VERTICAL ENGINE (CRANKS AT 90°)

If a point C midway between the centre lines of engine cylinders be chosen, the force at A may be replaced by an equal force at C, and a couple of transference $\frac{M\omega^2 r}{g} \cdot \frac{b}{2}$.

Similarly for the replacement of the force acting at B.

Then we have two equal couples, whose axes are represented in direction by the barred lines, of magnitudes $\frac{M\omega^2 r}{g} \cdot \frac{b}{2}$ and two forces of magnitude $M\omega^2 r$ acting parallel to the original forces upon the cranks at A and B.

The two couples may be replaced by a couple of magnitude $\frac{M\omega^2 r . b\sqrt{2}}{2g}$

acting at 45° with the crank arms whose axis is also perpendicular to the resultant force $\frac{M\omega^2 r\sqrt{2}}{g}$ of the two separate forces acting at C.

The resultant force due to the combined inertia effects of the two pistons, and their parts, will be the resolute of the above force in the directions of the cylinder axes and will be a maximum when the cranks are symmetrical with the plane of the lines of stroke, and its value will then be $\frac{M\omega^2 r\sqrt{2}}{g}$. At any other moment its value is $\frac{M\omega^2 r\sqrt{2}\cos(\theta - 45°)}{g}$, where θ is the angle made by the leading crank with line of stroke.

Similarly, the resolved part of the resultant couple in a plane perpendicular to the resultant force will give the resultant inertia couple effect due to the unbalanced reciprocating parts.

The unbalanced force gives rise to vertical vibrations of a frequency equal to the piston's frequency, whilst the unbalanced couple effect will tend to rock the engine about a horizontal axis, from side to side, the frequency of the angular oscillation being the same as that of the piston.

The maximum value of the unbalanced couple is $\frac{M\omega^2 rb\sqrt{2}}{2g}$, and occurs when the resolved part of the resultant force in the cylinder axes direction is a minimum, or when the leading crank has gone through 135° from its inner dead centre.

The Harmonics

The position of the second, fourth, and sixth harmonics is shown in the lower part of the figure, from which it will be seen that the octave or secondary harmonics are equivalent to a couple of magnitude $\frac{M\omega^2 rb}{n}$ rotating at twice engine speed, the resolved part of which in the plane perpendicular to cylinder axes will represent the resultant effect.

This couple sometimes enhances and sometimes detracts from the effect of the primary couple.

Thus these two couples may be compounded into a single resultant couple rotating at engine speed, the resolute of which in the perpendicular plane to cylinder axes will give the resultant couple effect.

A good exercise for the student would be to find the resultant couple due to the primary and octave harmonics, and to plot the magnitude

of this resultant as a radius vector, the vectorial angle representing the direction of the axis of couple (not the crank angle).

The fourth harmonics give rise to a hammering action of four times the fundamental frequency, and of maximum magnitude $\frac{M\omega^2 r}{4n^3 g}$. The sixth harmonics will give rise to a couple rotating at six times crankshaft speed, and of maximum value $\frac{36M\omega^2 r}{512n^5 g} \cdot b$, which will sometimes

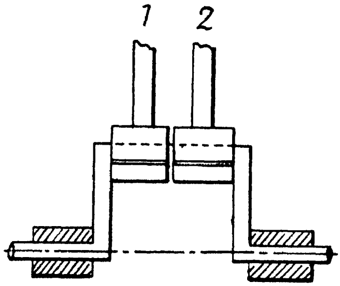


FIG 195

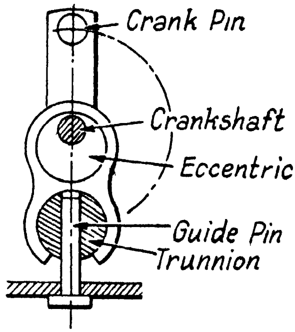


FIG. 196. METHOD OF BALANCING RECIPROCATING FORCES IN SINGLE- OR TWO-CYLINDER ENGINE

oppose and sometimes enhance the resultant of the primary and octave couples.

The octave couple may become very appreciable for small connecting rod crank ratios; this is a point that is occasionally overlooked. Thus, for $n = 4$, its maximum value is $\frac{M\omega^2 r \cdot b}{4g}$, whilst that of the primary is $\frac{M\omega^2 rb}{1{\cdot}414g}$, so that the octave effect is about half that of the fundamental couple.

A Method of Balancing a Two-cylinder Vertical Engine

The two-cylinder vertical engine, with a single crank upon which both connecting rods operate, so that the pistons move up and down together (Fig. 195), possesses the advantage over a single-cylinder engine of equal capacity of giving twice the number of firing strokes, and these occur at equal intervals, so that the value of the maximum torque is reduced and a smoother performance results. The engine balance, however, is no better than that of the single-cylinder engine.

In order to overcome this drawback a method of balancing the reciprocating forces, used by H. Ricardo on a petrol engine, was to

provide on either side of the crank webs an eccentric secured to the crankshaft with its "throw" arranged in the opposite direction from that of the crank, as depicted in Fig. 196. The eccentrics engaged with weights of suitable magnitude, so that as the main crank moved down, these two weights moved up, and *vice versa*. Thus, the reciprocating force due to the piston could be balanced by the pair of weights at all times. In order to allow for the sideways tilting of the balance weights as they were driven by their eccentrics an internal rocking

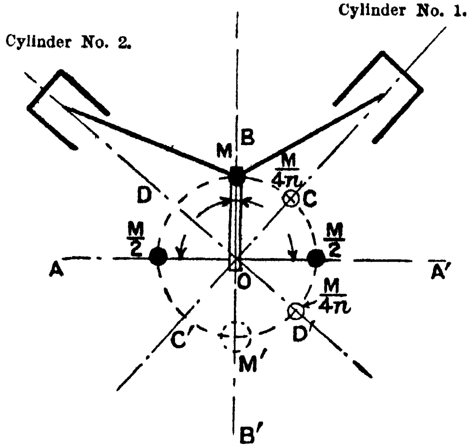


FIG. 197. THE TWO-CYLINDER 90° ENGINE

trunnion working on a pin fixed to the crankcase was provided in each case. In this arrangement the ratio of the length of the link of the balance weight to its vertical movement is equal to the connecting rod to crank ratio.

Two-cylinder V Engine, Axes at 90°

In this arrangement the two pistons drive one crank, as shown in Fig. 197.

The relation of the inertia forces in this type can best be studied by the principle of reverse cranks, in which a simple harmonic reciprocating motion can be represented by two equal rotating masses, of one-half the reciprocating weight, rotating in opposite directions as indicated in Fig. 197. The C.G. of these weights oscillates with a S.H.M. along the line of piston stroke and in the same phase as the piston, and exactly represents the piston's motion and inertia forces, for the fundamental harmonic.

For the secondary or octave components, the equivalent masses at crank radii will rotate (in proper phase relative to the fundamental

harmonic) at twice crankshaft speed, and so on for the higher harmonics.

Referring to Fig. 197, it will be evident that the primary reciprocating motion may be represented by a mass $\frac{M}{2}$ at A and one $\frac{M}{2}$ at B for each cylinder, revolving in opposite directions. Now the two masses $\frac{M}{2}$ at AA_1 balance, whilst the two masses $\frac{M}{2}$ may be regarded as a total mass M always at the crank pin B.

This may be balanced by a mass M′ at a radius R′ upon the opposite side of the crankshaft, such that

$$M'R' = MR$$

The Secondary Harmonics

A little consideration will show that when the crank is in the direction OC, the direct and reverse cranks for cylinder 1 will be at C, whilst those for cylinder 2, having rotated through twice the crank angle, will both be at D′, and hence the octave forces will not be in balance, in fact the C.G. of the two secondary forces oscillates in a S.H.M. along AA′. This can be easily understood if the positions of the direct and reverse cranks be taken for several main crank angles. The effect of these unbalanced octaves is to give to the engine a horizontal vibration of twice the frequency of the main harmonic.

The maximum value of the unbalanced secondary force will be the resultant of the two separate maximum forces $\frac{M}{ng} \cdot \omega^2 R$ each at 90° to the other, that is, a resultant maximum force of $\frac{M}{ng} \omega^2 R \cdot \sqrt{2}$ acting along AA′.

Two-cylinder 180° Opposed Type

This type of engine, which is illustrated in Fig 198, has the cylinders placed upon opposite sides of the crankshaft, and the cranks are at 180° with each other The firing strokes are evenly spaced, thus yielding a torque of regular character.

By arranging the cylinders so that the two lines of stroke coincide, which can be accomplished in practice by employing a pair of connecting rods for one cylinder symmetrically placed upon either side of the other, then the fundamental and octave motions, etc., of the two cylinders balance each other, and no rocking moments occur.

The balance is, in fact, as nearly perfect as it is possible to get in any type of engine. The only unbalanced factors that occur are those due to the variation in torque owing to piston inertia and to the explosion impulses. With this type of engine, owing to its excellent

balance, petrol-engine speeds of over 8000 r.p.m. have been attained at the time of writing, although the actual working speeds are appreciably lower.

It is often impossible to arrange the cylinders co-axially for manufacturing reasons, so that the axis of each cylinder is situated opposite to its own crank, the two axles being out of line, as shown in Fig. 199.

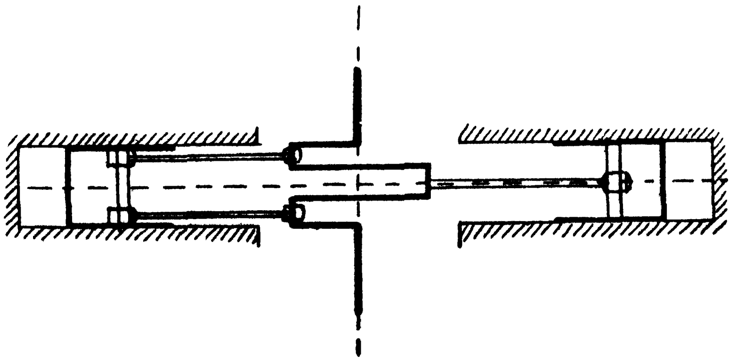


FIG. 198. THE 180° TWO-CYLINDER OPPOSED ENGINE, CO-AXIAL TYPE

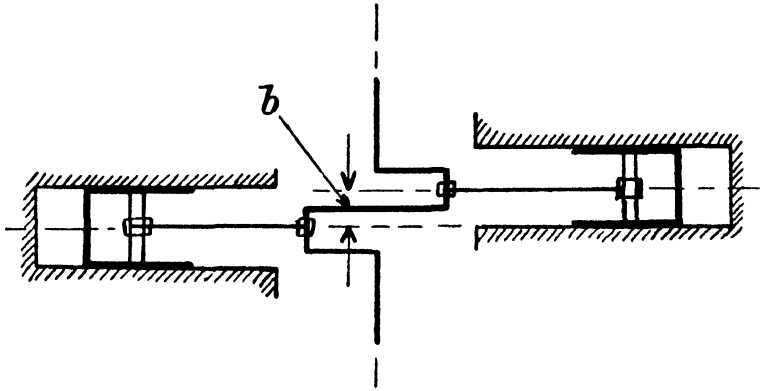


FIG. 199. THE 180° TWO-CYLINDER OPPOSED ENGINE, OFFSET TYPE

In this case there will exist rocking couples of moment equal to the primary force multiplied by the axial separation, i.e. $\dfrac{M\omega^2 r \,.\, b}{g}$, and similarly for the secondary and higher harmonics.

Four-cylinder Opposed Engine

In this favoured arrangement for aircraft engines used on light planes, perfect balance of the primary inertia forces can be obtained

by the use of a four-throw crankshaft, as shown in Fig. 200. In this manner the out-of-balance rocking couple (Fig. 199) is exactly balanced by that due to the other pair of opposed cylinders. The secondary inertia forces can be shown to be in balance, but there is an unbalanced secondary couple.*

Other arrangements of opposed cylinder engines with four, eight, and twelve cylinders are dealt with in the table mentioned in the footnote.

Three-cylinder Engine, Cylinders at 120°

This type of engine has been employed in motor-car practice, and was once used

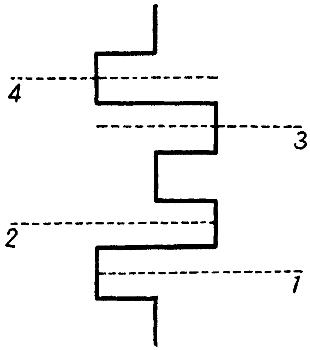


FIG. 200

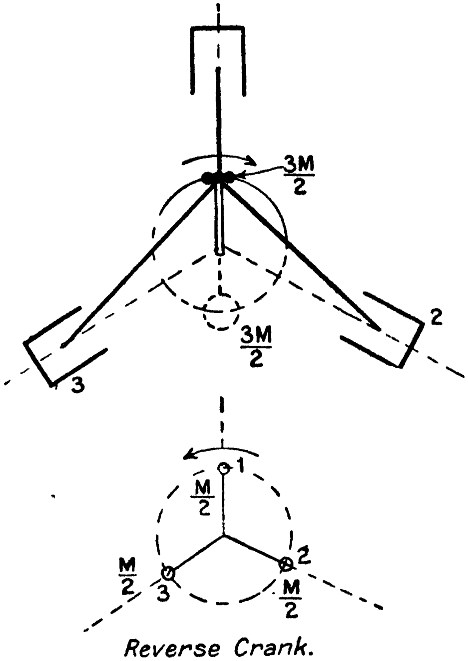


FIG. 201. THE THREE-CYLINDER RADIAL ENGINE

in the case of low-powered aeroplane engines, such as the 35 h.p. Anzani **Y**-type.

The principle of reverse cranks here provides a simple means of examining the engine balance.

Referring to the diagram given in Fig. 201, it will be seen that the three fundamental direct cranks will all be in the top vertical position, whilst the three reverse cranks balance each other.

Hence, the fundamental harmonic forces may be balanced by a mass $\frac{3M}{2}$ at an equal radius, but in a direction opposite to the main crank, as shown by the dotted circle.

* *Vide* Table XIII on page 280.

The Secondary Harmonics

Referring to Fig. 202, it will be seen that the three direct octave cranks will be in positions OA, OB, and OC respectively, and will therefore balance each other, whilst the reverse cranks will all be in the position OA (shown on the right in Fig. 202), and can only be

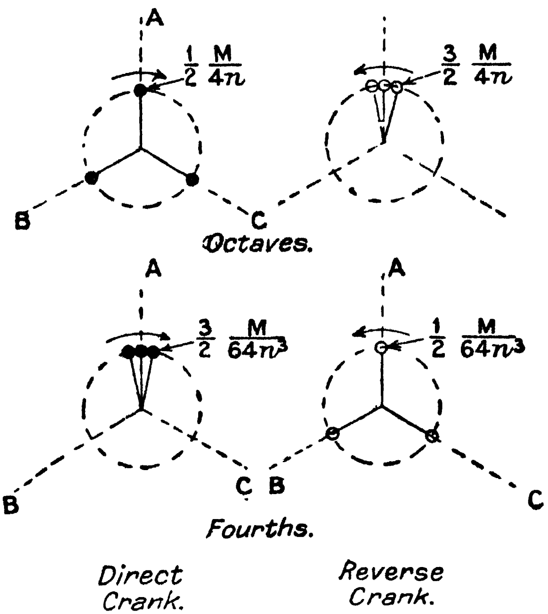


FIG 202 BALANCE OF RADIAL ENGINE (SIX-CYLINDER TYPE)

balanced by a mass $\frac{3}{2} \cdot \frac{M}{4n}$ placed opposite to OA (Fig. 202) and revolving counter-clockwise at twice engine speed.

The Fourth Harmonics

Similarly, it will be found that the three reverse cranks for the fourth harmonics all balance, whilst the direct cranks are all in position OA, and are only capable of balance by a mass $\frac{3}{2}\frac{M}{64_n{}^3}$ placed opposite, and rotating at four times engine speed.

The higher harmonics are of much less importance, since they give rise to unbalanced forces of proportionately smaller magnitude, but higher frequency.

The same method of analysing the inertia forces in other unicrank multi-cylinder type of radial engine applies.

Five-cylinder Radial Engine

Assuming that the inter-cylinder angles are equal and the five connecting rods act on a single crank—this being the maximum number of cylinders for such an arrangement—then it can readily be demonstrated by employing the same method as that used for the three-cylinder radial that the direct cranks are all in the top position (Fig. 203), whilst the reverse cranks are balanced. The out-of-balance

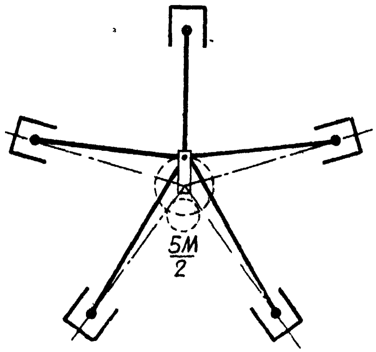


FIG 203. FIVE-CYLINDER RADIAL ENGINE

force is equal to $\frac{5M}{2}$, and can be balanced by a weight of equal amount and radius of action on the opposite side, as indicated by the dotted circle in Fig. 203.

The secondary harmonics can be shown to be balanced, both for the direct and reverse cranks, but the reverse fourth harmonic is out of balance. The value of this unbalanced force, which rotates at four times engine speed in the reverse direction to that of the engine crankshaft, is given by—

$$\text{Unbalanced fourth harmonic} = \frac{5}{2}\,\frac{M\omega^2 r}{g}\,\frac{m^3}{4} = \frac{5}{8}\cdot\frac{M\omega^2 r m^3}{g}$$

where M = mass of one reciprocating unit,

ω = angular velocity $\left(= \frac{2\pi N}{60},\ \text{where } N = \text{r.p.m.}\right)$,

r = crank radius in ft.,

m = ratio of crank to connecting rod,

and g = acceleration due to gravity = 32·19 f.s.s.

Radial Engines. General Case of Balance. Odd Cylinders

(1) When all the connecting rods are assumed to operate on a single crank pin, if the number of cylinders be denoted by n' and the mass

of each cylinder's reciprocating weight by M, then it can be demonstrated by a similar method to that employed for the three- and five-cylinder radial engine that there will be an unbalanced primary direct force equal to $\frac{n'}{2} \cdot \frac{M\omega^2 r}{g}$ which can be balanced by a counterweight on the opposite side of the crankshaft to the single crank. The secondary harmonics are in balance.

(2) *For a double row radial engine*, with n' cylinders per row and two cranks at 180°, with a phase angle between the rows of $\frac{360^\circ}{2n'}$, there will be a total of $2n$ cylinders. The primary forces are out-of-balance by an amount equal to $\frac{n'}{2} \frac{M\omega^2 r}{g}$ for each row, and each of these unbalanced forces can be balanced by a counterweight on the opposite side to its crank equal to the value stated and acting at the same radius. If the C.G. of the balance weight is to be at any other radius the moment about the crankshaft axis must be equal to that of the out-of-balance forces on the crank pin.

The secondary harmonics are in balance.

(3) *For a radial engine with a master rod* and link rods acting on a single crank pin, if n' = no. of cylinders, r = distance of crank pin to knuckle pin, l = length of link rod, L = length of master rod, then, assuming r is constant, $L = r + l$, and that the angle between the knuckle pins is the same as the angle between the cylinders, it can be shown that the primary forces or fundamental forces can be balanced by a counterweight equal to $\frac{n'}{2} \cdot \frac{M\omega^2 r}{g}$ at a radius r, on the opposite side to the crank pin.

There is an unbalanced secondary force equal to $n' \cdot \frac{r}{l} \cdot m \frac{M\omega^2 r}{g}$ on the same side as the crankpin when the master rod is on its top dead centre position. Here m = ratio of crank to connecting rod (L). There are *no unbalanced rocking couples* for any of the three general examples of radial engine that have here been considered.

Higher Harmonics in Radial Engines

Although, in general, any harmonics above the octave are unlikely to have any appreciable significance in practice, it may be of interest to note that in the case of a single crank radial engine having n' cylinders arranged at equal radial angles and with equal reciprocating masses for the respective cylinders, the following harmonics are unbalanced—

(1) *Direct.* Harmonics denoted by $(n' + 1)$, $(3n' + 1)$, $(5n' + 1)$, etc.

(2) *Reverse.* Harmonics denoted by $(n' - 1)$, $(3n' - 1)$, $(5n' - 1)$, etc.

Thus, in the case of a three-cylinder engine the unbalanced direct harmonics are the 4th, 10th, 16th, etc., and reversed harmonics the 2nd (or octave), 8th, 14th, etc.

For the five-cylinder engine the unbalanced direct harmonics are the 6th, 16th, 26th, etc., and reversed the 4th, 14th, 24th, etc.

It follows that with the increase in the number of cylinders, the lower harmonics die out progressively, so that for the more common seven and nine-cylinder radial engines, even with the master connecting rod arrangement, the effects are practically negligible.

Radial Type Engines, Even Number of Cylinders

In the case of engines with four cylinders the axes of which are in one plane and at 90°, it will be found that the fundamental harmonics require balancing by a mass at crank radius 2M. The direct and reverse cranks for the secondary or octave masses are each in perfect balance.

Similarly, the fourth harmonics are also perfectly balanced in themselves.

By multiplying the number of cylinders in radial types of engine similar to the preceding examples, the unbalanced harmonics can frequently be made to neutralize each other, but if the cylinder axes are not in the same plane (as in many multi-cylinder aircraft engines), rocking couples are introduced even when the hammering actions are avoided.

It will generally be found that in cases of radial engines with an even number of cylinders, spaced at equal intervals, the harmonics are all in perfect balance and that the fundamental harmonics can all be balanced by means of a suitable balance-weight placed opposite the single crank.

The firing intervals of even cylinder radial engines being unequal, this type is not now employed for single-crank engines.

Radial Engine Firing Intervals

With the usual four-cycle system, giving one firing stroke in every two revolutions, it will readily be seen that the firing order of a four-cylinder radial, assuming equal cylinder axis angles, and numbering the cylinders, clockwise, 1, 2, 3, and 4 (Fig. 204 (*A*) are: 1, 3, 2, 4, 1, and the corresponding angular intervals, 180°, 270°, 180°, and 90°.

For a six-cylinder single crank radial (Fig. 204 (*B*)), the firing order would be 1, 3, 5, 2, 4, 6, and the firing intervals 120°, 120°, 180°, 120°, 120°, and 60°. In general, it can thus be deduced that for a radial engine with any even number n' cylinders there will be three different firing intervals, namely, $\frac{720°}{n'}$, $\frac{1080°}{n'}$, and $\frac{360°}{n'}$, and that the first will occur $(n' - 2)$ times per complete cycle, and the other two once each.

It follows, therefore, that it *is impossible to obtain equal firing intervals with a single-row radial engine* having an *even number* of cylinders. If, however, the even number of cylinders be split into two odd numbers and arranged as a two-row radial with cranks at 180°—as in the case of the six-cylinder engine considered below, then equal firing intervals can be arranged, but the engine balance will not be very good. In general, modern radial engines of the single-row pattern

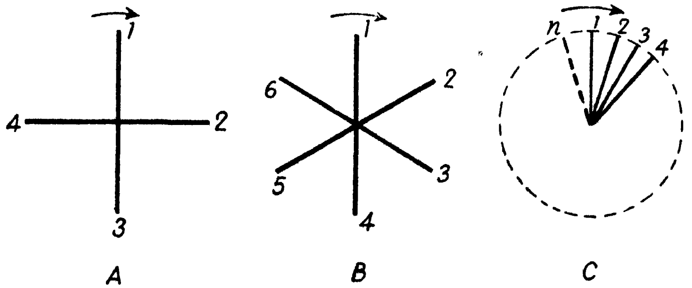


FIG. 204. ILLUSTRATING FIRING ORDERS OF RADIAL ENGINES

for automobile and aircraft purposes have an odd number of cylinders, e.g. 3, 5, 7, or 9, and the firing intervals are always equal, so that the torsional impulses occur regularly.

For a radial engine having n' cylinders in single-row arrangement *when n' is odd*, the firing intervals will all be equal to $\frac{720^\circ}{n'}$. The firing orders for the cylinders numbered consecutively from 1 to n (Fig. 204 (C)) will be 1, 3, 5, 7, and so on to n' for the first revolution of the single crank, and 2, 4, 6, 8, and so on to $(n' - 1)$ for the second revolution. For example, a seven-cylinder radial will have the following firing order, namely, 1, 3, 5, 7, 2, 4, 6, and a nine-cylinder one, 1, 3, 5, 7, 9, 2, 4, 6, 8.

Six-cylinder Radial Engine

In the case of this once popular type of aircraft engine, the usual arrangement is to split the engine up into two pairs of three cylinders each; each set is similar to that shown in Fig. 202, namely, with its cylinders at 120°; but one of the sets is arranged to have its cylinder lines at 60° to the other set, so as to fill in the gaps, as it were, and give a symmetrical radial arrangement.

Each of these three cylinder sets has its own crank, the two cranks being at 180°.

In effect, then, if Fig. 202 represents one set, the position of the second set is obtained by rotating the first through 180°.

Considering the balance of this type, it will at once be evident from the preceding example that the primary harmonic forces of one set which produce an unbalanced force of magnitude $\frac{3M}{2}$ will be exactly balanced by those of the second set, since they occur in mutually opposite directions. Owing, however, to the non-axiality of the two sets due to the double crank, there will be a primary rocking couple of magnitude $\frac{3M}{2} \cdot b$, where b is the perpendicular distance between the centre lines of the two sets.

Similarly, in the case of the secondary harmonics for the two sets, there is balance of the forces, but a rocking couple of magnitude $\frac{3}{2} \cdot \frac{M}{4n} \cdot b$ occurs, which changes in direction at the rate of twice the engine speed, but in the reverse direction.

The fourth harmonics also balance, except for a small rocking couple of magnitude $\frac{3}{2} \frac{M}{64n^3} \cdot b$ rotating at four times engine speed, but in the reverse direction.

Six-cylinder Radial Firing Order

If only one crank is employed, the firing intervals cannot be made equal, as previously mentioned.

In the usual arrangement of two sets of three cylinders, viz. 1, 3, 5, and 2, 4, 6, with a separate crank for each set and the two cranks at 180°, it is possible to obtain equal firing intervals of 120°. In this case, numbering the cylinders clockwise, 1, 2, 3, 4, 5, and 6, there will be 60° angular intervals between the consecutive cylinder axes. The firing order will be 1, 3, 5, 4, 6, 2, and the firing intervals will all be 120°.

This arrangement of 180° cranks has been used on radial six-cylinder engines.

Four-cylinder In-line Engine

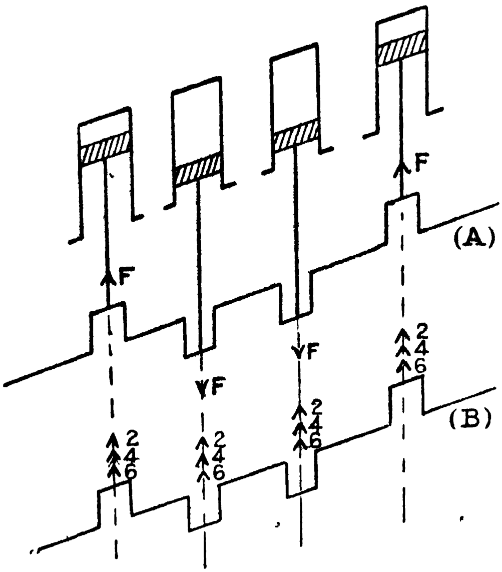
This is the ordinary type of motor-car engine with cranks arranged as shown in Fig. 205.

It will be seen later that the chief advantages of this type of engine consist in the regular firing intervals, and in the fundamental forces and couples being perfectly in balance.

However, it should be remembered that all of the secondary harmonics synchronize, and therefore give rise to an unbalanced vibration of twice the frequency of the engine revolutions.

If the connecting rods of this four-cylinder type were of infinite

length, the engine would be in perfect balance as regards the whole of the harmonics. Since, however, on account of the angularity or obliquity of the connecting rod the inertia forces are greater for the top-centre crank positions than for the bottom positions, it follows that when two pistons are passing through the upper dead centre positions, the two other pistons will be moving through the lower centre crank positions, with the net result that there will be an upward



(A). *Fundamental.* (B). *Harmonics*

FIG. 205. FOUR-CYLINDER VERTICAL ENGINE

resultant force due to the difference between the inertias of the pistons at the top and bottom positions.

Further, this upward force will occur every half-revolution of the crankshaft, with similarly occurring downward forces at intermediate positions. The net result is that the engine will experience a vibration of twice the frequency of the engine revolutions, as previously stated.

There is also another method of considering the balance of the four-cylinder engine, which is due to Lanchester.*

The effect of the angularity of the connecting rod is to cause each piston to reach its mid-position before the crank has moved through 90° from its top-centre position, or, stated in another manner, when

* "Engine Balancing," F. W. Lanchester, *Proc. I.A.E.*, 1914.

the crank pin is at 90° from the top or in-centre, all four pistons are somewhat below their mid-stroke positions.

In the case of a 20 h.p. rating engine, the position error a (Fig. 206), due to angularity, amounts to about a quarter of an inch.

Further, the pistons will be symmetrical with regard to the mid-stroke positions twice every revolution at the dead centres, and unsymmetrical by the downward position error twice every revolution for the 90° crank positions. The effect of this is that the displacement of the whole equivalent reciprocating mass occurs twice every revolution and gives rise to vibrations of the same periodicity, which are really the same thing as the octave vibrations.

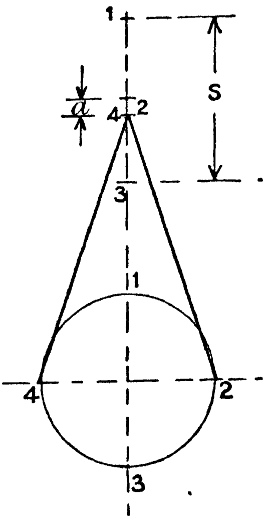


FIG. 206

Generally, the position error is approximately equal to the crank throw divided by the connecting rod length or $\frac{r}{2l}$, that is, $\frac{1}{2n}$.

It has further been shown that for a 20 h.p. rating engine with a connecting rod over crank ratio of $4\frac{1}{2}$, the octave amplitude is $\frac{1}{18}$ of the stroke; and since the disturbing force is proportional to the square of the periodic speed, then in the case of the octave harmonics, the forces called into play will be $\frac{1}{18}$ of that due to the main component of each piston.

Hence, considering the whole four pistons, the unbalanced force due to the octave harmonic will be $\frac{16}{18}$ of the unbalanced fundamental piston force for a single-cylinder engine, and, of course, gives rise to vibrations of twice the frequency of those occurring in the case of the single cylinder.

In the case of a four-cylinder engine of 4 in. bore and stroke, the weight of the piston being $4\frac{1}{4}$ lb. and the connecting rod 4 lb., and for a ratio of $n = 4$, the maximum value of the unbalanced vibrating forces at 1000 r.p.m. is about 320 lb.

At higher speeds, the forces will be very much greater, since they vary as the square of the speed. It will, therefore, be evident that the secondary forces in a four-cylinder engine are quite appreciable; although, as mentioned later, a method for absorbing these vibrations is possible.

The matter of the balance of this type of engine can also be dealt with by the same method as in the preceding examples. Examining first the *fundamentals* of the piston's motion, it will be evident that at all times the two outer crank harmonic forces balance in direction and magnitude the two inner crank forces, and further that the

moments of the primary couples for the two left-hand cranks always balance those due to the right-hand cranks, so that the primary balance is perfect.

Dealing next with the *secondary* or *octave components** of the piston's motion, a little consideration will show that the whole of the octave components for the crank position shown synchronize and act vertically upwards. When the main crank has turned through 90°, these octave components will have moved relatively through 180°, and will always be in synchronism They will, therefore, give rise to vibrations along the line of stroke of twice the main harmonic's frequency. Further, the value of the maximum octave unbalanced force will be $F_2 = \frac{4M \cdot \omega^2 \cdot r}{g \cdot n}$.

The Higher Harmonics

As regards the fourth harmonics, it will be noted that these all synchronize and give rise to vibrations of four times the frequency of the primary harmonic; the maximum value of the fourth harmonic's unbalanced force will be

$$F_4 = \frac{M\omega^2 r}{gn^3}$$

Similarly, it can be shown that the sixth and higher harmonics each synchronize for the four pistons, and give rise to vibrations of higher frequencies and with correspondingly smaller maximum forces.

Further, it should be mentioned that since each of the harmonics synchronize for all the pistons, and act in the same direction, there will be no resultant unbalanced couples for these harmonics.

In practice, the effects of the unbalanced octave and fourth harmonics are noticeable—it is quite possible to observe in some car engines the vibrations due to the former, since it synchronizes in frequency with the explosions. In addition, when a four-cylinder engine of this type is raced under no-load conditions since the torque vibrations are practically negligible, the higher period vibrations are quite apparent. When it is remembered that in the case of a 25 h.p. engine the maximum value of the unbalanced secondary force may be as much as half a ton alternately acting in an upward and downward direction, it will be seen that the effects must become of serious magnitude at high speeds. It is also a fact that at certain speeds of a motor-car engine the resonance, or sympathetic vibration effect between it and the body of the car itself, more especially limousines and landaulettes, accentuates this secondary vibration effect. By using rubber engine mountings, however, this effect can be eliminated, in practice.

* See also page 236.

Method of Balancing Secondary Forces

Lanchester has devised, and applied to four-cylinder car engines, an interesting piece of mechanism termed an "Anti-Vibrator," for the purpose of balancing the secondary harmonic effects. The principle

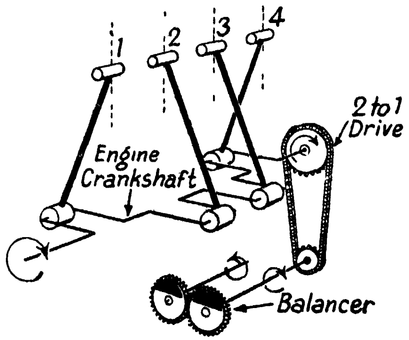


FIG 207. PRINCIPLE OF BALANCING THE SECONDARY HARMONICS IN FOUR-CYLINDER-IN-LINE ENGINE

of this arrangement consists in the introduction of an equivalent harmonic effect, but opposed in direction, to the secondary harmonic effect, as indicated, schematically, in Fig. 207.

The mechanism employed, which is illustrated in Fig. 208, employs two cranks, driven at twice crankshaft speed, and each rotating in

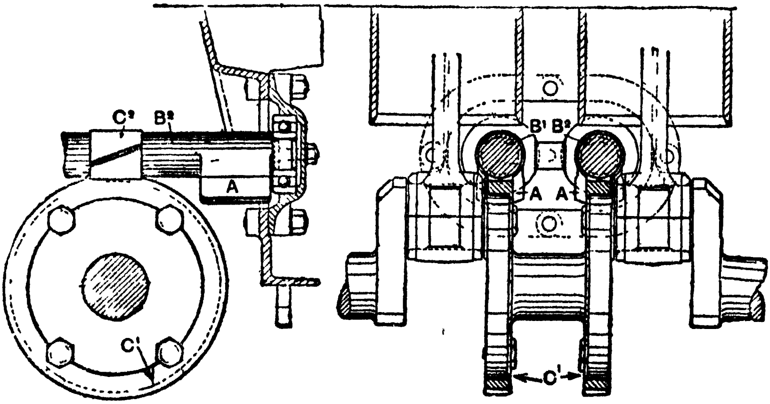


FIG 208. THE LANCHESTER METHOD OF BALANCING SECONDARY FORCES

opposite directions. In reality, it is the practical application of reverse crank method, suitable balance weights being caused to give an exactly equal and opposite force at all times to the secondary forces due to the pistons' motion.

In the diagram, the balance weights A are attached to shafts B_1, B_2, which are driven by means of helical gearing C_1, C_2 from the main crankshaft. The phase of these balance weights is the same, and it follows that a vertical harmonic reciprocating mass effect is obtained, opposed to the secondary piston reciprocating effect. The apparatus has been fitted successfully to certain British car engines.

Another design of Lanchester secondary harmonic balancer is shown in Fig. 209, the various items being described in the caption below.

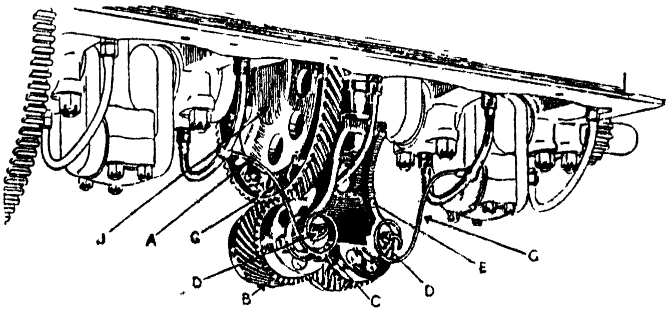


FIG. 209. ANOTHER VARIATION OF THE LANCHESTER METHOD OF BALANCING SECONDARY FORCES IN FOUR-CYLINDER ENGINES

A—helical gear; B—driven helical gear, C—out-of-balance driven wheel; D—similar geared out-of-balance wheel, E—bracket mounting for C and D, G—lubrication pipe to driven wheel bearings, J—mounting of A to web of crankshaft.

Other methods of attempting to balance the octave component have employed reciprocating masses of suitable frequency, but their non-success is attributable to the heavy stresses and consequent wear and tear involved.

Obviously in the method described, the energy of the balancing mechanism system remains constant for constant engine speed, and no appreciable stresses are introduced: but the centrifugal component due to each revolving weight will naturally result in a corresponding load being thrown upon the balance shaft bearings, the direction of which is continually changing as the weight revolves.

The reduction in the weight of the reciprocating parts, such as the piston and connecting rod will tend to reduce the magnitude of the unbalanced harmonic forces.

Another Method of Balancing Secondary Forces

A method proposed and used by W. Pilcher* for balancing a reciprocating engine consists in extending the connecting rod beyond the crank pin and so arranging the weight of the overhung portion that

* *Balancing of Engines*, A. Sharp, p. 167 (Longmans, Green, Ltd.).

the C.G. of the whole rod is on the crank pin centre (Fig. 210). For a single-cylinder engine there is perfect balance of the translational forces, but not in regard to the transverse couple,* which is actually increased by this method. In the case of *a two-cylinder vertical engine*

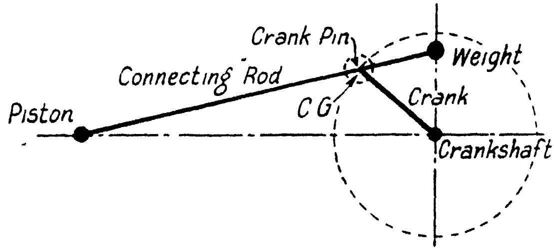


FIG. 210. METHOD OF BALANCING SECONDARY FORCES

with cranks at 180°, the transverse couples will be in perfect balance and the engine is then mathematically in perfect balance with the Pilcher arrangement.

In a four-cylinder vertical engine, also, the secondary and higher harmonic forces and transverse couples will also be in balance. A

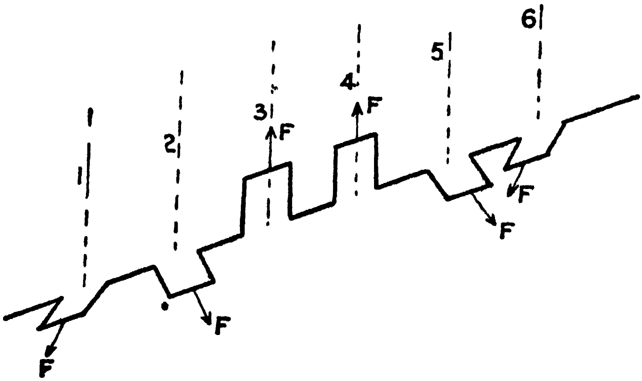


FIG. 211. THE SIX-CYLINDER ENGINE (VERTICAL), CRANKS AT 120°

proof of this statement is given in Appendix II in the paper referred to in the footnote.†

Six-cylinder Engine, Cranks at 120°

The general arrangement of the six-cylinder engine, as used in car practice, is shown in Fig. 211.

* *Vide* page 229.

† "The Four-cylinder Engine," G. F. Gibson, *Proc. Inst. Autom. Engrs.*, January, 1937.

It will be noticed that if the crankshaft be bisected by a plane perpendicular to the axis, the two halves of the crankshaft, each consisting of three cranks at 120°, will be symmetrically disposed with reference to this plane, and it is by viewing the matter by this method that it is proposed to investigate the problem.

Dealing first with the fundamental forces as represented by the components of rotating masses M upon the cranks, the three primary forces upon one side of the imaginary central plane will exactly balance the three *primary* forces upon the other side, or, stated in another manner, each set of three will be in balance so far as direction is concerned. And again, the moments of the primary forces about the central plane of the two sets of three cranks each, upon either side, will balance each other, so that the natural balance of the primary or fundamentals is perfect.

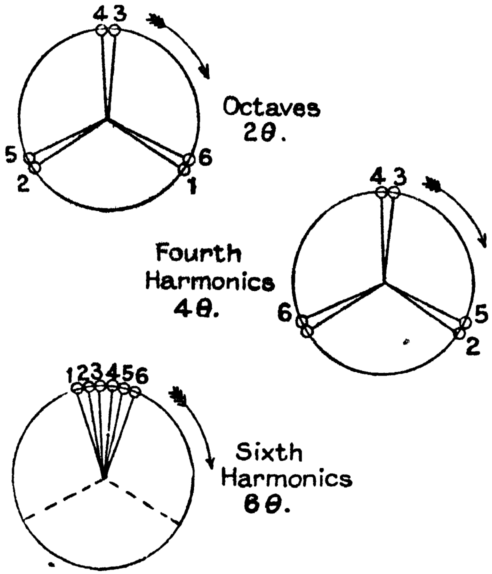


FIG. 212. HARMONICS OF SIX-CYLINDER ENGINE

Secondary Harmonics

Reference to the diagram shown in Fig. 212, the upper circle of which represents the crank arrangement referred to, a transverse perpendicular plane, will explain the position of the equivalent rotating masses, revolving at twice crankshaft speed for the octave harmonics indicated. These octave components will be seen to be in perfect balance. Similarly the fourth harmonics can be shown to be in perfect balance, both for magnitude and direction, and also moments. If the same method be applied to obtain the arrangement of the equivalent cranks for the sixth harmonics (that is, the 6θ term in the general expression for the acceleration force), it will be found that for the given crank position shown in Fig. 212, the arrangement of the sixth harmonic cranks will be as shown in the lower circle in Fig. 212, that is, the whole of these six harmonics of the piston's motion will synchronize and give rise to forces, the maximum value of which will be $6 \cdot \frac{36}{512n^5} \frac{M\omega^2 r}{g}$ or $\frac{27}{64} \cdot \frac{M\omega^2 r}{n^5 g}$, this causes a vibration of frequency

six times that of the primary, that is, occurring six times per revolution. There is, however, no resultant couple brought into play.

The practical effect of this rapidly occurring vibration is negligible as the magnitude of the maximum forces occurring due to this harmonic is very small indeed.

Three-cylinder Engine, Cranks at 120°

Resulting from this analysis of the balance of the forces occurring in the case of the six-cylinder engine, it will be apparent that if one-half of the crankshaft, as divided by the transverse central plane, be considered alone, then, so far as the primary, octave, and fourth harmonics are concerned, the forces are in perfect balance, but that owing to their lines of action being separated, *unbalanced couples* will occur in each of these cases, giving rise to a "rocking" or "plunging" vibration.

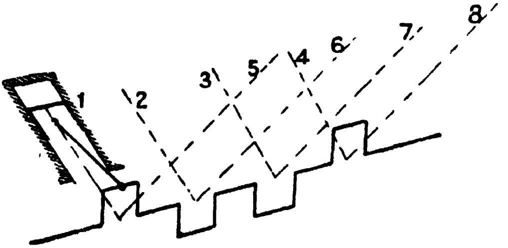


FIG. 213. THE EIGHT-CYLINDER 90° V-TYPE ENGINE

The sixth harmonic is unbalanced both as regards the forces and the couples, due to axial separation of the cylinders.

Eight-cylinder V Engine

This type of engine is of interest both as regards its torque and balance.

In passing, it is not out of place to note that it possesses marked advantages for motor-car use over other arrangements for the same number of cylinders. It enables one to get twice the power compared with a four-cylinder engine for a similar longitudinal dimension, that is, for a definite available length of bonnet space. It is lighter than a vertical in-line engine of the same number of cylinders and same size of bore and stroke; it is also lighter in proportion than a six-cylinder engine and, in addition, since the length of the crankshaft is less, the latter is relatively stiffer and less subject to whip and torsional vibrations.

Reverting to the question of engine balance, the arrangement of this type of engine is exactly similar in transverse view to that of the 90° twin-cylinder type already discussed, and in longitudinal view to that of the four-cylinder vertical engine. For the sake of simplicity, then, the eight-cylinder type may be considered to be made up of four sets of twin engines, cylinders being set at 90°, as shown in Fig. 213.

It has already been shown that in the case of a single 90° twin engine, the primary forces can be balanced by means of a counterbalance weight upon the opposite side of the crankshaft to the crank pin, but that the secondary forces are unbalanced, and give rise to a horizontal vibration. If now we consider the second set of twin engines with the crank at 180° to the first set, it will be seen that the unbalanced forces of the secondary harmonics synchronize with those of the first set. Hence for the whole set of four pairs of 90° twin cylinders with cranks arranged as in the four-cylinder vertical car engine, there will be four unbalanced secondary forces acting together or in synchronism.

The maximum value of the total unbalanced secondary forces will be $\frac{1}{\sqrt{2}}$ or 0·707 times that of a four-cylinder engine having the same reciprocating mass.

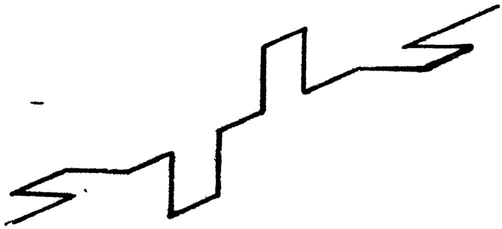


FIG. 214. ALTERNATIVE ARRANGEMENT FOR EIGHT-CYLINDER V-TYPE ENGINE

An alternative method of arranging the cranks of an eight-cylinder V-type engine is given in Fig. 214, the two centre cranks being at 180° with each other instead of together as before, and at 90° to the outer cranks.

In this case the unbalanced secondary forces of the second crank are twice 90°, or 180°, out of phase with those of the first crank, or opposed to them. Similarly, for the third and fourth cranks the secondary forces will be opposed, the net result being that the four secondary forces are in perfect balance, and that there is no resultant couple. In respect of the fourth harmonics, these will be seen to be in synchronism, since they are at four times 90° to each other, or 360°, and hence it will give rise to unbalanced forces of four times the frequency of the fundamental, but their magnitude being equal to $8 \cdot \left(\frac{M\omega^2 R}{4n^3 g}\right)\sqrt{2}$ or $2\,\frac{M\omega^2 R}{n^3 g}\sqrt{2}$ is very small compared with the unbalanced secondary forces in the other arrangements of crank.

The primary forces, it will be observed, are 90° out of phase and can be balanced by suitable counter-weights in order to avoid rocking couples. The centrifugal forces can also be balanced in a similar manner.

It has been estimated that the unbalanced forces of the fourth order in this case constitute only one-eightieth part of those of the unbalanced secondary forces in the ordinary type of eight-cylinder V-engine.

The Straight Eight Engine

This type of eight-cylinder engine was once popular for automobiles and is still employed in certain makes. It is here proposed to consider the question of engine balance from the point of view of selecting the best arrangement of cranks.

Unfortunately for the designer, there are several possible arrangements of crankshaft for this type of engine to investigate; for example, no fewer than *nine* different crankshaft schemes have been used or shown to be possible. Before considering any particular arrangement, it is proposed to examine the requirements of successful designs.

Firstly, it is necessary for satisfactory torque conditions that the firing intervals be equal. In the eight-cylinder engine there are eight explosions every two revolutions, i.e. four per revolution. This means that the cranks must be at 90° to one another for even firing intervals.

A little consideration will show that there is a number of different crank arrangements that comply with this requirement.

Secondly, for proper engine balance it is necessary that the primary forces and secondary forces shall be in balance.

With certain eight-cylinder 90° crank arrangements these conditions can be fulfilled.

Thirdly, it is essential that primary or secondary force rocking couples shall not occur, as these give rise to unpleasant vibrations at high speeds. It is this requirement that actually reduces the possible crank arrangements to a small number.

In order to obtain freedom from inertia couples of the first and second order, it is a condition that the resultant of the inertia forces of all the reciprocating parts moving downward at any instant shall be equal and opposite in direction to the resultant of the inertia forces due to the reciprocating parts moving upwards at the same instant.

As we have already seen, this condition is only obtained when the two halves of the crankshaft have "looking-glass" symmetry, i.e. when one-half is the reflected image of the other, as seen in a looking-glass.

Another way of expressing this condition is that the crank pins equally spaced from the ends of the crankshaft shall be in the same line and direction.

Figs. 215 and 216 show seven different eight-throw crankshaft arrangements which fulfil the three sets of conditions that we have enumerated.

The actual choice from these alternatives is largely a question of ease of manufacture, and of the number of main bearings that are to be employed. From the manufacturing viewpoint, the arrangements shown in Nos. 5 and 6 are the cheapest, since, in the usual method of making the crankshaft by twisting the cranks from a flat stamping,

it is only necessary to make two twists, as against four for types Nos. 3 and 4, and six for No. 2.

From the point of view of the light eight-cylinder engine with five main bearings, the arrangements shown in Nos. 5 and 6 are the most suitable ones; these have also a further advantage from the point of view of the centrifugal forces on the bearings.

The ideal arrangement is, of course, to have a bearing between each

1

1A

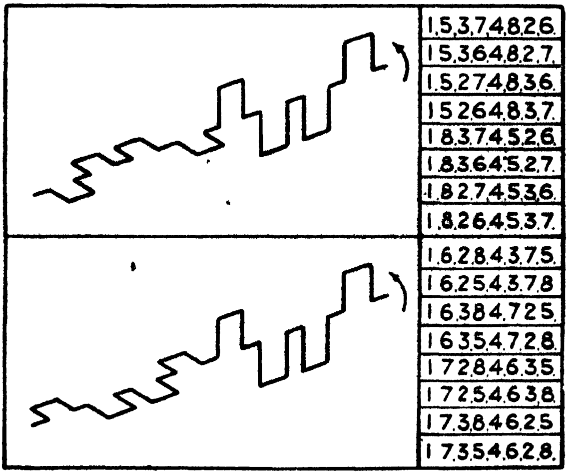


FIG. 215. TWO POSSIBLE CRANK ARRANGEMENTS, WITH ALTERNATIVE FIRING ORDERS FOR STRAIGHT-EIGHT ENGINE

pair of cranks, i.e. nine in all. It may be mentioned, here, that the crank arrangements Nos. 5 and 6 represent those which have been the most used.

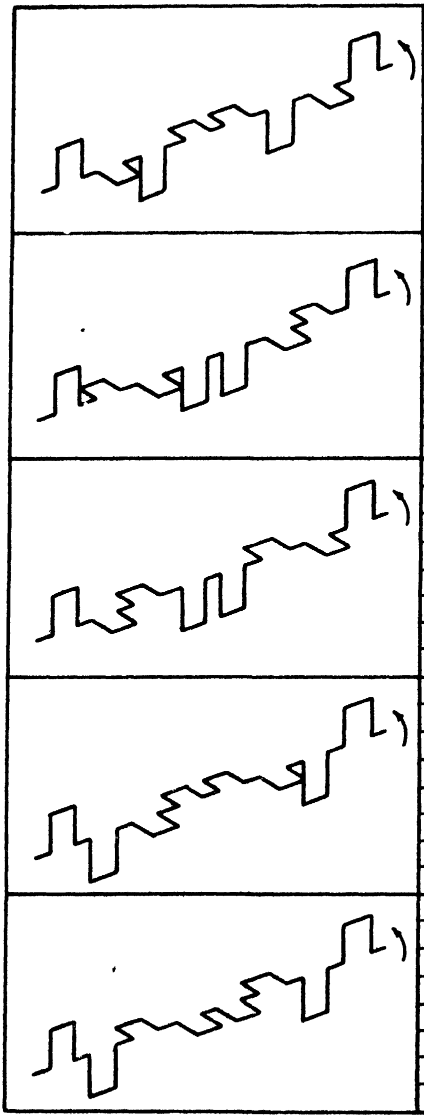
Alternative Crank Arrangements

The crank arrangements shown in Fig. 215 are seldom used, on account of their lack of "looking-glass" symmetry, and therefore of proper balance.

It will be evident, from the lack of "looking-glass" symmetry that, although these arrangements give equal firing intervals, the inertia force couples will not be in balance. If the two halves of the crankshaft be regarded as two four-cylinder vertical engine crankshafts, it will at once be seen that there will be a rocking couple in the longitudinal plane due to the secondary forces.

The crankshaft arrangements are therefore inferior to those shown in Fig. 216, but they are easier to manufacture, and when two carburettors are fitted to the engine give a better gas distribution.

CYLINDER FIRING ORDERS



2	17358264
	17348265
	17658234
	17648235
	12358764
	12348765
	12658764
	12648765
3	13428657
	13478652
	13528647
	13578642
	16428357
	16478352
	16528347
	16578342
4	12438756
	12468753
	12538746
	12568743
	17438256
	17568243
	17538246
	17568243
5	13248675
	13748625
	13258674
	13758624
	16248375
	16748325
	16258374
	16758324
6	14238576
	14268573
	14738526
	14768523
	15238476
	15738426
	15268473
	15768423

FIG 216 SOME OTHER ALTERNATIVE CRANK ARRANGFMENTS FOR STRAIGHT-EIGHT ENGINFS

Narrow Angle Vee-eight Engine

In order to effect a kind of compromise between the shorter Vee-eight and the "in-line" eight-cylinder engines, an engine has been proposed which consists of two sets of four cylinders staggered slightly in relation to each other, each set being at a narrow angle, namely 20°, to the other.

The advantages of such an arrangement are that a single monobloc casting of much lighter constructional weight than the eight cylinders (or two monoblocs of four cylinders each) of the two alternatives mentioned can be employed. Moreover, the overall length is little more than that of a Vee-eight engine, whilst the overall width is far less. The use of a relatively short stiff crankshaft, less prone to torsional vibrations, is an added advantage.

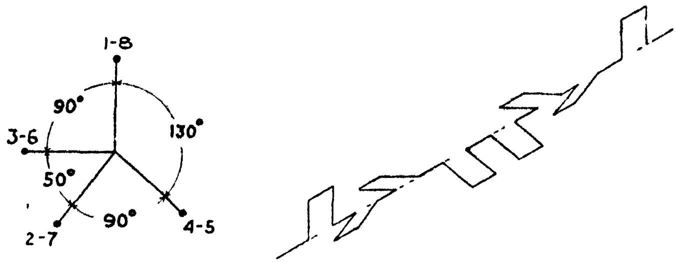


FIG. 217. CRANK ARRANGEMENT FOR NARROW ANGLE VEE-EIGHT MONOBLOC CYLINDER ENGINE

It can be shown that with the eight-throw crankshaft shown in Fig. 217, with the webs and crank pins suitably balanced for centrifugal force effects, the primary and secondary forces are in proper balance. If the left bank of cylinders viewed from the radiator end (for a car engine) is numbered 1, 3, 6, and 8, and the right bank 2, 4, 5, and 7, then the appropriate firing order is 1, 3, 2, 4, 8, 6, 7, 5.

The only slight disadvantage of this engine lies in the fact that the firing intervals are not quite the same, the intervals being 90°, 70°, 90°, 110°, 90°, 70°, 90°, 110°, etc.

Firing Order of Straight-eight Engines

For each of the different arrangements of cranks shown in Figs. 215 and 216, there are eight different firing orders. It is not necessary to give a list of these, as they are simple to work out, but it may be mentioned that the best firing order is that giving uniform spacing between the firing strokes.

A careful examination of the crankshaft and engine firing orders of the majority of straight-eight automobile engines reveals that the

most popular crankshaft arrangements are those shown in Fig. 216, and that the most favoured firing order is as follows—

1, 6, 2, 5, 8, 3, 7, 4

In about 80 per cent of the eight-cylinder engines in question, the above crank arrangement and firing orders were adopted.

Comparison of Eight-cylinder Engines

The two most commonly used types of eight-cylinder engines are the V-type and the "straight-eight." Each of these has certain theoretical and practical advantages, which may be enumerated, briefly as follows.

The V-type Engine

ADVANTAGES—

(1) More compact design, and therefore takes up less room fore-and-aft under the bonnet of a car or small aircraft.

(2) Shorter and therefore stiffer crankshaft.

(3) Fewer main bearings required.

(4) Torsional vibrational effects not so pronounced.

DISADVANTAGES—

(1) Heavy loading of crank-pin bearings due to big-end bearings of two rods on one pin, unless a master rod and articulated link is used.

(2) Inaccessibility of valves, tappets, and carburettor parts.

(3) Duplication of certain parts, e.g. exhaust water pipes and hose connexions.

(4) Much greater overall width of engine, bringing cylinder heads close to bonnet, or cowling.

The Straight-eight Engine

DISADVANTAGES—

(1) Overall length nearly twice that of V-type engine.

(2) Greater number of main bearings.

(3) Heavier in weight than V-type.

(4) Greater torsional vibrational tendency.

(5) More difficult to obtain uniform gas distribution.

(6) Crankshaft more complex and costly.

ADVANTAGES—

(1) Better wearing properties in regard to main and big-end bearings.

(2) Better accessibility of parts requiring attention.

(3) Smaller number of parts, and simpler design of many, e.g. exhaust and water connexions.

(4) Design of complete engine more straightforward.

In regard to the principal disadvantage of the straight-eight engine, namely, that of torsional vibrations due to the relatively long shaft, this was the cause of several cases of crankshaft fractures in the heavier types of internal combustion engine, but with a better knowledge of design requirements and with the introduction of vibration dampers or flexible couplings between the engine and the driven member, e.g. the propeller in the case of aircraft, and the gearbox in the case of cars, this trouble has been largely overcome.

Balancing of Other Types

It has been possible only to deal in the present chapter with a few of the more important types of engine, in respect to balancing; having become acquainted with the methods employed in the foregoing considerations, there should be no difficulty in applying these to any type of engine.

A necessary course to pursue is to consider both the torque curves and engine balance for any particular type of engine under investigation, and then it is possible to form a fairly definite opinion as to the suitability or otherwise of this given type for the purposes required.

For more detailed treatment, with wider scope, of the subject of engine balance, reference should be had to textbooks on the subject, such as *Engine Balancing*, by Prof. A. Sharp, Dalby's *Balancing of Engines*, etc., the "Proceedings of the Institute of Automobile Engineers," and to the papers referred to in the footnotes.

Classified Examples of Engine Balance

As it is not possible, owing to space considerations, to deal in detail with each and every example of engine balance for cylinder arrangements applicable to automobile and aircraft engines, a useful series of classified results* is given in tabular form on the following pages, so that the balance of any particular type of engine can at once be analysed in so far as the primary and secondary (shaking) forces and the primary and secondary rocking couples are concerned.

The general types of engines considered include: (*A*) *Vertical In-line with* 1 *to* 8 *Cylinders.* (*B*) *Opposed Engines with* 2 *to* 16 *Cylinders.* (*C*) *Vee-type Engines with* 2 *to* 16 *Cylinders.* (*D*) *W-type Engines with* 3 *to* 16 *Cylinders.* The radial type engines have already been dealt with in this chapter and are therefore omitted here.

The firing intervals, which should all be equal in actual engines, are given in the tables that follow.

* Based on information given in the *Society of Automotive Engineers' Journal*, October, 1934 (A. J. Meyer).

Notation Used. The following symbols and notation have been used in connexion with the tables—

C = Centrifugal force, in lb., that would result from the rotation of the reciprocating mass (M) of a single-cylinder unit with crankshaft speed at crank radius (r).

Thus $$C = \frac{M\omega^2 r}{g} \text{ lb}$$

where ω = angular velocity in radians $= \frac{2\pi N}{60}$ where N = r.p.m., r is in feet, and $g = 32{\cdot}19$ ft. per sec.^{2}

W = Value of the centrifugal force produced by the counterweight necessary to cancel all or part of the inertia forces. The primary weight is supposed to run with the same speed and in the same direction as the crankshaft. The secondary weight runs twice as fast and in the same direction, unless otherwise noted. The angular position of the weights with respect to the crank pin is indicated in the diagram by P and by S, respectively.

H = The horizontal unbalanced inertia resultant after the counterweight indicated has been incorporated.

V = The vertical unbalanced inertia resultant after the counterweight indicated has been incorporated.

Θ = Crank angle, measured from the crank position as shown, in a clockwise direction.

a = Distance in inches as shown.

b = Distance in inches as shown.

c = Distance in inches as shown.

M_v = Rocking couple, in lb.-in., in the vertical plane. A positive couple turns clockwise in the top view or the side view, as shown.

M_h = Rocking couple, in lb.-in., in the horizontal plane. A positive couple turns clockwise in the top view or the side view, as shown.

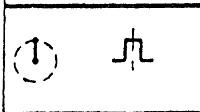
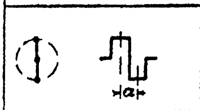
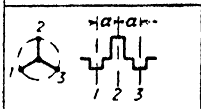
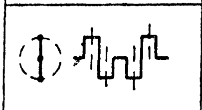
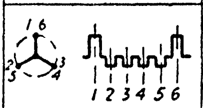
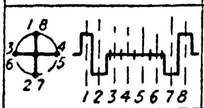
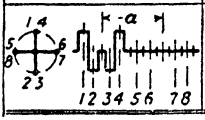
λ = Ratio of crank radius divided by connecting-rod length, i.e. is the same as m (or q) used in the previous calculations given in this chapter.

In regard to the results given in the tables, the best arrangements of the cylinders are those corresponding to zero values for the primary and secondary shaking forces and couples, with equal firing intervals.

The engines given in Table (XII) which fulfil these conditions are the six-cylinder vertical and the first of the two eight-cylinder arrangements. In the other examples, unbalanced rocking couples or shaking forces occur.

TABLE XII

FOUR-CYCLE IN-LINE ENGINES

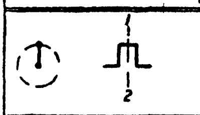
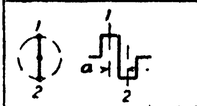
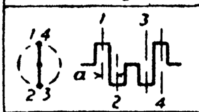
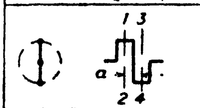
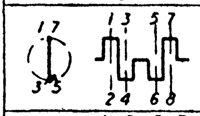
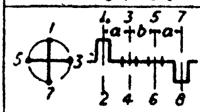
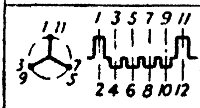
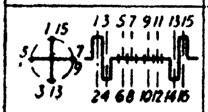
INERTIA BALANCE AND FIRING INTERVALS						
ARRANGEMENT OF CRANKSHAFT AND CYLINDERS	NO OF CYL	PRIMARY SHAKING FORCE	PRIMARY ROCKING COUPLE	SECONDARY SHAKING FORCE	SECONDARY ROCKING COUPLE	FIRING INTERVAL DEG
FOUR-CYCLE IN-LINE ENGINES						
	1	$W=0$ $V=C\cos\theta$ $H=0$	$M=0$	$W=0$ $V=\lambda C\cos 2\theta$ $H=0$	$M=0$	720
	2	$W=0$ $V=0$ $H=0$	$M_V=aC\cos\theta$	$W=0$ $V=2\lambda C\cos 2\theta$ $H=0$	$M=0$	180 540
	3	$W=0$ $V=0$ $H=0$	$M_V=a\sqrt{3}C\sin\theta$	$W=0$ $V=0$ $H=0$	$M_V=a\sqrt{3}\lambda C\sin 2\theta$	240
	4	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=4\lambda C\cos 2\theta$ $H=0$	$M=0$	180
	6	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	120
	8	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	90
	8	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M_V=4\lambda C\cos 2\theta$	90

In the case of the opposed engines the only two instances of perfect balance given are those of the eight-cylinder with four cranks in the same plane and the twelve-cylinder arrangement shown. The former engine, however, has the disadvantage that two cylinders must fire simultaneously so that the firing intervals are 180° instead of the (possible) 90° with the second arrangement.

It may here be mentioned that it is possible to balance a six-cylinder opposed engine, using a six-throw crankshaft with consecutive pairs at 180°, the three pairs being 120° apart.

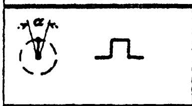
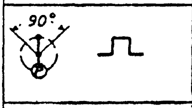
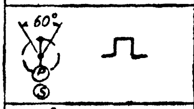
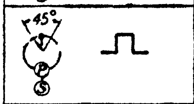
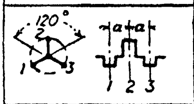
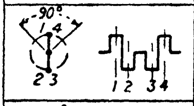
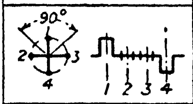
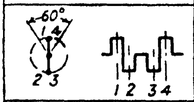
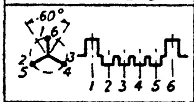
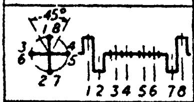
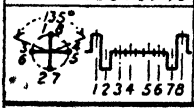
TABLE XIII

FOUR-CYCLE OPPOSED ENGINES

INERTIA BALANCE AND FIRING INTERVALS						
Arrangement of crankshaft and cylinders	No. of cyl.	Primary		Secondary		Firing interval, deg
		Shaking force	Rocking couple	Shaking force	Rocking couple	
FOUR-CYCLE OPPOSED ENGINES						
	2	$W=0$ $V=2C\cos\theta$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	180 540
	2	$W=0$ $V=0$ $H=0$	$M_V=$ $aC\cos\theta$	$W=0$ $V=0$ $H=0$	$M_V=$ $a\lambda C\cos 2\theta$	360
	4	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M_V=$ $2a\lambda C\cos 2\theta$	180
	4	$W=0$ $V=0$ $H=0$	$M_V=$ $2aC\cos\theta$	$W=0$ $V=0$ $H=0$	$M=0$	180
	8	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	180 2 CYLS SIMULT
	8	$W=0$ $V=0$ $H=0$	$M_V=2(2a+b)$ $\times C\cos\theta$ $-2bC\sin\theta$	$W=0$ $V=0$ $H=0$	$M=0$	90
	12	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	60
	16	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	90 2 CYLS SIMULT

Of the V-type engines, there are four examples giving perfect engine balance, in so far as the primary and secondary factors are concerned, combined with equal firing intervals. These include the second of the three eight-cylinder arrangements, with suitable

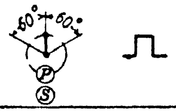
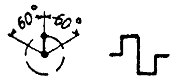
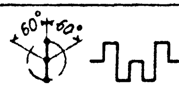
TABLE XIV.—Four-cycle V-type Engines

INERTIA BALANCE AND FIRING INTERVALS						
ARRANGEMENT OF CRANKSHAFT AND CYLINDERS	NO. OF CYL	PRIMARY		SECONDARY		FIRING INTERVAL DEG
		SHAKING FORCE	ROCKING COUPLE	SHAKING FORCE	ROCKING COUPLE	
FOUR-CYCLE V-TYPE ENGINES						
	2	$W=0$ $V=2C\cos\theta \times\cos^2\alpha/2$ $H=2C\sin\theta \times\sin^2\alpha/2$	$M=0$	$W=0$ $V=2\lambda C\cos 2\theta \times\cos\frac{\alpha}{2}\cos\alpha$ $H=2\lambda C\sin 2\theta \times\sin\frac{\alpha}{2}\sin\alpha$	$M=0$	
	2	$W=C$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=\sqrt{2}\lambda C \times\cos 2\theta$	$M=0$	450 270
	2	$W=\frac{1}{2}C$ $V=C\cos\theta$ $H=0$	$M=0$	$W=\frac{1}{2}\sqrt{3}\lambda C$ $V=0$ $H=0$	$M=0$	420 300
	2	$W=0.293C$ $V=\sqrt{2}C \times\cos\theta$ $H=0$	$M=0$	$W=0.541\lambda C$ $H=0$ $V=0.765\lambda C \times\cos 2\theta$	$M=0$	405 315
	6	$W=0$ $V=0$ $H=0$	$M_V=\frac{1}{2}a\sqrt{3} \times C\sin\theta$ $M_H=\frac{3}{2}a\sqrt{3} \times C\cos\theta$	$W=0$ $V=0$ $H=0$	$M_V=\frac{1}{2}a\sqrt{3} \times\lambda C\sin 2\theta$ $M_H=\frac{3}{2}a\sqrt{3} \times\lambda C\cos 2\theta$	120
	8	$W=C$ * $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=4\sqrt{2}\lambda C \times\cos 2\theta$	$M=0$	90
	8	$W=C$ * $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	90
	8	$W=0$ $V=0$ $H=0$ * OPPOSITE EACH CRANK	$M=0$	$W=2\sqrt{3}\lambda C$ $V=0$ $H=0$	$M=0$	60 120 60 120 ETC.
	12	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	60
	12	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	45 75 45 75 ETC.
	16	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	45
	16	$W=0$ $V=0$ $H=0$	$M=0$	$W=0$ $V=0$ $H=0$	$M=0$	45

counterweight, the first of the two twelve-cylinder ones and both sixteen-cylinder ones.

It may be mentioned that there are two different twenty-four cylinder aircraft engines, namely, the Rolls Royce Vulture and the Allison V–3420, which utilize two sets of twelve-cylinder V-engine units geared together to drive the propeller shaft in a common crank-

TABLE XV
FOUR-CYCLE W-TYPE ENGINES

INERTIA BALANCE AND FIRING INTERVALS						
ARRANGEMENT OF CRANKSHAFT AND CYLINDERS	NO OF CYL	PRIMARY		SECONDARY		FIRING INTERVAL, DEG
		SHAKING FORCE	ROCKING COUPLE	SHAKING FORCE	ROCKING COUPLE	
FOUR-CYCLE W-TYPE ENGINES						
	3	$W = 1\frac{1}{2}C$ $V = 0$ $H = 0$	$M = 0$	$W = \frac{1}{2}\lambda C$ $V = 0$ $H = \lambda C \sin 2\theta$	$M = 0$	120 300 300
	6	$W = 1\frac{1}{2}C$ * $V = 0$ $H = 0$	$M = 0$	$W = \lambda C$ $V = 0$ $H = 2\lambda C \times \sin 2\theta$	$M = 0$	120
	12	$W = 0$ $V = 0$ $H = 0$	$M = 0$	$W = 2\lambda C$ $V = 0$ $H = 4\lambda C \times \cos 2\theta$	$M = 0$	60
	16	$W = 0$ $V = 0$ $H = 0$ * OPPOSITE EACH CRANK	$M = 0$	$W = 0$ $V = 0$ $H = 0$	$M = 0$	45

case unit. Each of the two V-engine units in such arrangements can be perfectly balanced.

Of the engine arrangements shown in Table XV only the sixteen one gives perfect balance with equal firing intervals.

Elimination of Rocking Moments in Engines

As pointed out originally by Lanchester, the rocking moment which occurs, due to the axial separation of the cylinders in different types of engine may be eliminated in multi-cylinder types by the method of "looking-glass" symmetry. The meaning of this phrase can be made quite clear by considering a particular case. It is known that in the two-cylinder vertical type with cranks at 180° there is a primary rocking couple. If now we imagine a transversely situated mirror at the end of crankshaft, then the reflected image of the crankshaft represents an arrangement of cranks, which with the original cranks will give no rocking couple—or in other words, the rocking couple of

the original type exactly balances that of the reflected type. This combination of object and image is, of course, the ordinary four-cylinder arrangement.

Again, the six-cylinder 120° vertical engine can be divided by a central transverse plane, one half of the crankshaft being an exact reflection of the other half—and it is known that there is no resultant rocking couple in this type. Similarly the two-cylinder 180° opposed type represents a direct and reflected image of the single-cylinder type, and no rocking moment occurs.

Generally speaking, then, if there exists in any arrangement of cylinders an optical symmetry about a symmetrical plane, there will be no rocking couples.

Another way of expressing this result is that whatever rocking couple is set up by the direct or object half of the engine will be neutralized or balanced by the reflected image half.

The Effect of Cylinder Offset upon Engine Balance

The practice of offsetting the crank* in the direction of rotation has been adopted in the past by several makers of car, cycle, and aircraft engines, with a view to reducing the thrust upon the cylinder walls during the earlier part of the explosion stroke when such a thrust is a maximum.

This arrangement will cause a modification in the torque diagram, and also affect the balance of the engine.

It is not proposed here to deal with the question of offset in relation to engine balance in detail, as this would involve a tedious mathematical analysis out of proportion to the value resulting, but to indicate briefly the actual results of this practice, and to point out the lines along which to deal with problems relating to offsetting the crankshaft.

The matter has been treated in detail in articles by Prof. A. Sharp in *The Automobile Engineer*, November, 1910, and April, 1912, and F. W. Lanchester in a paper read before the Institution of Automobile Engineers, February, 1914, entitled "Engine Balancing," and it is to the latter that we owe the following treatment.

Referring to Fig. 218, the line of stroke for an offset engine is represented by OA_1, and of a normal type by CS. When the crank is perpendicular to the line of stroke, the connecting rod is in the positions PA_2 and P_1A_1 respectively for the offset type and PC and P_1C for the normal type, so that in the former case the points A_1 and A_2 correspond to the point C in the latter case.

Further, since the angle A_1CA_2 is equal to the angle PCP_1, and this latter angle is a maximum for the crank positions shown, it will be evident that the distance A_1A_2 is a maximum as shown.

* Known as the *desaxé* arrangement.

Hence the motion of the offset piston may be considered to be equivalent to the normal piston motion and an approximately harmonic motion superimposed of maximum amplitude AA^1, or alternatively the offset piston's motion may be represented approximately by the motion of the small end of the connecting rod driven by an

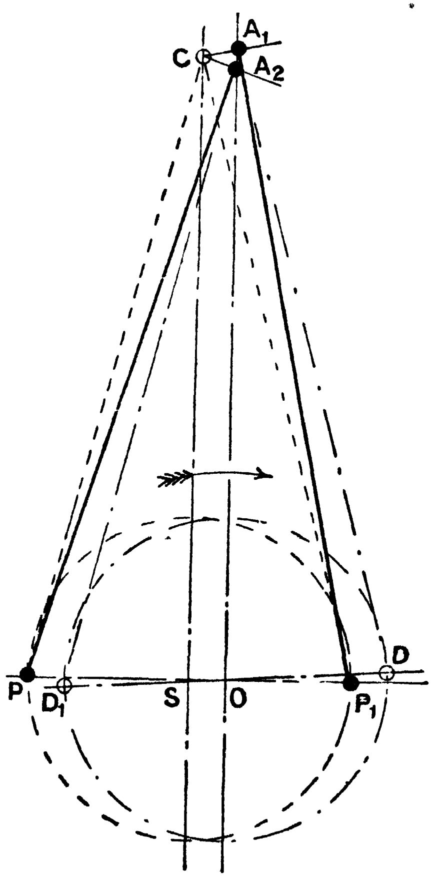


Fig. 218. The Offset or Desaxé Engine

imaginary crank OD with offset centre O, and it will be seen to be equivalent to an alteration in the phase of the normal type by an amount D_1OD.

The effect of offset then is equivalent to a retardation of the phase by a small angle, both for the finite and infinite connecting rods; but in the latter case the piston's motion will be strictly simple harmonic.

In the case of the finite connecting rod length, it can be shown that the secondary vibration is out of phase with the main vibration by

the offset angle, and in such a direction as to oppose or counteract the effect of the offset angle, that is, in advance of the fundamental harmonic. Hence the secondary vibration will be in the same phase and in the same direction as in the normal type of engine.

Analytical Method

The question of the influence of offset upon engine balance can also be considered analytically, and the general method adopted consists in obtaining an expression for the offset piston's position in terms of the crank angle, and by double differentiation deducing the acceleration expression.

The expression demonstrates that the offset piston's motion is due to a number of harmonic motions of different frequencies, the amplitudes and maximum values being dependent upon the connecting rod crank ratio and upon the degree of offset.

The expression for the acceleration force in the normal type engine is given by

$$\frac{M}{g}\frac{d^2x}{dt^2} = -\frac{M}{g}\,\omega^2 r\left[\cos\theta + C_2\cos 2\theta + C_4\cos 4\theta + C_6\cos 6\theta + \ldots\right]$$

and in the offset type of engine by

$$\frac{M}{g}\frac{d^2x_1}{dt^2} = -\frac{M}{g}\,\omega^2 r\left[\cos\theta + A_2\cos 2\theta + A_4\cos 4\theta + A_6\cos 6\theta + \text{etc}\right]$$
$$+\frac{M}{g}\,\omega^2 r\left[\sin\theta + B_3\sin 3\theta + B_5\sin 5\theta + \text{etc.}\right]$$

where C_2, C_4, C_6, etc., are constants depending upon the crank connecting rod ratio, and A_2, A_4, A_6, etc., B_3, B_5, B_7, etc , are constants depending upon both the crank connecting rod ratio and the amount of offset, which can generally each be expressed as an algebraic series in n, the crank-connecting-rod ratio, and a the offset.

The effect of offset is to introduce odd harmonics into the piston's motion, so that for symmetrical crank positions the piston's position is not the same (as in the normal type).

The quantitative effect of offset upon engine balance will be rendered more intelligible if a concrete example be given.

Taking the connecting rod crank ratio $n = 5$, and the amount of offset a = crank radius, the expression for the offset piston displacement is

$$x = r\cos\theta + l\{0{\cdot}9700 + 0{\cdot}0106\cos 2\theta - 0{\cdot}00001\cos 4\theta + \ldots$$
$$+\, 0{\cdot}0415\sin\theta - 0{\cdot}0002\sin 3\theta + \ldots\}$$

and the accelerating force by

$$\frac{M}{g}\frac{d^2x}{dt^2} = -\frac{M}{g}\,\omega^2 r\,\{\cos\theta + 0{\cdot}0424\cos 2\theta - 0{\cdot}0001\cos 4\theta + \ldots$$
$$+ 0{\cdot}0415\sin\theta - 0{\cdot}0018\sin 3\theta + \ldots$$

the higher harmonics becoming of less importance.

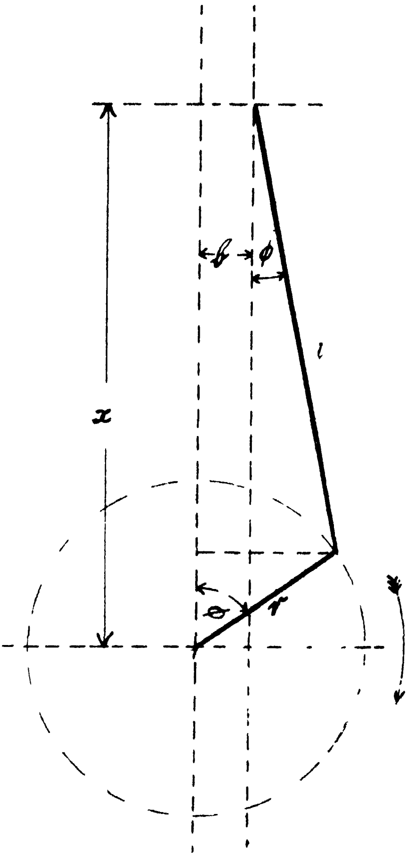


FIG. 219

It has been shown that in the case of a single cylinder with a connecting rod crank ratio of 5 and an offset equal to half the crank radius that the increase in the fundamental unbalanced force is only 0·0002 of its normal value, and in the case of a four-cylinder offset engine that the secondary unbalanced force is about 1·5 per cent greater than in the normal four-cylinder type. Generally speaking, with the degree of offset employed in practice the effect upon engine balance is practically negligible.

Approximate Method for Offset Cylinder Engine

The following method is an approximate one but giving fairly accurate results. It consists in finding an expression for the piston's displacement, and by the addition of a small quantity to the expression which is under the surd sign, so as to make it a perfect square, in simplifying the expression so that it can be readily differentiated in order to obtain the piston acceleration.

Referring to Fig. 219, and employing the notation shown thereon, the following relations are obtained—

$$\text{Piston displacement } x = l\cos\phi + r\cos\theta \quad . \quad . \quad (1)$$

and
$$r\sin\theta = l\sin\phi + b \quad . \quad . \quad (2)$$

from which $\sin\phi = \dfrac{r\sin\theta - b}{l}$ and $\cos\phi = \dfrac{x - r\cos\theta}{l}$

The term ϕ can now be eliminated by using the relation—

$$\sin^2\phi + \cos^2\phi = 1 = \left(\frac{r\sin\theta - b}{l}\right)^2 + \left(\frac{x - r\cos\theta}{l}\right)^2$$

This expression simplifies to the following—

$$x^2 - 2xr\cos\theta + (b^2 + r^2 - l^2 - 2br\sin\theta) = 0$$

which is a quadratic in x

The real solution of this equation is—

$$x = r\cos\theta + l\sqrt{1 - \left(\frac{r\sin\theta - b}{l}\right)^2} \quad . \quad . \quad (3)$$

To make the surd expression a perfect square, add to it the small quantity $\dfrac{1}{4}\left(\dfrac{r\sin\theta - b}{l}\right)^4$. No appreciable error is introduced by this addition of a second order term. Then Equation (3) becomes

$$x = r\cos\theta + l\left[1 - \frac{1}{2}\left(\frac{r\sin\theta - b}{l}\right)^2\right]$$

$$= r\cos\theta - \frac{(r\sin\theta - b)^2}{2l} + l$$

This expression simplifies to—

$$x = r\cos\theta - \frac{r^2}{2l} \cdot \sin^2\theta + \frac{br}{l}\sin\theta$$

Differentiating for the velocity, we get—

$$V_P = \frac{dx}{dt} = -r\left[\sin\theta + \frac{r}{l}\cos 2\theta - \frac{b}{l}\cos\theta\right] \cdot \frac{d\theta}{dt}$$

For $\frac{d\theta}{dt}$, the angular velocity $\omega = \frac{2\pi N}{60} = \frac{\pi N}{30}$ may be written

Then $$V_P = r\cdot\frac{\pi N}{30}\left[\sin\theta + \frac{r}{l}\sin 2\theta - \frac{b}{l}\cos\theta\right]$$

The piston acceleration is now obtained by differentiating the velocity with respect to the time.

Thus, $$A_P = \frac{dV_P}{dt} = \frac{d^2x}{dt^2} = -r\cdot\left(\frac{\pi N}{30}\right)^2\left[\cos\theta + \frac{r}{l}\cos 2\theta + \frac{b}{l}\sin\theta\right]$$

This expression is almost identical with that given on page 234 for the engine with no offset, but there is an additional term

$$r\left(\frac{\pi N}{30}\right)^2\cdot\frac{b}{l}\sin\theta$$

The normal engine acceleration can be obtained by putting $b = 0$ in the above expression.

The reciprocating force acting in the case of the offset engine is given by—

$$F = \frac{M}{g}\cdot A_P = \frac{Mr}{g}\cdot\left(\frac{\pi N}{30}\right)^2\left[\cos\theta + \frac{r}{l}\cos 2\theta + \frac{b}{l}\sin\theta\right]$$

If M is in lb., and r and l are in ft., F will be in lb.

It will be seen that the reciprocating force may be considered as being made up of a primary force $k\cos\theta$, with another smaller superimposed one $k^1\sin\theta$, both following the same kind of variation, and a harmonic of the form $k_2\cos 2\theta$.

The additional harmonic $k^1\sin\theta$ modifies the inertia force diagram. It has the effect of increasing the inertia force during the first part of the downstroke, then reducing it during the second part of the stroke. It then increases the inertia force during the first part of the upstroke, and finally diminishes it again during the second part.

The Four-cylinder Offset Engine

In the case of the four-cylinder vertical engine, the effect of the offset upon the inertia forces can be conveniently studied by considering the expression for the reciprocating force given above, namely—

$$F = \frac{Mr}{g}\left(\frac{\pi N}{30}\right)^2\left[\cos\theta + \frac{b}{l}\sin\theta + \frac{r}{l}\cos 2\theta\right]$$

where r, l, and b are the crank radius, connecting-rod length, and amount of offset respectively; θ, the crank angle; and M, the mass of the reciprocating parts.

Referring to Fig. 205, it will be seen that the two outer cranks can be treated as one, and also the two inner cranks.

The total primary force for the two sets will then be given by—

$$F_1 = \frac{2Mr}{g}\left(\frac{\pi N}{30}\right)^2\left\{\cos\theta + \frac{b}{l}\sin\theta + \cos(180^\circ + \theta) + \frac{b}{l}\sin(180^\circ + \theta)\right\}$$

Since $\cos(180 + \theta) = -\sin\theta$, the above expression becomes zero, so that the primary forces are in perfect balance.

The total secondary forces for the two sets of cranks is given by

$$F_2 = \frac{2Mr^2}{gl}\left(\frac{\pi N}{30}\right)^2\left\{\cos 2\theta + \cos 2(180 + \theta)\right\}$$

$$= \frac{4 \,.\, Mr^2}{gl}\left(\frac{\pi N}{30}\right)^2 \cos 2\theta$$

so that, as in the case of the normal four-cylinder engine, the secondary forces are unbalanced, and by the same amount

It will also be observed that the amount of the offset, b, does not enter into the result, so that the balance of the offset engine is identical with that of the normal type. This result can also be readily shown graphically from the inertia diagrams, by super-position for the two sets of parallel crank forces.

CHAPTER X

SOME MISCELLANEOUS CONSIDERATIONS

The Flywheel

THE function of the flywheel is to minimize the fluctuation of engine speed caused by the variation of the crank effort and by the variation in load upon the engine.

The actual crank effort at any moment, in the case of an internal combustion engine, depends upon the piston position, connecting rod-to-crank ratio, number of cylinders, etc., and is always varying, sometimes the energy imparted to the flywheel and crankshaft is in excess of the mean requirements of the engine, and sometimes below it, so that the speed of the flywheel will tend to undergo corresponding fluctuations.

Alternatively, the actual load upon the engine may vary periodically, but this condition does not often occur in the case of petrol engines, unless they be employed for driving pumps or compressing air.

Thus, in the former mode of variation of output energy during a revolution, if the line MN (Fig. 220) represents the mean height of the torque diagram, this line may be regarded as the mean torque equivalent to the steady load, or resistance, imposed upon the engine.

Then the shaded area above the line MN will represent the excess of energy obtained during the explosion stroke and imparted to the flywheel, thereby causing an increase in its speed.

Similarly, the shaded area below the line MN shows the energy obtained at the expense of the kinetic energy of rotation of the flywheel and required for the idle strokes (exhaust, suction, and compression) of the engine.

This absorption of energy from the flywheel will cause a reduction in the speed of the flywheel.

Further, at the points A and B the rates at which work is being done on and by the crankshaft is the same, and hence the flywheel's speed will be neither increasing nor decreasing at these points.

At A the flywheel will just have ceased to supply energy to the crankshaft, and therefore its speed will be a minimum at this point, as indicated by the lower diagram in Fig. 220, which represents the speed fluctuation curve for a working cycle.

Similarly, at B the flywheel will have just finished receiving surplus energy from the crankshaft, and so its speed will be a maximum here, as shown in the lower diagram.

Obviously, the excesses and deficiencies of energy are equal for a

cycle of operations, or, in other words, the algebraic sum of the positive and negative areas for a complete cycle is zero.

If the area of the torque curve for one complete cycle of operations (for any type of engine) be denoted by E, and the greatest of the separate areas above or below the mean torque line be denoted by δE, then the ratio $\frac{\delta E}{E}$ is termed the *Coefficient of Fluctuation of Energy.*

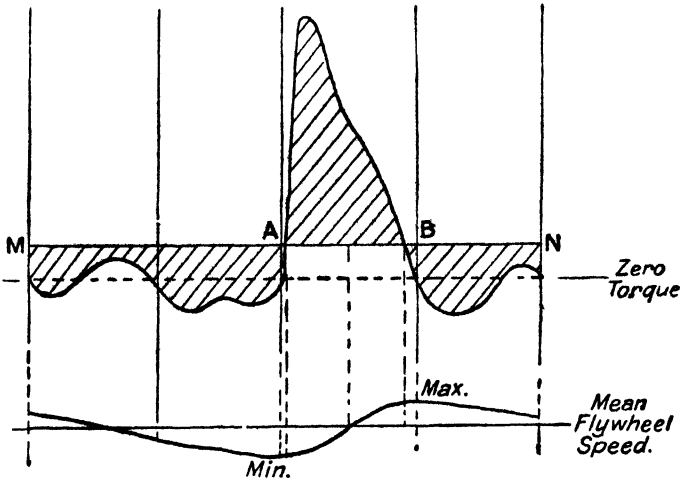


FIG. 220. CONSTANT LOAD FLYWHEEL EFFECT

The following table gives some actual values for the energy fluctuation coefficient in different types of petrol engines.

TABLE XVI

Type of Engine	Energy Coefficient
	Per cent
Single-cylinder	97
Two-cylinder, vertical cranks at 180°	60
Two-cylinder, 90° twin	74
Four-cylinder vertical	5
Six-cylinder vertical	2·4
Seven-cylinder radial	2·6

We have assumed the load upon the engine, until now, to be constant during a revolution, but more generally if this load be regarded as variable, such as when the engine drives a pump (hydraulic or pneumatic) or either the clutch or tyres slip during a cycle of operations,

etc., then the curve of load torque would be no longer represented by a line such as MN in Fig. 220, but by a curve such as M′N′ as shown in Fig. 221, then the fluctuation of energy will be represented by the shaded areas, and the points A and B will still represent the places of minimum and maximum speeds, respectively, of the flywheel, but their actual maximum values will not be the same.

The ratio of the extreme range of speed to the mean speed of the flywheel, resulting from the variation in the engine effort and load

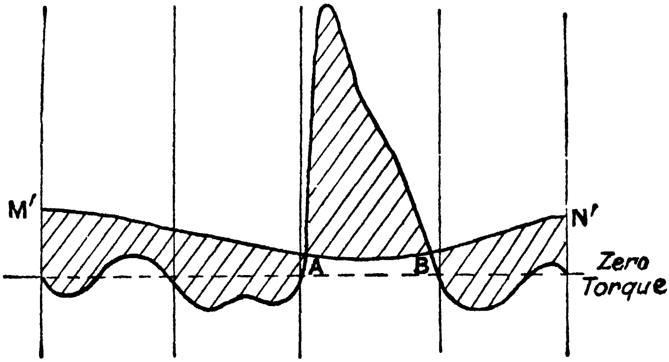


FIG. 221. VARIABLE LOAD FLYWHEEL EFFECT

"effort," is termed the *Coefficient of Speed Fluctuation*, and it is the function of the flywheel to keep this coefficient down to an assigned limit.

Since the energy which can be stored in a flywheel is proportional to its moment of inertia and to the square of its angular velocity, there is considerable latitude in which to design an appropriate flywheel to fulfil its functions, for the mass, diameter, and disposition of the material can be varied, as desired, so as to obtain an appropriate moment of inertia.

As an illustration of the values of the speed fluctuation coefficient (expressed as a percentage) employed in practice the following list is given—

TABLE XVII

Type of Engine	Coefficient of Speed Fluctuation
Single-cylinder petrol engine	5 to 10
Four-cylinder petrol engine	1 to 2
Six-cylinder petrol engine	0·5 to 1 0
Seven-cylinder radial engine	0·1 to 0·4
Steam engine driving machine tools	2 8
Steam engine driving pump	5
Steam engine driving electrical machinery	0·6

Determination of Flywheel Proportions

It is required to find the size of flywheel which will give a certain limiting speed variation under working conditions.

Let the mean speed of the flywheel, expressed as an angular velocity, be ω_0, and q the coefficient of speed fluctuation given.

If ω_1 and ω_2 be the maximum and minimum speeds, then $q = \frac{\omega_1 - \omega_2}{\omega_o}$.

If I be the moment of inertia of the flywheel and E_0 the energy of rotation at its mean speed of the flywheel

Then $$E_o = \frac{1}{2} I\omega_o^2$$

and the fluctuation of energy δE will be given by

$$\delta E = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} I(\omega_1 - \omega_2)(\omega_1 + \omega_2)$$
$$= I(\omega_1 - \omega_2)\omega_o - 2E_o\left(\frac{\omega_1 - \omega_2}{\omega_o}\right)$$
$$= 2E_o q$$

Whence $$E_o = \frac{\delta E}{2q}$$

From this result the energy E_0 of the flywheel necessary to keep the speed variation within the assigned limits is known, since the fluctuation δE is known from the torque diagram.

If the mean speed ω_0 be known, we have

$$I = \frac{2E_o}{\omega_o^2} = \frac{\delta E}{q\omega_o^2}$$

and since $I = Mk^2$, where M is the mass and k the radius of gyration of the flywheel, either of these quantities may be variable in choosing an appropriate size of flywheel.

Example. If the flywheel be made up of an annular rim connected to the central boss by arms, then if W_R and W_S be their respective weights, and k_r and k_s their respective polar radii of gyration, we have $I = W_R k_r^2 + W_S k_s^2$.

If the rim of flywheel be of rectangular section, and of internal and external radii R_1 and R_2, then we have $k_r^2 = \frac{R_1^2 + R_2^2}{2} = R^2$ approximately in practice.

Usually $$k_s^2 = \frac{R^2}{3}$$

so that $$I = \left(W_R + \frac{1}{3} W_S\right) R^2 = \frac{\delta E}{q\omega_0^2}$$

Hence it will be evident that in order to make q, the speed variation, a minimum for a given energy fluctuation, the weight and radius of the flywheel rim should be as large as conveniently possible.

Peripheral Stresses

The radius of the flywheel is limited in practice by the centrifugal tension or stresses produced in the material of the flywheel itself by the centrifugal force due to rotation.

Thus it can readily be shown that the centrifugal tension in the rim of a flywheel is $\frac{\omega v^2}{g}$ lb. per sq. in., where v is the peripheral velocity in feet per second and ω the weight per foot of a bar 1 sq. in. in section.

Now, for plain cast-iron, such as is used in flywheels, $\omega = 3{\cdot}37$ lb. per ft. per sq in. section, and the ultimate tensile strength is about 8 tons to the square inch, or about 18,000 lb. per sq in , i.e. $f = \frac{3{\cdot}3v^2}{g} = 0{\cdot}1024v^2$.

The bursting speed is then given by

$$v^2 = \frac{32{\cdot}2 \times 18{,}000}{3{\cdot}37}$$

from which $$v = 415 \text{ ft. per sec.}$$

The safe working peripheral speed is usually obtained with a factor of safety allowance for ordinary cast iron of about 18, and will be seen to be equivalent to a peripheral speed, in cast-iron, of 97 ft. per sec.

Much, however, depends upon the actual design of the flywheel, and upon the frequency and magnitude of the energy fluctuations, etc., and it has been found, in practice, inadvisable to exceed a peripheral speed, in plain cast-iron, of 80 ft. per sec. and 120 for alloy cast-iron flywheels.

Numerous cases of "burst flywheels" have resulted from employing peripheral speeds exceeding this figure.

Another factor concerned in flywheel strength is that due to initial casting stresses, which may very appreciably weaken the rims and their supports.

Further, with cast iron, the metal is not usually homogeneous, air-bubbles being often interspersed with the metal, and the balance may be thereby impaired, thus causing additional stresses on the flywheel rim. Centrifugally cast-iron wheels are not, however, subject to this drawback.

The adoption of forged steel and cast steel, for flywheels, is becoming the practice for high-speed engines and machinery, and in certain cases steel rims have been shrunk on to the rims of cast-iron flywheels,

and even cast-steel wire rope has been employed with cast-iron flywheels, enabling the working speed with the latter material to be trebled.

With special alloy-steel flywheels the working peripheral speed should not exceed 200 to 250 ft. per sec., and for forged or stamped mild steel or iron wheels, 140 ft. per sec.

For any other material, the safe working rim speed will vary as the square root of its ultimate tensile strength, and inversely as the square root of its density or weight per linear foot.

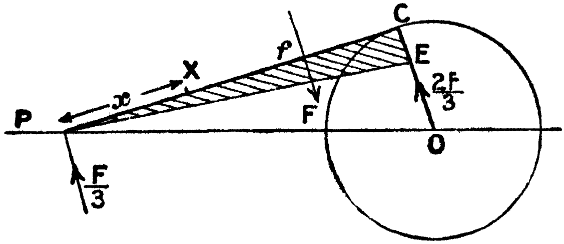


FIG. 222. INERTIA FORCES IN CONNECTING ROD

Thus, if f_t = tensile strength in lb. per sq. in.; w_t = weight of the metal per cubic in., in lb., k = factor of safety and v = safe rim speed in feet per sec., then—

$$v = \sqrt{\frac{2 \cdot 6835 f_t}{w \,.\, k}}$$

Inertia Forces producing Bending Action on Connecting Rod

Let the connecting rod be situated as shown in Fig. 222, so that it is at right angles to the crank.

Then the resultant acceleration of the whole connecting rod in a direction parallel to OC will cause an inwardly-acting inertia force F upon the rod, acting at some point f, and causing a bending action to occur.

Let m^1 be the weight per unit volume of the rod, then, assuming the section of the rod to be constant throughout its length, the weight per unit length is given by $m = m^1\text{A}$, where A is the sectional area of the rod and l = PC.

(1) Dealing with this simplest case of a uniform section rod first, the inward acceleration at C is given by $\omega^2 r$, where ω is the crank pin angular velocity.

At any point X situated at a distance x from P the normal acceleration will be $\frac{x}{l} \,.\, \omega^2 r$, and hence the acceleration is proportional to the distance

from P, and can therefore be represented by the triangle PCE, whose breadth at any part represents the normal acceleration at that place.

The resultant force normal to the rod is evidently the integral of all the component accelerations similar to the elementary length δx at x from P.

The force causing acceleration of such an element is

$$\delta F = \frac{m}{g} \quad \omega^2 r \,.\, \delta x \quad \frac{x}{l}$$

Hence the resultant force $F = \int_{x=0}^{x=l} \frac{m}{g} \frac{\omega^2 r}{l} \quad x dx$

$$= \frac{m}{2g} \quad \omega^2 l r$$

and acts at $\frac{2}{3}\, l$ from P

The reactions at P and C are therefore $\frac{F}{3}$ and $\frac{2}{3}$ F respectively.

The rod may now be treated as a beam supported at the ends and loaded with a force or weight F

The bending moment at any given point such as X is then given by

$$M = \frac{F}{3} \quad x - \frac{m}{g}\, \omega^2 r \,.\, \frac{x^3}{6l}$$

or

$$M = \frac{m\omega^2 r}{6lg}\, (l^2 x - x^3) \quad . \quad . \quad . \quad . \quad (1)$$

It will be seen from this that the bending moment varies not as the distance x from B, but as $x(l^2 - x^2)$, and that the curve of bending moment has the equation $y = kx(l^2 - x^2)$, where y is the B M and k a constant.

The maximum B M. occurs where $\frac{dy}{dx} = 0$

that is

$$l^2 - 3x^2 = 0$$

or

$$x = \frac{l}{\sqrt{3}} \quad . \quad . \quad . \quad . \quad (2)$$

Substituting this value of x in (1) we have

$$M_{max} = \frac{m\omega^2 r}{9\sqrt{3}} \,.\, \frac{l^2}{g} \quad .$$

For the purposes of design it is necessary to be able to compute M_{max} in this way, in order to determine the maximum bending stress

in the rod. Thus, if Z be the "strength modulus" of the section of the rod, the maximum stress is given by

$$f = \frac{M_{max}}{Z}$$

For a round rod $f = \dfrac{M_{max}}{\dfrac{\pi d^3}{32}} = \dfrac{32M_{max}}{\pi d^3}$, d = diameter of rod at maximum B.M. place, and for a rectangular rod of breadth b and depth d

$$f = \frac{6M_{max}}{bd^2}$$

(2) If the section of the rod be not uniform, but varies all along the rod, as in Fig. 223, according to some law of sectional variation $K = \phi(x)$, where x is the distance from P.

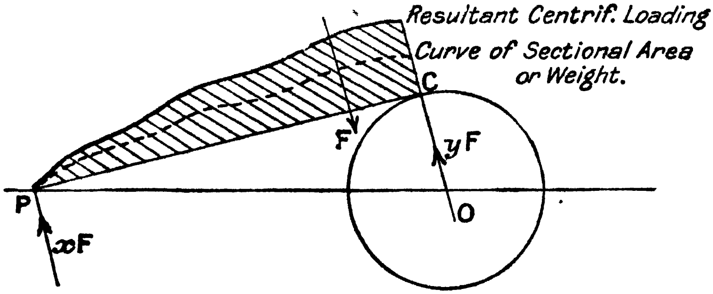


FIG. 223. BENDING OF IRREGULAR SECTION CONNECTING ROD

Then the centrifugal force upon an elementary length δx at distance x from P is given by

$$\delta F = m^1 . \frac{\omega^2 r}{g} . \phi(x)\delta x . \frac{x}{l}$$

where l = PC.

The resultant force F upon the rod will then be given by

$$F = \frac{m^1\omega^2 r}{gl}\int_{x=0}^{x=l} \phi(x) . x . dx$$

the value of which will depend upon the law of variation of the section.

Thus, if the section increases uniformly from P to C (neglecting the big and small ends)

then $K = a . x$, where a is a constant

and
$$F = \frac{m^1\omega^2 r}{gl} \cdot \int_{x=0}^{x=l} ax^2 \, . \, dx$$
$$= \frac{m^1\omega^2 r}{gl} \cdot \frac{al^3}{3}$$

For more complex variations of section it is simpler to perform the integration graphically by dividing the rod up into a number of parts by parallel planes to the crank OC, and finding first the mass of each part from the average sectional area, and then the resultant force upon each mass due to its acceleration in the direction parallel to OC.

The resultant of all of these component forces can be then easily obtained.

The Connecting-rod as a Strut

Besides being subjected to inertia bending forces, the connecting rod is also subject to axial thrust due to the piston and piston inertia pressures, which will create buckling stresses quite apart from the inertia stresses

The rod may be treated as a strut hinged at each end and loaded with a load equal to the greatest thrust which can occur, due to the piston pressure corrected for inertia effects

Thus, if T be the maximum axial thrust in rod,

$$T = \frac{1}{\cos\phi}\left[P - \frac{M\omega^2 r}{g}\left(1 \pm \frac{1}{n}\right)\right]$$

where P is the *total* piston load and M the reciprocating mass, ϕ being the angle CPO (Fig. 222) and n = ratio of connecting rod to crank, r = crank radius.

The maximum value of this expression can easily be determined by foregoing methods

Knowing T, then the stress to which the rod is subjected, as a strut, is given by*

$$f_c = \frac{T}{K \, . \, A} + \frac{\pi^2 EI}{l^2}$$

where K = the factor of safety, usually lying between 6 and 10, under the above conditions of load application,

A = the sectional area (assumed uniform at the centre),

E = modulus of elasticity of material of rod,

* Where $\frac{l}{k}$ is greater than 60 or 70, k being the least radius of gyration. For smaller values of $\frac{l}{k}$ the Rankine, Gordon, or similar type of formula must be employed.

I = least moment of inertia of cross-section at middle of rod, about an axis through its C.G.

The value of the working stress $f_c = \frac{T}{KA}$ can thus be calculated.

Resultant Connecting Rod Stress

In order to determine the strength of the connecting rod under the combined influence of bending and end thrust, it is necessary to calculate the stresses due to these two causes separately, and to add them together for the resultant stress.*

Thus $$\text{Resultant stress } f_R = f + f_C$$

and from this value f_R the dimensions of the rod are determined, knowing the properties of the material employed.

Thus if the rod be made of mild steel, f_R should not exceed 7000 lb. per sq. in., more especially on account of the alternating action of the loading, which is compressive for the firing, exhaust and compression strokes, and tensile for the suction and any misfiring or idle strokes.

For cast-steel rods, f_R should not be greater than about 12,000 lb. per sq. in., whilst for the alloy steels, such as nickel steel, nickel chrome, chrome vanadium, etc., the value of the permissible working stresses will in general be much higher, and will depend upon the ultimate breaking stress, elastic limit, and fatigue-resisting properties of these materials.

Connecting Rod Stress in Aircraft Engine

In the case of the twelve-cylinder V-engine, particulars of the inertia and pressure forces of which are considered on page 27, the connecting rod is treated as a strut in direct compression under a load due to the maximum explosive pressure, which is equivalent to a total load of 8830 lb.

The section of the rod is an I-beam, of area 0·3529 sq. in., so that the unit compression stress is given by—

$$F_c = \frac{8830}{0{\cdot}3527} = 25{,}000 \text{ lb. sq. in.}$$

The Rankine formula for hinged struts may be written—

$$F = F_c + \frac{c \,.\, P \,.\, L^2}{I}$$

* This approximate method is usually sufficiently accurate for most purposes, but it does not take account of eccentricity of loading, non-uniformity of material, etc.

TABLE XVIII*

The Firing Order of Various Types of Petrol Engines

Type of Engine	Firing Order	No of Explosions in 2 revs	Maximum Explosion Interval in degrees	Minimum Explosion Interval in degrees
1. Single cylinder	1–1–1, etc	1	720°	720°
2 Two-cylinder vertical opp cranks	1 2–1–2, etc	2	360°	360°
3. Two-cylinder vertical cranks at right angles	1–2–1–2, etc	2	450°	270°
4. Two-cylinder 90° V-type	1–2 1–2, etc	2	450°	270°
5 Two-cylinder horizontal opp, with cranks opp	1–2–1–2, etc	2	360°	360°
6 Three-cylinder Y type, rad	1–3–2–1, etc	3	240°	240°
7. Five-cylinder radial type	1–3–5–2–4–1, etc	5	144°	144°
8 Six-cylinder radial type	1–3–5–2–4–6–1, etc	6†	180°	60°
9 Seven-cylinder radial type	1–3–5–7–2–4–6–1, etc	7	102°	$102\frac{6}{7}$°
10. Eight-cylinder radial type	1–3 5–7–2–4 6–8–1, etc	8‡	135°	45°
11 Nine-cylinder radial type	1 3 5–7–9–2–4 6–8 1	9	80°	80°
12 *n*-cylinder radial type, where *n* is even	1–3–5–7–9, etc, to $(n-1)$–2–4–6 8, etc, to *n*	*n*	$\frac{3}{2}\ \frac{720°}{n}$	$\frac{1}{2}\ \frac{720°}{n}$
n cylinder radial type, where *n* is odd	1–3–5–7–9, etc to *n* – 2–4–6–8, etc, to $(n-1)$	*n*	$\frac{720°}{n}$	$\frac{720°}{n}$
13. Four cylinder vertical	1–3–4–2–1 1–2–4–3 1	4§	180°	180°
14 Six-cylinder vertical (cranks in 3 pairs – 120°)	1–4–2–6–3–5	6	120°	120°
	1–5–3–6–2 4	6	120°	120°
	1–3–2–6–4–5–	6	120°	120°
15. Eight-cylinder V-type (two sets of four cyldrs)	1R–4L–3R–2L 4R–1L–2R–3L	8§	90°	90°
	Alternative 1R–4L–2R–3L 4R–1L–3R–2L	8	90°	90°
16. Twelve-cylinder V type cranks – 120° (two sets of six cyldrs)	1R–6L–4R–3L–2R–5L–6R–1L–3R–4L–5R–2L	12	60°	60°
	Alternative 1R–6L–5R–2L–3R–4L–6R–1L–2R–5L–4R–3L	12	60°	60°
	Another Altern 1R–6L–3R–4L 2R–5L–6R–1L–4R–3L–5R–2L	12	60°	60°
17 Twelve-cylinder Arrow type (four cylinders in three rows)	Propeller end 7 2 9 4 11 6 10 5 12 1 8 3	12	—	—

* See also Tables XII, XIII, XIV and XV

† Mean interval for other periods, 120°.

‡ Mean interval for other periods, 90°.

§ This is the usual order adopted in practice.

where P = the total end load in lb. = 8830 lb.,

L = length between hinge centres in inches = 12 in ,

I = least moment of inertia of I-beam = $0{\cdot}1038$ in ^{5},

c = a constant = $0{\cdot}000526$ for hinged struts.

Hence

$$F = 25{,}000 + \frac{0{\cdot}000526 \times 8830 \times 144}{0{\cdot}1038}$$
$$= 25{,}000 + 6450$$
$$= 31{,}450 \text{ lb.}$$

The connecting rods are of high tensile strength alloy steel, namely, heat-treated nickel chrome or chrome-vanadium steel.

With an ultimate compressive strength of 65 tons per sq. in., the above stress corresponds to a factor of safety of about 9·25.

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